Advances and Applications in Mathematical Sciences
Volume 20, Issue 6, April 2021, Pages 1115-1128

SEPARATION AXIOMS IN SUPRA NEUTROSOPHIC CRISP TOPOLOGICAL SPACES

V. AMARENDRA BABU and J. ASWINI

Department of Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar, A.P., India
E-mail: amarendrab4@gmail.com
aswinijampana9@gmail.com

Abstract

In this paper, we extend concept of neutrosophic crisp sets into supra neutrosophic crisp sets and supra neutrosophic crisp limit points and we study the separation axioms in supra neutrosophic crisp topological spaces and some of their properties are investigated.

Introduction

Smarandache [6, 7, 8] introduced the notions of neutrosophic theory and introduced the neutrosophic components (T, I, F) which represent the membership, indeterminacy, and non membership values respectively, where $\left]^{-}0, 1^{+}\right[$ is a non standard unit interval. The supra topological spaces had been introduced by A. S. Mashhour at [3] in 1983. So the supra open sets are defined where the supra topological spaces are presented. B. K. Mahmoud [5] introduced on supra-separation axioms for supra topological spaces. A. A. Salama and S. A. Alblowi [2] introduced the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces and Ahmed B. Al-Nafee, Riad K. Al-Hamido, Florentin Smarandache [4] introduced separation axioms in neutrosophic crisp topological spaces.

Finally, we introduce the definitions of supra neutrosophic crisp topological spaces, we used these points $[N_C P_N]$ to define the concept of

2010 Mathematics Subject Classification: 06D72, 03E72.
Keywords: neutrosophic crisp topological spaces, neutrosophic crisp limit point, supra neutrosophic crisp topological spaces, supra neutrosophic crisp limit point.
Received January 24, 2020; Accepted May 13, 2020

supra neutrosophic crisp limit point $[SN_C P_N]$ with some of its properties and separation axioms in supra neutrosophic crisp topological spaces.

1. Preliminaries and Basic Definitions

Definition 1.1[3]**.** The spaces considered in this paper are supra topological spaces. (X, τ) is said to be a supra topological space if it is satisfying these conditions:

1. $\phi, X \in \tau$

2. The union of any number of sets in τ belongs to τ

Each element $A \in \tau$ is called a supra open set in (X, τ), and A^C is called a supra closed set in (X, τ) [3].

The supra closure of a set A is denoted by supra $cl(A) = \bigcap\{B : B$ is a supra closed and $A \subseteq B\}$.

The supra interior of a set A is denoted by supra $\operatorname{int}(A) = \bigcup\{B : B$ is a supra open and $A \supseteq B\}$ [3].

Definition 1.2[1]**.** Let X be a non empty fixed set. A neutrosophic crisp set $[N_C S]B$ is an object having the form $B = \langle B_1, B_2, B_3 \rangle$ where B_1, B_2 and B_3 are subsets of X.

Definition 1.3[1]**.** The object having the form $B = \langle B_1, B_2, B_3 \rangle$ is called:

1. A neutrosophic crisp set of type 1 if satisfying

$B_1 \cap B_2 = \phi, B_1 \cap B_3 = \phi$ and $B_2 \cap B_3 = \phi$.

2. A neutrosophic crisp set of type 2 if satisfying

$B_1 \cap B_2 = \phi, B_1 \cap B_3 = \phi$ and $B_2 \cap B_3 = \phi, B_1 \cup B_2 \cup B_3 = X$.

3. A neutrosophic crisp set of type 3 if satisfying

$B_1 \cap B_2 \cap B_3 = \phi$ and $B_1 \cup B_2 \cup B_3 = X$.

Definition 1.4[1]**.** Types of $N_C Ss \phi_{NC}$ and X_{NC} in X as follows:

1. ϕ_{NC} may be defined in many ways as a $N_C S$ as follows:

Type 1: $\phi_{NC} = \langle \phi, \phi, X \rangle$

Type 2: $\phi_{NC} = \langle \phi, X, X \rangle$

Type 3: $\phi_{NC} = \langle \phi, X, \phi \rangle$

Type 4: $\phi_{NC} = \langle \phi, \phi, \phi \rangle$

2. X_{NC} may be defined in many ways as a $N_C S$ as follows:

Type 1: $X_{NC} = \langle X, \phi, \phi \rangle$

Type 2: $X_{NC} = \langle X, X, \phi \rangle$

Type 3: $X_{NC} = \langle X, \phi, X \rangle$

Type 4: $X_{NC} = \langle X, X, X \rangle$.

Definition 1.5[1]**.** Let X be a non empty set and the $N_C Ss$ M and N in the form $M = \langle M_1, M_2, M_3 \rangle, N = \langle N_1, N_2, N_3 \rangle$ then we may consider two possible definitions for subsets $M \subseteq N$, may be defined in two ways:

1. $M \subseteq N \Leftrightarrow M_1 \subseteq N_1, M_2 \subseteq N_2$ and $N_3 \subseteq M_3$

2. $M \subseteq N \Leftrightarrow M_1 \subseteq N_1, N_2 \subseteq M_2$ and $N_3 \subseteq M_3$.

Definition 1.6[1]**.** Let X be a non empty set and the $N_C Ss$ M and N in the form $M = \langle M_1, M_2, M_3 \rangle, N = \langle N_1, N_2, N_3 \rangle$

1. $M \cap N$ may be defined in two ways as a $N_C N$ as follows:

$M \cap N = [M_1 \cap N_1], [M_2 \cup N_2]$ and $[M_3 \cup N_3]$

$M \cap N = [M_1 \cap N_1], [M_2 \cap N_2]$ and $[M_3 \cup N_3]$

2. $M \cup N$ may be defined in two ways as a $N_C S$ as follows:

$M \cup N = [M_1 \cup N_1], [M_2 \cup N_2]$ and $[M_3 \cap N_3]$

$M \cup N = [M_1 \cup N_1], [M_2 \cap N_2]$ and $[M_3 \cap N_3]$

2. Supra Neutrosophic Crisp Topological Spaces

In this section, we will introduce the supra neutrosophic crisp topological spaces $[SN_CT_s]$ and SN_C interior and SN_C closure and discuss their basic properties.

Definition 2.1. A supra neutrosophic crisp topology $[SN_CT]$ on a nonempty set X is a family τ^{μ} of supra neutrosophic crisp subsets in X satisfying the following axioms

1. $\phi_{NC}, X_{NC} \in \tau^{\mu}$

2. $\cup A_j \in \tau^{\mu} \forall \{A_j : j \in J\} \subseteq \tau^{\mu}$.

The pair (X, τ^{μ}) is said to be a supra neutrosophic crisp topological space $[SN_CT_s]$ in X. Moreover, the elements in τ^{μ} are said to be supra neutrosophic crisp open sets $[SN_CO_s]$. A supra neutrosophic crisp set E is closed $[SN_CC_s]$ if and only if its complement E^C is an open supra neutrosophic crisp set.

Example 2.2. Let $X = \{a, b\}, \tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B\}, A = \langle \phi, \{a\}, \phi \rangle$, $B = \langle \{b\}, \phi, \{a\} \rangle$

$\therefore (X, \tau^{\mu})$ is a supra neutrosophic crisp topological space.

Definition 2.3. Let X be a non empty set, and the $SN_CSs\ A$ be in the form $A = \langle A_1, A_2, A_3 \rangle$. Then A^C may be defined in three ways as an SN_CS

1. $A^C = \langle A_1^C, A_2^C, A_3^C \rangle$ or

2. $A^C = \langle A_3, A_2, A_1 \rangle$ or

3. $A^C = \langle A_3, A_2^C, A_1 \rangle$.

Definition 2.4. Let (X, τ^{μ}) be a SN_CT_s and $A = \langle A_1, A_2, A_3 \rangle$ be a

SN_CS on X. Then the supra neutrosophic crisp closure of $A[SN_C \inf(A)]$ and supra neutrosophic crisp interior of $A[SN_C cl(A)]$ are defined by

1. $SN_C \inf(A) = \cup\{G : G \subseteq A$ and G is a SN_C-open set in $X\}$.

2. $SN_C cl(A) = \cap\{F : A \subseteq F$ and F is a SN_C-closed set in $X\}$.

Remark 2.5. For any supra neutrosophic crisp set A in X, we have

1. $SN_C cl(A) \supseteq A$.

2. A is a SN_C closed set in X iff $SN_C \operatorname{int}(A) = A$.

Proposition 2.6. *Let* (X, τ^{μ}) *be any* $SN_C T_s$.

1. $SN_C \operatorname{int}(X_N) = X_N$.

2. $SN_C cl \operatorname{int}(\phi_N) = \phi_N$.

Proof. Obvious.

Proposition 2.7. *Let* (X, τ^{μ}) *be any* $SN_C T_s$. *If A and B are any two* SN_C *sets in* (X, τ^{μ}). *Then the* $SN_C \operatorname{int}(A)$ *operator satisfies the following properties*:

1. $SN_C \operatorname{int}(A) \subseteq A$.

2. $A \subseteq B \Rightarrow SN_C \operatorname{int}(A) \subseteq SN_C \operatorname{int}(B)$.

3. $SN_C \operatorname{int}(A \cap B) - SN_C \operatorname{int}(A) \cap SN_C \operatorname{int}(B)$.

4. $(SN_C cl(A))^C = SN_C \operatorname{int}(A)^C$.

5. $(SN_C \operatorname{int}(A))^C = SN_C cl(A)^C$.

Proof.

1. $SN_C \operatorname{int}(A) = \cup\{G : G$ is a SN_C-open set in X and $G \subseteq A\}$.

Thus, $SN_C \operatorname{int}(A) \subseteq A$.

2. $SN_C \operatorname{int}(B) = \cup\{G : G$ is a SN_C-open set in X and

$G \subseteq B\} \supseteq \bigcup\{G : G$ is a SN_C-open set in X and $G \subseteq A\} \supseteq SN_C \operatorname{int}(A)$.

Thus, $SN_C \operatorname{int}(A) \subseteq SN_C \operatorname{int}(B)$.

3. $SN_C \operatorname{int}(A \cap B) = \bigcup\{G : G$ is a SN_C-open set in X and $A \cap B \supseteq G\} = (\bigcup\{G : G$ is a SN_C-open set in X and $A \supseteq G\} \cap (\bigcup\{G : G$ is a SN_C-open set in X and $B \supseteq G\} = SN_C \operatorname{int} \cap SN_C \operatorname{int}(B)$.

Thus, $SN_C \operatorname{int}(A \cap B) = SN_C \operatorname{int}(A) \cap SN_C \operatorname{int}(B)$.

4. $SN_C cl(A) = \bigcap\{G : G$ is a SN_C-closed set in X and $A \subseteq G\}$, $(SN_C cl(A))^C = \bigcup\{G^C : G^C$ is a SN_C-open set in X and $A^C \supseteq G^C\} = SN_C \operatorname{int}(A)^C$.

Thus, $(SN_C cl(A))^C = SN_C \operatorname{int}(A)^C$.

5. $SN_C \operatorname{int}(A) = \bigcup\{G : G$ is a SN_C-open set in X and $A \supseteq G\}$, $(SN_C \operatorname{int}(A))^C = \bigcap\{G : G$ is a SN_C-closed set in X and $A^C \supseteq G^C\} = SN_C cl(A)^C$.

Thus, $(SN_C \operatorname{int}(A))^C = SN_C cl(A)^C$.

Proposition 2.8. *Let (X, τ^{μ}) be any $SN_C T_s$. If A and B are any two SNc sets in (X, τ^{μ}). Then the $SN_C cl(A)$ operator satisfies the following properties*:

1. $A \subseteq SN_C cl(A)$.
2. $A \subseteq B \Rightarrow SN_C cl(A) \subset SN_C cl(B)$.
3. $SN_C cl(A \cup B) = SN_C cl(A) \cup SN_C cl(B)$.

3. Supra Neutrosophic Crisp Limit Point

In this section, we will introduce the supra neutrosophic crisp limit points with some properties. This work contains an adjustment for the above-mentioned definitions 1.5 and 1.6, this was necessary to homogeneous suitable results for the upgrade of this research.

Definition 3.1. Let X be a nonempty set and the SN_CSs M and N in the form $M = \langle M_1, M_2, M_3\rangle$, $N = \langle N_1, N_2, N_3\rangle$ then the additional new ways for the intersection, union and inclusion between M and N are

$$M \cap N = [M_1 \cap N_1], [M_2 \cap N_2], \text{ and } [M_3 \cap N_3]$$

$$M \cup N = [M_1 \cup N_1], [M_2 \cup N_2] \text{ and } [M_3 \cup N_3]$$

$$M \subseteq N \Leftrightarrow M_1 \subseteq N_1, M_2 \subseteq N_2 \text{ and } M_3 \subseteq N_3.$$

Definition 3.2. For all a, b, c belongs to a non empty set X. Then the supra neutrosophic crisp points related to a, b, c are defined as follows:

- $a_{N_1} = \langle\{a\}, \phi, \phi\rangle$, is called a supra neutrosophic crisp point $(SN_CP_{N_1})$ in X.
- $b_{N_2} = \langle\phi, \{b\}, \phi\rangle$, is called a supra neutrosophic crisp point $(SN_CP_{N_2})$ in X.
- $c_{N_3} = \langle\phi, \phi, \{c\}\rangle$, is called a supra neutrosophic crisp point $(SN_CP_{N_3})$ in X.

The set of all supra neutrosophic crisp points $(SN_CP_{N_1}, SN_CP_{N_2}, SN_CP_{N_3})$ is denoted by SN_CP_N.

Definition 3.3. Let X be a non empty set and $a, b, c \in X$. Then the supra neutrosophic crisp point:

- a_{N_1} is belonging to the supra neutrosophic crisp set $B = \langle B_1, B_2, B_3\rangle$, denoted by $a_{N_1} \in B$, if $a \in B_1$, wherein a_{N_1} does not belongs to the supra neutrosophic crisp set B denoted by $a_{N_1} \notin B$, if $a \notin B_1$.
- b_{N_2} is belonging to the supra neutrosophic crisp set $B = \langle B_1, B_2, B_3\rangle$, denoted by $b_{N_2} \in B$, if $b \in B_2$. In contrast b_{N_2} does not belongs to the supra neutrosophic crisp set B, denoted by $b_{N_2} \notin B$, if $b \notin B_2$.

- c_{N_3} is belonging to the supra neutrosophic crisp set $B = \langle B_1, B_2, B_3 \rangle$, denoted by $c_{N_3} \in B$, if $c \in B_3$. In contrast c_{N_3} does not belongs to the supra neutrosophic crisp set B, denoted by $c_{N_3} \notin B$, if $c \notin B_3$.

Definition 3.4. Let (X, τ^{μ}) be SN_CT_s, $p \in SN_CP_N$ in X, a supra neutrosophic crisp set $B = \langle B_1, B_2, B_3 \rangle \in \tau^{\mu}$ is called supra neutrosophic crisp open nbd of p in (X, τ^{μ}), if $p \in B$.

Definition 3.5. Let (X, τ^{μ}) be SN_CT_s, $p \in SN_CP_N$ in X, a supra neutrosophic crisp set $B = \langle B_1, B_2, B_3 \rangle \in \tau^{\mu}$ is called supra neutrosophic crisp nbd of p in (X, τ^{μ}), if there is supra neutrosophic crisp open set $A = \langle A_1, A_2, A_3 \rangle$ containing p such that $A \subseteq B$.

Note 3.6. Every supra neutrosophic crisp open nbd of any point $p \in SN_CP_N$ in X is supra neutrosophic crisp nbd of p, but the converse need not be true following the example.

Example 3.7. Let $X = \{a, b, c\}$, $\tau^{\mu} = \{X_{NC}, \phi_{NC}, A, B, G\}$, $A = \langle \{a, b\}, \phi, \phi \rangle$, $B = \langle \phi, \{b\}, \phi \rangle$, $G = \langle \{a, b\}, \{b\}, \phi \rangle$.

If we take $U = \langle \{a, b\}, \phi, \{c\} \rangle$.

Then $G = \langle \{a\}, \phi, \phi \rangle$ is an open set containing $p = a_{N_1} = \langle \{a, b\}, \phi, \phi \rangle$ and $G \subseteq U$. That is U is a supra neutrosophic crisp nbd of p in (X, τ^{μ}), while it is not a supra neutrosophic crisp open nbd of p.

Definition 3.8. Let (X, τ^{μ}) be a SN_CTs and $B = \langle B_1, B_2, B_3 \rangle$ be SN_C set of X. A supra neutrosophic crisp point $p \in SN_CP_N$ in X is called a supra neutrosophic crisp limit point of $B = \langle B_1, B_2, B_3 \rangle$ iff every supra neutrosophic crisp open set containing p must contains at least one supra neutrosophic crisp point of B different from p. It is easy to say that the point p is not supra neutrosophic crisp limit point of B if there is a supra neutrosophic crisp open set G of p and $B \cap (G \setminus p) = \phi_{NC}$.

Definition 3.9. The set of all supra neutrosophic crisp limit points of a supra neutrosophic crisp set B is called supra neutrosophic crisp derived set of B, denoted by $SN_CD(B)$.

Example 3.10. Let $X = \{a, b, c, d\}$, $\tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G\}$, $A = \langle\{a, b\}, \phi, \{d\}\rangle$, $B = \langle\phi, \{b\}, \phi\rangle$, $G = \langle\{a, b\}, \{b\}, \phi\rangle$. If we take $D = \langle\{a, b\}, \{b\}, \phi\rangle$. Then $P = C_{N_1} = \langle\{c\}, \phi, \phi\rangle$ is the only supra neutrosophic crisp limit point of D, i.e., $SN_CD(D) = \{C_{N_1}\}$.

Theorem 3.11. *Let* (X, τ^{μ}) *be* SN_CT_s *and* $B = \langle B_1, B_2, B_3\rangle$ *be a supra neutrosophic crisp set of* X, *then* B *is supra neutrosophic crisp closed set* (SN_CC_s) *iff* $SN_CD(B) \subseteq B$.

Proof. Let B be SN_CC_s, then $(X \setminus B)$ is supra neutrosophic crisp open set (SN_CO_s) this implies that for each supra neutrosophic crisp point $p \in SN_CP_N$ in $(X \setminus B)$, $p \notin B$, there is a supra neutrosophic crisp open set G of p and $G \subseteq (X \setminus B)$.

Since $B \cap (X \setminus B) = \phi_{NC}$, then p is not supra neutrosophic crisp limit point of B, thus $G \cap B = \phi_{NC}$, which implies that $p \notin SN_CD(B)$. Hence $SN_CD(B) \subseteq B$.

Conversely: Assume that $p \notin SN_CD(B)$, implies that p is not supra neutrosophic crisp limit point of B, hence, there is a supra neutrosophic crisp open set G of p and $G \cap B = \phi_{NC}$, which means that $G \subseteq (X \setminus B)$ and since $(X \setminus B)$ is a supra neutrosophic crisp open set.

Hence B is supra neutrosophic crisp closed set.

Theorem 3.12. *Let* (X, τ^{μ}) *be* SN_CT_s *and* B, G *be a supra neutrosophic crisp sets of* X, *then the following properties hold*:

(i) $SN_CD(\phi_{NC}) = \phi_{NC}$,

(ii) *If* $B \subseteq G$, *then* $SN_CD(B) \subseteq SN_CD(G)$,

(iii) $SN_CD(B \cap G) \subseteq SN_CD(B) \cap SN_CD(G)$,

(iv) $SN_C D(B \cup G) \subseteq SN_C D(B) \cup SN_C D(G)$.

Proof. (i) the proof is directly.

(ii) Assume that $SN_C D(B)$ be a supra neutrosophic crisp set containing a supra neutrosophic crisp point $p \in SN_C P_N$. Then by definition 3.8, for each supra neutrosophic crisp open set V of p, we have $B \cap V \setminus p \neq \phi_{NC}$, but $B \subseteq G$, hence $G \cap V \setminus p \neq \phi_{NC}$, this means that $p \in SN_C D(G)$.

Hence, $SN_C D(B) \subseteq SN_C D(G)$.

(iii) Since $B \cap G \subseteq B$, then by (ii) $SN_C D(B \cap G) \subseteq SN_C D(B)$ (i)

$B \cap G \subseteq G$, implies $SN_C D(B \cap G) \subseteq SN_C D(G)$ (ii)

From (i) and (ii) $SN_C D(B \cap G) \subseteq SN_C D(B) \cap SN_C D(G)$.

(iv) Let $p \in SN_C P_N$ such that $P \notin SN_C D(B) \cup SN_C D(B) \cup SN_C D(G)$, then either $P \notin SN_C D(B)$ and $P \notin SN_C D(G)$, then there is a supra neutrosophic crisp open set K of P and $B \cap K \setminus P = \phi_{NC}$ and $G \cap K \setminus P = \phi_{NC}$ this implies that $(B \cup G) \cap K \setminus P = \phi_{NC}$, i.e. $P \notin SN_C D(B \cup G)$. Hence $SN_C D(B \cup G) \subseteq SN_C D(B) \cup SN_C D(G)$. (iii)

Conversely, since $B \subseteq B \cup G, G \subseteq B \cup G$, then by property (ii) $SN_C D(B) \subseteq SN_C D(B \cup G)$, and $SN_C D(G) \subseteq SN_C D(B \cup G)$. Thus $SN_C D(B \cup G) \supseteq SN_C D(B) \cup SN_C D(G)$.

(iv) From (iii) and (iv) we have $SN_C D(B \cup G) = SN_C D(B) \cup SN_C D(G)$.

4. Separation Axioms in a Supra Neutrosophic Crisp Topological Spaces

In this section we introduce separation axioms in a supra neutrosophic crisp topological spaces and we study its characterizations.

Definition 4.1. The supra neutrosophic crisp topological space (X, τ^{μ}) is said to be $SN_C T_0$ space, if $\forall x, y \in X, x \neq y$ and there exists supra neutrosophic crisp open set G such that $x \in G$ and $y \notin G$.

Example 4.2. Let $X = \{a, b, c\}$, $\tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G\}$, $A = \langle \{a\}, \phi, \phi \rangle$, $B = \langle \phi, \{b\}, \phi \rangle$, $G = \langle \phi, \phi, \{a\} \rangle$.

$\therefore (X, \tau^{\mu})$ is a $SN_C T_0$ space.

Definition 4.3. The supra neutrosophic crisp topological space (X, τ^{μ}) is said to be SN_C subspace of (X, τ^{μ}), if $A \subseteq X$, $A \neq \phi$, τ^{μ}_A is the class of all intersection of A with each element in τ^{μ}.

Theorem 4.4. *Let (X, τ^{μ}) is a $SN_C T_0$ space and (Y, τ^{μ}) is a SN_C subspace of (X, τ^{μ}). Then (Y, τ^{μ}_y) is a $SN_C T_0$ space.*

Proof. Suppose that $x, y \in Y$, $x \neq y$, since $Y \subseteq X$ then $x, y \in X$.

Since (X, τ^{μ}) is a $SN_C T_0$ space means that $\exists$ a SN_C open set $G \subseteq X \ni x \in G$ and $y \notin G$.

We have that $G_y = Y \cap G$, G_y is a SN_C open set in Y and $x \in G_y$ but $y \notin G_y$, so we found a SN_C open set $G_y \subseteq Y$ which it contained x and not contained y.

Hence (Y, τ^{μ}_y) is a $SN_C T_0$ space.

Theorem 4.5. *Let (X, τ^{μ}), (X_1, τ^{μ}_1) are two $SN_C T_s$, (X, τ^{μ}) is a $SN_C T_0$ space and f is a SN_C open function and bijective. Then (X_1, τ^{μ}_1) is a $SN_C T_0$ space.*

Proof. Suppose that (X, τ^{μ}) is a $SN_C T_0$ space.

Now we have to prove that (X_1, τ^{μ}_1) is a $SN_C T_0$ space.

Let $x_1, y_1 \in X_1$, $x_1 \neq y_1$, since f is a bijective function, then $\exists x, y \in X \ni x_1 = f(x)$, $y_1 = f(y)$ and $x \notin G$.

Since (X, τ^{μ}) is a $SN_C T_0$ space, then $\exists\, G \subseteq X$ is a SN_C open set $\ni x \in G$ and $y \notin G$.

We obtain that $f(G) \subseteq X_1$ is a SN_C open set in X_1 because f is a SN_C open function.

So $x_1 \in f(G)$ and $y_1 \notin f(G)$. Hence, (X_1, τ_1^{μ}) is a $SN_C T_0$ space.

Definition 4.6. The supra neutrosophic crisp topological space (X, τ^{μ}) is said to be $SN_C T_1$ space, if $\forall x, y \in X, x \neq y$ and $\exists G, H \subseteq X$ are SN_C open sets $\ni x \in G, y \notin G$ and $y \in H, x \notin H$.

Example 4.7. Let $X = \{a, b, c\}, \tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G, H\}$, $A = \langle \{a\}, \phi, \phi \rangle$ $B = \langle \phi, \{b\}, \phi \rangle$, $G = \langle \phi, \phi, \{a\} \rangle$, $H = \langle \phi, \phi, \{b\} \rangle$.

$\therefore (X, \tau^{\mu})$ is a $SN_C T_1$ space.

Theorem 4.8. *Let (X, τ^{μ}) is a $SN_C T_S$ and is a $SN_C T_1$ space if and only if for every $x \in X$, $\{x\}$ is a SN_C closed set.*

Proof. Let (X, τ^{μ}) be a $SN_C T_s$, we show that $\{x\}^c$ is a SN_C open set in X. Suppose that $a \in \{x\}^c, a \neq x$ then by (def. 3.6) $\exists G_a$ is a SN_C open set in X where G_a does not contain x. Hence $a \in G_a \subseteq \{x\}^c$ and $\{x\}^c = \{G_a : a \in \{x\}^c\}$.

This means $\{x\}^c$ is a union of all SN_C open sets and by def. $SN_C T_s$.

$\therefore \{x\}^c$ is a SN_C open set. Hence $\{x\}$ is a SN_C closed set.

Conversely suppose that $\{x\}$ is a SN_C closed set in X and let $a, b \in X$ where $a \neq b$ then $a \in \{x\}^c, b \in \{y\}^c$ and $\{x\}^c, \{y\}^c$ are SN_C open sets in X.

Hence (X, τ^{μ}) is a $SN_C T_1$ space.

Theorem 4.9. *Let (X, τ^{μ}) is a $SN_C T_1$ space and (Y, τ^{μ}) is a SN_C subspace of (X, τ^{μ}). Then (Y, τ_y^{μ}) is a $SN_C T_1$ space.*

Theorem 4.10. *Let $(X, \tau^{\mu}), (X_1, \tau_1^{\mu})$ are two $SN_C T_s, (X, \tau^{\mu})$ is a*

SN_CT_1 space and f is a SN_C open function and bijective. Then (X_1, τ_1^{μ}) is a SN_CT_1 space.

Remark 4.11. Every SN_CT_1 is a SN_CT_0 space, but the converse is not true as in the following example.

Example 4.12. Let $X = \{a, b\}, \tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G\}, A = \langle\phi, \{a\}, \phi\rangle$, $B = \langle\phi, \phi, \{a\}\rangle$, $G = \langle\{a\}, \phi, \phi\rangle$.

$\therefore (X, \tau^{\mu})$ is a SN_CT_0 but not SN_CT_1 space.

Definition 4.13. The supra neutrosophic crisp topological space (X, τ^{μ}) is said to be SN_CT_2 space, if $\forall x, y \in X, x \neq y$ and $\exists G, H \subseteq X$ are SN_C open sets $\ni x \in G, y \in H$ and $G \cap H = \phi$.

Example 4.14. Let $X = \{a, b, c\}, \tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G, H\}$, $A = \langle\{a\}, \{b\}, \phi\rangle$, $B = \langle\phi, \{b\}, \{a\}\rangle$, $G = \langle\phi, \{a\}, \phi\rangle$, $H = \langle\phi, \phi, \{b\}\rangle$ and also $G \cap H = \phi$.

$\therefore (X, \tau^{\mu})$ is a SN_CT_2 space.

Theorem 4.15. *Let (X, τ^{μ}) is a SN_CT_2 space and (Y, τ^{μ}) is a SN_C subspace of (X, τ^{μ}). Then (Y, τ_y^{μ}) is a SN_CT_2 space.*

Proof. Suppose that $x, y \in Y, x \neq y$, since $Y \subseteq X$ then $x, y \in X$ which means that $\exists$ two SN_C open sets $G, H \subseteq X \ni x \in G$ and $y \in H$ and $G \cap H = \phi$.

Now $G_y = G \cap Y, H_y = H \cap Y$ are two SN_C open sets in $Y \ni x \in G_y$ and $y \in H_y$.

Since $G \cap H = \phi$, then $G_y \cap H_y = \phi$. So (Y, τ_y^{μ}) is a SN_CT_2 space.

Hence, (Y, τ_y^{μ}) is a SN_CT_2 space.

Theorem 4.16. *Let $(X, \tau^{\mu}), (X_1, \tau_1^{\mu})$ are two SN_CT_s, (X, τ^{μ}) is a SN_CT_2 space and f is a SN_C open function and bijective. Then (X_1, τ_1^{μ}) is a SN_CT_2 space.*

Remark 4.17. Every SN_CT_2 space is a SN_CT_1 space, but the converse is not true as in the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau^{\mu} = \{\phi_{NC}, X_{NC}, A, B, G, H\}$, $A = \langle\{a\}, \phi, \{b\}\rangle$, $B = \langle\phi, \{a\}, \{b\}\rangle$, $G = \langle\phi, \{a\}, \phi\rangle$, $H = \langle\phi, \{b\}, \phi\rangle$.

$\therefore$ (X, τ^{μ}) is a SN_CT_1 space but not SN_CT_2 space.

5. Conclusion

In this paper, we introduced the supra neutrosophic crisp topological spaces with some definitions and properties. We have introduced supra neutrosophic crisp limit points and some definitions and properties. We have introduced separation axioms in supra neutrosophic crisp topological spaces and examine the relationship between them in details.

References

[1] A. A. Salama, F. Smarandache and Valeri Kroumov, Neutrosophic crisp sets and neutrosophic crisp topological spaces, Neutrosophic Sets and Systems (2) (2014), 25-30.

[2] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics, Vol. 3, Issue 4 (Sep-Oct 2012), 31-35.

[3] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, On supra topological spaces, Indian J. Pure and Appl. Math. 4, 14(4) (1983), 502-510.

[4] Ahmed B. AL-Nafee and Riad K. AI-Hamido, Florentin Smarandache, separation axioms in neutrosophic crisp topological spaces, Neutrosophic Sets and Systems, Vol. 25, 2019, University of New Mexico.

[5] B. K. Mahmoud, On supra-separation axioms for supra topological spaces, Tikrit Journal of Pure Science 22(2) (2017), Issn-1813-1662.

[6] F. Smarandache, An introduction to the neutrosophy probability applied in Quantum Physics, International Conference on Introduction Neutrosophic Physics, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA, 2-4 Dec. 2011.

[7] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, 1999.

[8] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic logic, Probability, and Statistics, University of New Mexico, NM 87301, USA (2002).