Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4389-4398

ON DOUBLE DIVISOR CORDIAL LABELING OF STAR RELATED GRAPHS

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Abstract

A double divisor cordial labeling of a graph $G_g(V_g, E_g)$ is a bijective function Ψ from V_g to $\{1, 2, 3, \ldots, |V_g|\}$ such that each edge 'ab' is given label 1 if $2\psi(a) \mid \psi(b)$ or $2\psi(b) \mid \psi(a)$ and 0 otherwise, then the number of edges given 0 and 1 differ by a maximum of 1. If G_g admits a double divisor cordial labeling, then it is said to be a double divisor cordial graph. This paper is focused on deriving certain results of high interest on star, bistar, and their related graphs under some well-known graph operations for double divisor cordial labeling.

1. Introduction

For the last few decades, graph theory has developed at impressive pace. By graph $G(V, E)$, we mean a relationship between set of nodes/vertices represented by V and edges/lines represented by E. Graph labeling means an allocation of labels (usually integers) to nodes or edges or both under some frame. Graph labeling is a beautiful interference of graph theory and number theory. By joining these two fields, it has become one of the powerful tools for researchers and software developers. Around 3000 research papers can be found in Gallian [4] where different graph labeling techniques are explored. We are referring to [5] and [2] for basic terminology of graph theory and

2020 Mathematics Subject Classification: 05C78.

Keywords: Geometric labeling, Double divisor cordial labeling, Double divisor cordial graph, Star, Bistar, Corona Product.

Received January 10, 2022; Accepted March 2, 2022

number theory, respectively. Before we move to main section, we recall that Cahit [3] came out with the idea of cordial labeling. After that new variants were introduced with a minor change in the cordial condition. Of all the variants, prime cordial [8] and divisor cordial [10] are famous. In the literature many variants of divisor cordial labeling can be found. Vishally et al. [7] defined double divisor cordial labeling and established the same for some well-known graphs. An in-depth look of divisor cordial labeling can be found in [6]. Throughout this article, we use DCL, DDCL, DCG and DDCG notations for divisor cordial labeling, double divisor cordial labeling, divisor cordial graph and double divisor cordial graph, respectively.

Definition 1 [10]**.** A DCL of G_g is a bijective map $\psi : V_g \rightarrow \{1, 2, 3, \ldots, |V_g|\}$ such that each edge 'ab' is marked with label 1 if $\psi(a) \mid \psi(b)$ or $\psi(b) \mid \psi(a)$ and 0 otherwise; then $|e_{\psi}(0) - e_{\psi}(1)| \leq 1,$ where $e_{\psi}(0)$ and $e_{\psi}(1)$ represent the number of edges having labels 0 and 1, respectively. If G_g permits a DCL, then it is called a DCG.

Definition 2 [7]**.** A DDCL of G_g is determined by a bijective map Ψ from V_g to $\{1, 2, 3, \ldots, |V_g|\}$ so that each edge "ab" is marked 1 if $2\psi(a) \mid \psi(b)$ or $2\psi(b) \mid \psi(a)$ and 0 otherwise, then the positive difference of lines having 1 and 0 does not exceed 1, i.e., $|e_{\psi}(0) - e_{\psi}(1)| \leq 1.$ A graph is named as DDCG if it allows a DDCL.

2. Preliminaries

In this section, a few star-related graphs for DDCL are investigated using various graph algorithms.

Definition 3 [1]**.** Let $e = rs$ be an edge of G_g. A line e is known as subdivided if it is replaced by edges $e' = rw$ and $e'' = ws.$ The graph is termed barycentric subdivision of $G_g,$ denoted by $S(G_g),$ if each edge of G_g is subdivided.

Theorem 1. *$S(K_{1,n})$ admits a DDCL* $\forall\ n \geq 1.$

Proof. Let $V(K_{1,n}) = \{x_0, x_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{x_0, x_i : 1 \leq i \leq n\}$. Let $S(K_{1,n})$ be the subdivision of $K_{1,n}$ with $V(S(K_{1,n})) = V(K_{1,n}) \cup \{y_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n})) = \{x_0 y_i : 1 \leq i \leq n\} \cup \{y_i x_i : 1 \leq i \leq n\}$. Clearly, $|V(S(K_{1,n}))| = 2n + 1$ and $|E(S(K_{1,n}))| = 2n$. Define a bijective function $\theta : V(S(K_{1,n})) \to \{1, 2, \ldots, 2n + 1\}$ in the following way; $\theta(x_0) = 1$, $\theta(y_1) = 2$, $\theta(y_i) = \theta(y_{i-1}) + 2$; $2 \leq i \leq n$ and $\theta(x_i) = \theta(y_i) + 1$; $1 \leq i \leq n$. Evidently, $|e_\theta(0) - e_\theta(1)| = 0$ which proves that $S(K_{1,n})$ is a DDCG.

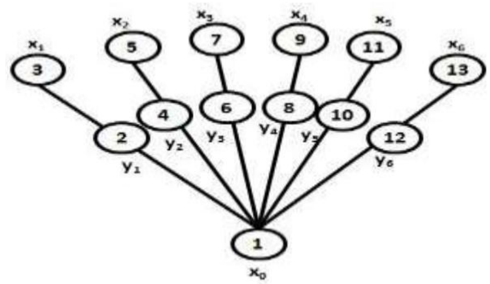

Figure 1. A DDCL of $S(K_{1,6})$.

Theorem 2. *$S(B_{n,n})$ admits a DDCL* $\forall\, n \geq 1$.

Proof. Let $V(B_{n,n}) = \{x_0, y_0, x_i, y_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{x_0 x_i : 1 \leq i \leq n\} \cup \{y_0 y_i : 1 \leq i \leq n\} \cup \{x_0 y_0\}$. Let $S(B_{n,n})$ be the subdivision of $B_{n,n}$ with $V(S(B_{n,n})) = V(B_{n,n}) \cup \{x_i', y_i' : 1 \leq i \leq n\} \cup \{u\}$ and $E(S(B_{n,n})) = \{x_0 x_i' : 1 \leq i \leq n\} \cup \{x_i' x_i : 1 \leq i \leq n\} \cup \{y_0 y_i' : 1 \leq i \leq n\} \cup \{y_i' y_i : 1 \leq i \leq n\} \cup \{x_0 u, u y_0\}$. Clearly, $|V(S(B_{n,n}))| = 4n + 3$ and $|E(S(B_{n,n}))| = 4n + 2$. Define a bijective function $\theta : V(S(B_{n,n})) \to \{1, 2, \ldots, 4n + 3\}$ by considering $\theta(x_0) = 1$, $\theta(u) = 4$, $\theta(y_0) = 2$, $\theta(x_1') = 6$, $\theta(y_1') = 3$, $\theta(y_2') = 8$, $\theta(y_1) = 5$, $\theta(x_i') = \theta(x_{i-1}') + 4$, $2 \leq i \leq n$, $\theta(x_i) = \theta(x_i') + 1$; $1 \leq i \leq n$, $\theta(y_i') = \theta(y_{i-1}') + 4$; $3 \leq i \leq n$, and $\theta(y_i) = \theta(y_i') + 1$; $2 \leq i \leq n$. Evidently, $|e_\theta(0) - e_\theta(1)| \leq 1$ which proves that $S(B_{n,n})$ is a DDCG.

Theorem 3. *Subdivision of path and cycle admits a DDCL.*

Proof. The proof follows from the reason that subdivision of path and cycle yields path and cycle again, which are DDCG, as proved in [7].

Definition 4 [9]**.** Splitting graph of G_g, denoted by $S'(G_g)$ is constructed by inserting a new node v' corresponding to every node v of G_g such that $N(v) = N(v')$.

Theorem 4. $S'(K_{1,n})$ *permits a DDCL for* $n \geq 2$.

Proof. Let $V(K_{1,n}) = \{x_0, x_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{x_0x_i : 1 \leq i \leq n\}$ and let $V(S'(K_{1,n})) = V(K_{1,n}) \cup \{u_0, u_i : 1 \leq i \leq n\}$ and $E(S'(K_{1,n})) = E(K_{1,n})$ $\cup \{x_0u_i : 1 \leq i \leq n\} \cup \{u_0x_i : 1 \leq i \leq n\}$. Clearly, $|V(S'(K_{1,n}))| = 2n + 2$ and $|E(S'(K_{1,n}))| = 3n$. Consider $\theta : V(S'(K_{1,n})) \to \{1, 2, \ldots, 2n + 2\}$ as follows; $\theta(x_0) = 1, \theta(u_0) = 2, \theta(x_1) = 4, \theta(x_i) = \theta(x_{i-1}) + 2; 2 \leq i \leq n$ and allocate unutilized odd labels to $u_i; 1 \leq i \leq n$. Observe that when n is even, $e_\theta(0) = n + \frac{n}{2} = \frac{3n}{2}$ and $e_\theta(1) = \frac{3n}{2}$, and when n is odd, $e_\theta(0) = n + \left\lfloor \frac{n}{2} \right\rfloor$ and $e_\theta(1) = n + \left\lceil \frac{n}{2} \right\rceil$. Thus $|e_\theta(0) - e_\theta(1)| \leq 1$.

Theorem 5. $S'(B_{n,m})$ *permits a DDCL for* $n, m \geq 2$.

Proof. Let $\{x_0, y_0, x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $\{x_0x_i : 1 \leq i \leq n\} \cup$ $\{y_0y_j : 1 \leq i \leq m\} \cup \{x_0y_0\}$ represent respectively the node set and edge set of $B_{n,m}$. Let $S'(B_{n,m})$ with $V(S'(B_{n,n})) = V(B_{n,n}) \cup \{x'_0, y'_0, x'_i, y'_j :$ $1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(S'(B_{n,n})) = E(B_{n,n}) \cup \{x_0x'_i : 1 \leq i \leq n\} \cup$ $\{x'_0x_i : 1 \leq i \leq n\} \cup \{y_0y'_i : 1 \leq j \leq m\} \cup \{y'_0y_j : 1 \leq j \leq m\} \cup \{x_0y'_0, y_0x'_0\}$. Clearly, $|V(S'(B_{n,m}))| = 2n + 2m + 4$ and $|E(S'(B_{n,m}))| = 3n + 3m + 3$. Define a bijective function $\theta : V(S'(B_{n,m})) \to \{1, 2, \ldots, 2n + 2m + 4\}$ under the below mentioned cases.

Case (i). When $n = m$.

Let $\theta(x_0) = 1$, $\theta(x'_0) = 2$, $\theta(y_0) = 4$, $\theta(x_1) = 8$, $\theta(x_i) = \theta(x_{i-1}) + 4; 2 \leq i \leq n$ and $\theta(y'_0) = p$, where p is the largest prime $\leq 4n + 4$. Assign the unutilized even labels to $x'_i; 1 \leq i \leq n$ and odd labels to y_j and $y'_i; 1 \leq j \leq n$, in any order. Observe that $2\theta(x_0) \mid \theta(x'_i)$, $2\theta(x_0) \mid \theta(x_i)$, $2\theta(x'_0) \mid \theta(x_i)$, $2\theta(x'_0) \mid \theta(y_0)$ and $2\theta(x_0) \mid \theta(y_0)$, see that $e_\theta(1) = 3n + 2$ and $e_\theta(0) = 3n + 1$.

Case (ii). When $n \neq m$.

Without loss of generality, suppose $n > m$. Let p_1, p_2 represents the first and second largest prime numbers such that $p_2 < p_1 \leq 2n + 2m + 4$. Fix $\theta(y_0) = p_1$, $\theta(y'_0) = p_2$, $\theta(x_0) = 1$, $\theta(x'_0) = 2$, $\theta(x_1) = 4$, $\theta(x_i) = \theta(x_{i-1}) + 4; i \geq 2$ such that $\theta(x_k) \leq 2n + 2m + 4$ for some $k \leq n$. Next allocate the unused even labels to $x'_i; i \geq 1$ so that $\theta(x'_t) \leq 2n + 2m + 4$ for some $t \leq n$, and, to unlabeled x_i, if any.

Next, assign the unconsumed labels to unlabeled nodes in any fashion. In both the cases, $| e_\theta(0) - e_\theta(1) | \leq 1$.

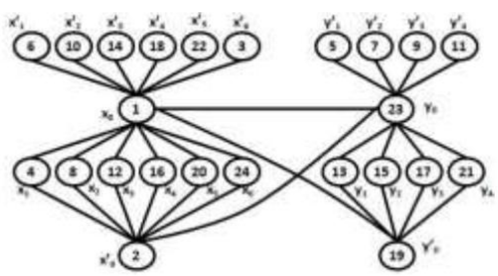

Figure 2. A DDCL of $S'(B_{6,4})$.

Theorem 6. *$S'(S(K_{1,n}))$ permits a DDCL for all $n \geq 1$.*

Proof. Let x, y_i and z_i, $1 \leq i \leq n$ represent the nodes of $S(K_{1,n})$ and x', y'_i and z'_i, $1 \leq i \leq n$ be the additional inserted nodes to construct $S'(S(K_{1,n}))$. Clearly, $| V(S'(S(K_{1,n}))) | = 4n + 2$ and $| E(S'(S(K_{1,n}))) | = 6n$. Consider the function $\theta : V(S'(S(K_{1,n}))) \to \{1, 2, \ldots, 4n + 2\}$ defined by $\theta(x) = 1$, $\theta(x') = 2$, $\theta(y_i) = 4i; 1 \leq i \leq n$, $\theta(y'_i) = 4i + 2; 1 \leq i \leq n$, $\theta(z_i) = 4i + 1;$

$1 \leq i \leq n, \theta(z_i') = 4i - 1; 1 \leq i \leq n.$ It follows that $e_\theta(0) = e_\theta(1) = 3n$ proving that $S'(S(K_{1,n}))$ a DDCG.

Definition 5 [11]. $\langle K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(m)} \rangle$ denotes the graph formed by connecting the apex nodes of $K_{1,n}^{(t-1)}$ and $K_{1,n}^{(t)}$, to a newly inserted node r_{t-1} where $2 \leq t \leq m.$

Theorem 7. $G_g = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)} \rangle$ *admits a DDCL.*

Proof. Let G_g represent the graph formed by joining the apex nodes, say, x_0 and y_0 respectively of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$, to a new node, say, w. The cardinality of node and edge set of G_g are respectively equal to $2n + 3$ and $2n + 2$. Consider a function $\theta : V(G_g) \to \{1, 2, \ldots, 2n + 3\}$ defined as $\theta(x_0) = 1, \theta(w) = 2$ and $\theta(y_0) = p$ where p is the largest prime $\leq 2n + 3$. Now allocate all the unused even labels to the pendant nodes of $K_{1,n}^{(1)}$ and the remaining labels to unlabeled nodes. Consequently, $|e_\theta(0) - e_\theta(1)| \leq 1$, establishing that G_g a DDCG.

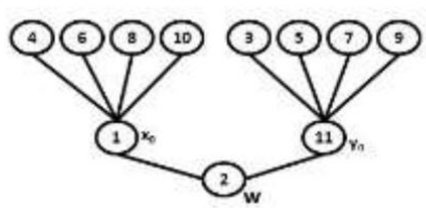

Figure 3. A DDCL of $\langle K_{1,4}^{(1)}, K_{1,4}^{(2)} \rangle$.

Theorem 8. $G_g = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} \rangle$ *permits a DDCL.*

Proof. Let $x_0^{(i)}, x_1^{(i)}, x_2^{(i)}, \ldots, x_n^{(i)}$ represent the nodes of $K_{1,n}^{(i)}$, where $x_0^{(i)}; 1 \leq i \leq 3$ stands for apex nodes. Let r_1 and r_2 be the newly introduced nodes such that $x_0^{(1)}$ and $x_0^{(2)}$ are adjacent to r_1, and $x_0^{(2)}$ and $x_0^{(3)}$ are adjacent to r_2. The cardinality of node and edge set of G_g, are respectively $3n+5$ and $3n+4$. Consider a function $\theta : V(G_g) \to \{1, 2, \ldots, 3n+5\}$ defined as $\theta(r_1) = 4$, $\theta(x_0^{(1)}) = 1$, $\theta(x_0^{(2)}) = 2$ and $\theta(x_0^{(3)}) = p$, where p is the largest prime $\leq 3n+5$. Now allocate all unused even labels of the type $4n; n \in \mathbb{N}$ to the pendant nodes of $K_{1,n}^{(2)}$ and remaining even labels to the unlabeled nodes of $K_{1,n}^{(1)}$. Now allocate the remaining labels simultaneously to the remaining unlabeled nodes. It is easy to see that G_g allows a DDCL.

Definition 6. $\langle B_{n,n}^{(1)}, B_{n,n}^{(2)}, \ldots, B_{n,n}^{(m)} \rangle$ denotes the graph constructed by connecting the apex nodes of $B_{n,n}^{(t-1)}$ and $B_{n,n}^{(t)}$, to new nodes r_{t-1}, s_{t-1} where $2 \leq t \leq m$.

Theorem 9. *$G_g = \langle B_{n,n}^{(1)}, B_{n,n}^{(2)} \rangle$ admits a DDCL.*

Proof. Let $V(B_{n,n}^{(i)}) = \{x_0^{(i)}, y_0^{(i)}, x_j^{(i)}, y_j^{(i)} : 1 \leq j \leq n\}$ and $E(B_{n,n}^{(i)}) = \{x_0^{(i)} y_0^{(i)}, x_j^{(i)} x_j^{(i)}, y_0^{(i)} y_j^{(i)} : 1 \leq j \leq n\}$. Let $G_g = \langle B_{n,n}^{(1)}, B_{n,n}^{(2)} \rangle$ and r, s be newly introduced nodes such that r is adjacent to $x_0^{(1)}$ and $x_0^{(2)}$, and s is adjacent to $y_0^{(1)}$ and $y_0^{(2)}$. Clearly the cardinality of node and edge set of G_g are $4n+6$ and $4n+6$ respectively. Define a map $\theta : V(G_g) \to \{1, 2, \ldots, 4n+6\}$ as follows; fix $\theta(x_0^{(1)}) = 1$, $\theta(x_0^{(2)}) = 2$, $\theta(y_0^{(1)}) = 6$, $\theta(y_0^{(2)}) = 3$, $\theta(r) = 4$, $\theta(s) = 9$. Assign even labels of the type $4t; t \in \mathbb{N}$ to $x_j^{(2)}; 1 \leq j \leq n$ and the remaining even labels to $x_j^{(1)}; 1 \leq j \leq n$. Next, allocate the remaining unused labels to unlabeled nodes simultaneously. By following this pattern, it follows that G_g a DDCG.

Definition 7 [1]**.** The corona product of H^* of order r with K^*, denoted by $H^* \odot K^*$ is a graph formed by taking a copy of H^* and r-copies of K^* thereby joining the r^{th} vertex of H^* by a line to each vertex in the r^{th} copy of K^*.

Theorem 10. *$K_{1,n} \odot K_1$ admits a DDCL.*

Proof. Let $k_0, k_1, k_2, \ldots, k_n$ denote the nodes of $K_{1,n}$. Let $G_g = K_{1,n} \odot K_1$ with $V(G) = V(k_{1,n}) \cup \{k'_0, k'_1, k'_2, \ldots, k'_n\}$ and $E(G_g) = E(K_{1,n}) \cup \{k_0k'_0, k_ik'_i : 1 \le i \le n\}$. The cardinality of node and edge set of G_g are respectively $2n+2$ and $2n+1$. Consider a map $\theta : V(G_g) \to \{1, 2, \ldots, 2n+2\}$ defined by fixing $\theta(k_0) = 1, \theta(k'_0) = 2n+2, \theta(k_i) = 2i;$ $1 \le i \le n$ and $\theta(k'_i) = \theta(k_i) + 1; 1 \le i \le n$. It is noteworthy here that $| e_\theta(0) - e_\theta(1) | \le 1$ which establishes that G_g is a DDCG.

Theorem 11. *$K_{2,n} \odot K_1$ admits a DDCL.*

Proof. Let U, V denote the node sets of $K_{2,n}$ where $U = \{x_1, x_2\}$ and $V = \{y_1, y_2, \ldots, y_n\}$. Let $G_g = K_{2,n} \odot K_1$ having $V(G_g) = V(K_{2,n}) \cup \{x'_1, x'_2, y'_1, y'_2, \ldots, y'_n\}$ and $E(G_g) = E(K_{2,n}) \cup \{x_1x'_1, x_2x'_2, y_iy'_i : 1 \le i \le n\}$. The cardinality of node and edge sets of G_g are respectively $2n+4$ and $3n+2$. Consider a map $\theta : V(G_g) \to \{1, 2, \ldots, 2n+4\}$ defined by fixing $\theta(x_1) = 1, \theta(x'_1) = 2n+4, \theta(x_2) = 2, \theta(x'_2) = 2n+3, \theta(y_i) = 2i+2; 1 \le i \le n$ and $\theta(y'_i) = \theta(y_i) - 1; 1 \le i \le n$. Observe that $| e_\theta(0) - e_\theta(1) | \le 1$, which implies that G_g is a DDCG.

Theorem 12. *$K_{3,n} \odot K_1$ admits a DDCL.*

Proof. Let U, V denote the node sets of $K_{3,n}$ where $U = \{x_1, x_2, x_3\}$ and $V = \{y_1, y_2, \ldots, y_n\}$. Let $G_g = K_{3,n} \odot K_1$ with $V(G_g) = V(K_{3,n}) \cup \{x'_1, x'_2, x'_3, y'_1, y'_2, \ldots, y'_n\}$ and $E(G_g) = E(K_{2,n}) \cup \{x_1x'_1, x_2x'_2, x_3x'_3, y_iy'_i : 1 \le i \le n\}$. The cardinality of node and edge set of G_g are respectively $2n+6$ and $4n+3$. Consider a map $\theta : V(G_g) \to \{1, 2, \ldots, 2n+6\}$ given by fixing $\theta(x_1) = 1, \theta(x'_1) = 6, \theta(x_2) = 2, \theta(x_3) = 2n+5, \theta(x'_2) = 4, \theta(x'_3) = 2n+3$ and $\theta(y_1) = 10$. There arise two cases.

Case (i). When n is odd.

Let $\theta(y_i) = \theta(y_{i-1}) + 4; 2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \theta(y_{\left\lceil \frac{n}{2} \right\rceil}) = 8,\ \theta(y_i) = \theta(y_{i-1}) + 4; \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n,\ \theta(y'_i) = \frac{\theta(y_i)}{2}; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$. Assign the remaining unused labels simultaneously to unlabeled nodes.

Case (ii). When n is even.

Let $\theta(y_i) = \theta(y_{i-1}) + 4; 2 \le i \le \frac{n}{2},\ \theta(y_{\frac{n}{2}+1}) = 8,\ \theta(y_i) = \theta(y_{i-1}) + 4;\ \frac{n}{2} + 2 \le i \le n,\ \theta(y'_i) = \frac{\theta(y_i)}{2}; 1 \le i \le \frac{n}{2}$. Assign the unconsumed labels simultaneously to unlabeled nodes. In both the cases, note that $|e_\theta(0) - e_\theta(1)| \le 1$. Hence G_g is a DDCG.

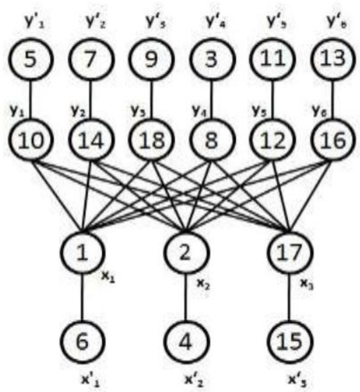

Figure 4. A DDCL of $K_{3,6} \odot K_1$.

Conclusion

In this article, we explored further results on DDCL. Various star related graphs are discussed for DDCL under numerous graph operations viz; splitting graph, barycentric subdivision and corona product. DDCL of other graph families under various graph operations is still an open area of research.

References

[1] M. I. Bosmia and K. K. Kanani, Divisor cordial labeling in the context of corona product, in 9th National Level Science Symposium 3 (2016), 178-182.

[2] D. M. Burton, Elementary Number Theory, seventh ed., McGraw-Hill, New York, 2011.

[3] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combinatoria (1987), 201-207.

[4] J. A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics 16, DS6, 2009.

[5] F. Hararay, Graph Theory, Addison Wesley Reading, 1972.

[6] A. Parthiban and Vishally Sharma, A comprehensive survey on prime cordial and divisor cordial labeling of graphs, Journal of Physics: Conference Series 1531 (2020), 012074.

[7] A. Parthiban and Vishally Sharma, Double divisor cordial labeling of graphs, accepted for publication in Journal of Physics: Conference Series, (2021).

[8] M. Sundaram, R. Ponraj and S. Somasundaram, Prime cordial labeling of graphs, Journal of Indian Academy of Mathematics 27 (2005), 373-390.

[9] S. K. Vaidya and N. H. Shah, Some star and bistar related divisor cordial graphs, Annals of Pure and Applied Mathematics 3 (2013), 67-77.

[10] R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Divisor cordial graphs, International Journal of Mathematical Combinatorics 4 (2011), 15-25.

[11] R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Special classes of divisor cordial graphs, International Journal of Mathematical Combinatorics 7 (2012), 1737-1749.