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AN OPTIMAL SOLUTION OF FUZZY TRANSPORTATION PROBLEM USING BEST CANDIDATE METHOD

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Abstract

Transportation problem is one of the sub classes of linear programming problem. The objective of the transportation problem is to minimize the transportation cost or maximize the profit. Fuzzy set theory has been applied in many fields of science, Engineering, and Management. In this article, a new ranking method is proposed for solving Hexadecagonal fuzzy transportation problem. Fuzzy transportation problem is transformed into crisp transportation problem and solved by Best candidate method. A numerical example is presented, and the optimal solution obtained by using the proposed method.

1. Introduction

Transportation problem is the special case of linear programming problem. The objective of the transportation problem is to minimize the transportation cost or maximizing the profit while satisfying the rim requirements. The methods for solving the transportation problem are available only for when the demand and supply quantities are exactly known. But in some situations the values of the problem are not exactly known. This uncertainty leads fuzziness. A fuzzy transportation problem is a fuzzy

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transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities

L. A. Zadeh [22] was first introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Bellmann and Zadeh [4] proposed the concept of decision-making in fuzzy environment. H. J. Zimmermann [23] showed that solutions obtained by fuzzy linear programming are always efficient. Stefan Chanas and Dorota Kuchta [6] were studied a concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Hlayel Abdallah Ahmad [9] solved the optimization problems using best candidates method. Stefan Chanas and Dorota Kuchta [7] were solving fuzzy integer transportation problem. Shiang-Tai Liu and Chiang Kao [10] was finding solution of fuzzy transportation problems based on extension principle. Stefan Chanas, Waldemar Kolodziejczyk and Anna Machaj [5] discussed a fuzzy approach for solving the transportation problem. H. Basirzadeh [3] had approached a new technique for solving fuzzy transportation problem. Abdallah A. Hlayel and Mohammad A. Alia [1] were solved the transportation problem using the best candidate method.

Pandian and G. Natarajan [14] proposed a new method called a fuzzy zero point method for finding optimal solution of fuzzy transportation problems. S. Solaiappan and K. Jeyaraman [17] investigated the fuzzy transportation problem by using zero termination method. A. Nagoor Gani and K. Abdul Razak [12] solved two stage fuzzy transportation problem. M. S. Annie Christi and D. Malini [2] were solving the octagonal fuzzy transportation problem using the best candidate method and centroid method. S. Renuka and C. Jenita Nancy [15] solved Nanogonal fuzzy transportation problem using Russell's Method. L. Sudha, R. Shanmugapriya and B. Rama [19] were finding the optimal solution of fuzzy transportation problem using nanogonal fuzzy number. D. Stephen Dinagar and R. Keerthivasan [18] discussed the comparative study of some optimization methods with the best candidate method for fuzzy transportation problem.

Jatinder Pal Singh and NehaIshesh Thakur [16] proposed a new method for solving fuzzy transportation problem using dodecagonal fuzzy number. L. Sujatha, P. Vinothini and R. Jothilakshmi [21] solved a fuzzy transportation problem using zero point maximum allocation method. S. U. Malini and

Felbin C. Kennedy [11] were solving the octagonal fuzzy transportation problem. V. J. Sudhakar and V. Navaneetha Kumar [20] solved the multi objective two stage fuzzy transportation problem by using zero suffix method. S. Narayanamoorthy, S. Saranya and S. Maheswari [13] solved a fuzzy transportation problem using fuzzy Russell's method. A. Felix, S. Christopher and A. Victor Devadoss [8] were discussing the arithmetic operations of Nonagonal Fuzzy Number.

In this paper, a new ranking method is proposed for ranking the Hexadecagonal fuzzy numbers. Fuzzy transportation problem can be converted in to crisp transportation problem using ranking method and an optimal solution is obtained by using the best candidate method.

2. Preliminaries

Definition 2.1. A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval $\{0, 1\}$. (i.e.) $A = \{x, \mu_A(x); x \in X\}$. Here $\mu_A(x) = 1$.

Definition 2.2. A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist at least one $x \in X$ Such that $\mu_A(x) = 1$.

Definition 2.3. The support of fuzzy set in the Universal set X is the set that contains all the elements of X that have anon-zero membership grade in $\widetilde{A}$. (i.e.) $\operatorname{Supp}(\widetilde{A}) = \{x \in X / \mu_{\widetilde{A}}(x) > 0\}$

Definition 2.5. A fuzzy set $\widetilde{A}$ defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A(x) : R \to [0, 1]$ has the following properties

(i) A must be a normal and convex fuzzy set

(ii) α_A must be a closed interval for every $\alpha \in (0, 1]$

(iii) The support of $\widetilde{A}$ must be bounded.

Definition 2.6. A fuzzy number $\widetilde{A}$ is called triangular function is denoted by $\widetilde{A} = (a_1, a_2, a_3)$ whose membership function is defined as follows

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases}$$

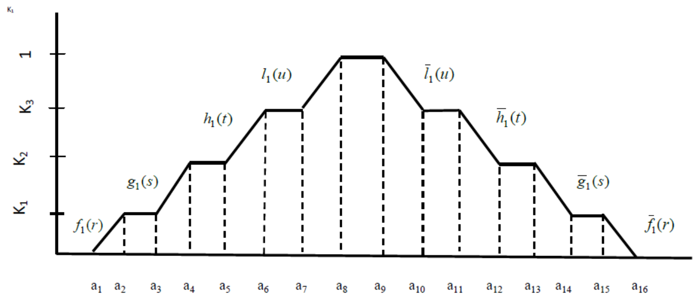
3. Hexadecagonal Fuzzy Number

A fuzzy number $\widetilde{A}$ is a Hexadecagonal fuzzy number defined by $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$ are real numbers and its membership function is given by

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ k_1\left(\dfrac{x - a_1}{a_2 - a_1}\right) & a_1 \leq x \leq a_2 \\ k_1 & a_2 \leq x \leq a_3 \\ k_1 + (k_2 - k_1)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & a_3 \leq x \leq a_4 \\ k_2 & a_4 \leq x \leq a_5 \\ k_2 + (k_3 - k_2)\left(\dfrac{x - a_5}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\ k_3 & a_6 \leq x \leq a_7 \\ k_3 + (1 - k_3)\left(\dfrac{x - a_7}{a_8 - a_7}\right) & a_7 \leq x \leq a_8 \\ 1 & a_8 \leq x \leq a_9 \\ k_3 + (1 - k_3)\left(\dfrac{a_{10} - x}{a_{10} - a_9}\right) & a_9 \leq x \leq a_{10} \\ k_3 & a_{10} \leq x \leq a_{11} \\ k_2 + (k_3 - k_2)\left(\dfrac{a_{12} - x}{a2 - a_{11}}\right) & a_{11} \leq x \leq a_{12} \\ k_2 & a_{12} \leq x \leq a_{13} \\ k_1 + (k_2 - k_1)\left(\dfrac{a_{14} - x}{a_{14} - a_{13}}\right) & a_{13} \leq x \leq a_{14} \\ k_1 & a_{14} \leq x \leq a_{15} \\ k_1\left(\dfrac{a_{16} - x}{a_{16} - a_{15}}\right) & a_{14} \leq x \leq a_{16} \\ 0 & a_{16} < x \end{cases}$$

where $0 < k_1 < k_2 < k_3 < 1$.

Graphical Representation of Hexadecagonal Fuzzy Number



Definition. The α-cut of Hexadecagonal fuzzy number

$\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ is defined as $\widetilde{A}_\alpha$

$$\left[a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1),\ a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right] \text{ for } \alpha \in [0, k_1]$$

$$\left[a_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3),\ a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13})\right] \text{ for } \alpha \in [k_1, k_2]$$

$$\left[a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5),\ a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right] \text{ for } \alpha \in [k_2, k_3]$$

$$\left[a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7),\ a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right] \text{ for } \alpha \in [k_3, 1]$$

3.1. Arithmetic Operations on Hexadecagonal Fuzzy Number

(a) Addition of Hexadecagonal Fuzzy Numbers: Let $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ and $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ be two Hexadecagonal fuzzy numbers. To calculate the multiplication of two fuzzy numbers first we add the α-cut $\widetilde{A}$ and $\widetilde{B}$ using interval arithmetic. Then $[\widetilde{A}]_\alpha + [\widetilde{B}]_\alpha$ is

$$\left[a_1 + b_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1 + b_2 - b_1), a_{16} + b_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15} + b_{16} - b_{15})\right]$$

for $\alpha \in [0, k_1]$

$$\left[a_3 + b_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3 + b_4 - b_3), a_{14} + b_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13} + b_{14} - b_{13})\right] \text{ for } \alpha \in [k_1, k_2]$$

$$\left[a_5 + b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5 + b_6 - b_5), a_{12} + b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11} + b_{12} - b_{11})\right] \text{ for } \alpha \in [k_2, k_3]$$

$$\left[a_7 + b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7 + b_8 - b_7), a_{10} + b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9 + b_{10} - b_9)\right] \text{ for } \alpha \in [k_3, 1]$$

(b) Subtraction of Hexadecagonal Fuzzy numbers: Let $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ and $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ be two Hexadecagonal fuzzy numbers. To calculate the subtraction of two fuzzy numbers first we add the α-cut $\widetilde{A}$ and $\widetilde{B}$ using interval arithmetic.

Then $[\widetilde{A}]_\alpha + [\widetilde{B}]_\alpha$ is for $\alpha \in [0, k_1]$

$$L = \min\begin{bmatrix} \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\} - \left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\} \\ - \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \\ \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\} - \left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\} \\ - \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \end{bmatrix}$$

$$R=\max\begin{bmatrix}\left\{a_1+\left(\frac{\alpha}{k_1}\right)(a_2-a_1)\right\}-\left\{b_1+\left(\frac{\alpha}{k_1}\right)(b_2-b_1)\right\},\left\{a_1+\left(\frac{\alpha}{k_1}\right)(a_2-a_1)\right\}\\-\left\{b_{16}-\left(\frac{\alpha}{k_1}\right)(b_{16}-b_{15})\right\}\\\left\{a_{16}-\left(\frac{\alpha}{k_1}\right)(a_{16}-a_{15})\right\}-\left\{b_1+\left(\frac{\alpha}{k_1}\right)(b_2-b_1)\right\},\left\{a_{16}-\left(\frac{\alpha}{k_1}\right)(a_{16}-a_{15})\right\}\\-\left\{b_{16}-\left(\frac{\alpha}{k_1}\right)(b_{16}-b_{15})\right\}\end{bmatrix}$$

For $\alpha\in[k_1,k_2]$

$$L=\min\begin{bmatrix}\left\{a_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_4-a_3)\right\}-\left\{b_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_4-b_3)\right\},\left\{a_4+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_4-a_3)\right\}\\-\left\{b_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_{14}-b_{13})\right\}\\\left\{a_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_{14}-a_{13})\right\}-\left\{b_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_4-b_3)\right\},\left\{a_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_{14}-a_{13})\right\}\\-\left\{b_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_{14}-b_{13})\right\}\end{bmatrix}$$

$$R=\max\begin{bmatrix}\left\{a_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_4-a_3)\right\}-\left\{b_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_4-b_3)\right\},\left\{a_4+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_4-a_3)\right\}\\-\left\{b_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_{14}-b_{13})\right\}\\\left\{a_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_{14}-a_{13})\right\}-\left\{b_3+\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_4-b_3)\right\},\left\{a_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_{14}-a_{13})\right\}\\-\left\{b_{14}-\left(\frac{\alpha-k_1}{k_2-k_1}\right)(b_{14}-b_{13})\right\}\end{bmatrix}$$

For $\alpha\in[k_2,k_3]$

$$L=\min\begin{bmatrix}\left\{a_5+\left(\frac{\alpha-k_2}{k_3-k_2}\right)(a_6-a_5)\right\}-\left\{b_5+\left(\frac{\alpha-k_2}{k_3-k_2}\right)(b_6-b_5)\right\},\left\{a_5+\left(\frac{\alpha-k_2}{k_3-k_2}\right)(a_6-a_5)\right\}\\-\left\{b_{12}-\left(\frac{\alpha-k_2}{k_3-k_2}\right)(b_{12}-b_{11})\right\}\\\left\{a_{12}-\left(\frac{\alpha-k_2}{k_3-k_2}\right)(a_{12}-a_{11})\right\}-\left\{b_5+\left(\frac{\alpha-k_2}{k_3-k_2}\right)(b_6-b_5)\right\},\left\{a_{12}-\left(\frac{\alpha-k_2}{k_3-k_2}\right)(a_{12}-a_{11})\right\}\\-\left\{b_{12}-\left(\frac{\alpha-k_2}{k_3-k_2}\right)(b_{12}-b_{11})\right\}\end{bmatrix}$$

$$R = \max\begin{bmatrix} \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\} - \left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\} \\ - \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \\ \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\} - \left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\} \\ - \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \end{bmatrix}$$

For $\alpha \in [k_3, 1]$

$$L = \min\begin{bmatrix} \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} - \left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} \\ - \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \\ \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} - \left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} \\ - \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \end{bmatrix}$$

$$R = \max\begin{bmatrix} \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} - \left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} \\ - \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \\ \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} - \left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} \\ - \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \end{bmatrix}$$

(c) Multiplication of Hexadecagonal Fuzzy numbers: Let $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ and $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ be two Hexadecagonal fuzzy numbers. To calculate the multiplication of two fuzzy numbers first we add the α-cut $\widetilde{A}$ and $\widetilde{B}$ using interval arithmetic. Then $[\widetilde{A}]_\alpha * [\widetilde{B}]_\alpha$ is

For $\alpha \in [0, k_1]$

$$L = \min \begin{bmatrix} \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\}\left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\} \\ \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \\ \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\}\left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\} \\ \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \end{bmatrix}$$

$$R = \max \begin{bmatrix} \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\}\left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1)\right\} \\ \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \\ \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\}\left\{b_1 + \left(\frac{\alpha}{k_1}\right)(b_2 - b_1)\right\}, \left\{a_{16} - \left(\frac{\alpha}{k_1}\right)(a_{16} - a_{15})\right\} \\ \left\{b_{16} - \left(\frac{\alpha}{k_1}\right)(b_{16} - b_{15})\right\} \end{bmatrix}$$

For $\alpha \in [k_1, k_2]$

$$L = \min \begin{bmatrix} \left\{a_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3)\right\}\left\{b_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_4 - b_3)\right\}, \left\{a_4 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3)\right\} \\ \left\{b_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_{14} - b_{13})\right\} \\ \left\{a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13})\right\}\left\{b_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_4 - b_3)\right\}, \left\{a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13})\right\} \\ \left\{b_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_{14} - b_{13})\right\} \end{bmatrix}$$

$$R = \max \begin{bmatrix} \left\{a_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3)\right\}\left\{b_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_4 - b_3)\right\}, \left\{a_4 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3)\right\} \\ \left\{b_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_{14} - b_{13})\right\} \\ \left\{a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13})\right\}\left\{b_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_4 - b_3)\right\}, \left\{a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13})\right\} \\ \left\{b_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(b_{14} - b_{13})\right\} \end{bmatrix}$$

For $\alpha \in [k_2, k_3]$

$$L = \min \left[\begin{array}{l} \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\}\left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\} \\ \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \\ \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\}\left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\} \\ \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \end{array}\right]$$

$$R = \max \left[\begin{array}{l} \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\}\left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5)\right\} \\ \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \\ \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\}\left\{b_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_6 - b_5)\right\}, \left\{a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11})\right\} \\ \left\{b_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(b_{12} - b_{11})\right\} \end{array}\right]$$

For $\alpha \in [k_3, 1]$

$$L = \min \left[\begin{array}{l} \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\}\left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} \\ \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \\ \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\}\left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} \\ \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \end{array}\right]$$

$$R = \max \left[\begin{array}{l} \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} - \left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7)\right\} \\ \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \\ \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\}\left\{b_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_8 - b_7)\right\}, \left\{a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9)\right\} \\ \left\{b_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(b_{10} - b_9)\right\} \end{array}\right]$$

3.2. Measure of fuzzy number

The measure of $\widetilde{A}_\omega$ is a measure is a function $M_0 : R_\omega(I) \to R^+$ which assign a non-negative real numbers $M_0^{HXDFN}(\widetilde{A}_\omega)$ that expresses the measure of

$$M_0^{HXDFN}(\widetilde{A}_\omega) = \frac{1}{2}\int_\alpha^{k_1}(f_1(r) + \bar{f}_1(r))\,dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s) + \bar{g}_1(s))\,ds$$

$$+ \frac{1}{2}\int_{k_2}^{k_3}(h_1(t) + \bar{h}_1(t))\,dt + \frac{1}{2}\int_{k_3}^{\omega}(l_1(w) + \bar{l}_1(w))\,dw$$

where $0 \leq \alpha < 1$.

4. Ranking Method

Let $\widetilde{A}$ be a normal Hexadecagonal fuzzy number. The measure of $\widetilde{A}$ is calculated as follows

$$M_0^{HXDFN}(\widetilde{A}_\omega) = \frac{1}{2}\int_0^{k_1}(f_1(r) + \bar{f}_1(r))\,dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s) + \bar{g}_1(s))\,ds$$

$$+ \frac{1}{2}\int_{k_2}^{k_3}(h_1(t) + \bar{h}_1(t))\,dt + \frac{1}{2}\int_{k_3}^{1}(l_1(w) + \bar{l}_1(w))\,dw$$

$$M_0^{HXDFN}(\widetilde{A}) = \frac{1}{4}\begin{Bmatrix}(a_1 + a_2 + a_{15} + a_{16})k_1 + (a_3 + a_4 + a_{13} + a_{14})(k_2 - k_1) \\ + (a_5 + a_6 + a_{11} + a_{12})(k_3 - k_2) + (a_7 + a_8 + a_9 + a_{10})(1 - k_3)\end{Bmatrix}$$

where $0 < k_1 < k_2 < k_3 < 1$

5. Fuzzy Transportation Problem

The mathematical form of fuzzy transportation problem is as follows

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}\widetilde{x}_{ij}$$

Subject to the constrains

$$\sum_{j=1}^{n}\widetilde{x}_{ij} = \widetilde{a}_i \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \qquad j = 1, 2, \ldots, n$$

where $\tilde{x}_{ij} \geq 0$ for all i and j where $\tilde{a}_i$ is the fuzzy availability of the product at the i^{th} source, $\tilde{b}_j$ is the fuzzy demand of the product at the j^{th} destination, $\tilde{c}_{ij}$ is the fuzzy transporting cost of one unit of the product from the i^{th} source to the j^{th} destination and $\tilde{x}_{ij}$ is the amount of units of the product that should be transported from the i^{th} source to j^{th} destination.

Generally the transportation problem is displayed in the following tabular form

		Destination				
Source		1	2	…	n	Fuzzy Supply
	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1n}$	$\tilde{a}_1$
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2n}$	$\tilde{a}_2$
	…	…	…		…	…
	m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	…	$\tilde{c}_{mn}$	$\tilde{a}_m$
	Fuzzy Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_n$	

The Best Candidates Method (BCM).

BCM's process includes three steps; these steps are shown as follows:

Step 1. Prepare the BCM matrix, if the matrix is unbalanced, then the matrix will be balanced without using the added row or column candidates in solution procedure.

Step 2. Select the best candidate, which is for minimizing problems to the minimum cost, and maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as, the columns must be checked such that if it is not having candidates, so that, the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.

Step 3. Find the combinations by determining one candidate for each row and column, this should be done by starting from the row that has the least candidates, and then delete that row and column. If there are situations that have no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step 3 by determining the next candidate in the row that started from. Compute and compare the summation of candidates for each combination. This is to determine the best combination that gives the optimal solution.

Proposed Method

In this study, we proposed a new solving method for transportation problems by using BCM. The proposed method must operate the as following:

Step 1. We must check the matrix balance, if the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make to supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Step 2. Appling BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.

Step 3. Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row, if it has an element with the lowest cost comparing to the determined element in the chosen combination, then we elect it.

Step 4. Elect the next least cost from the chosen combination and repeat Step 3 until all columns and rows is exhausted.

6. Numerical Example

In this section validity of the ranking method is explained by numerical examples.

Example 6.1. Consider the following Hexadecagonal fuzzy assignment problem which consists of four jobs and four machines. The cost matrix $\tilde{c}_{ij}$ those elements are Hexadecagonal fuzzy numbers. Here, our objective is to find the optimum assignment so as to minimize the cost (or time).

Table 1. Hexadecagonal Fuzzy Assignment Problem.

	D_1	D_2	D_3	Supply
S_1	(1,2,3,4,5,6,7, 8,9,10,11,12, 13,14,15,16)	(0,1,2,3,4,5,6, 7,8,9,10,11,12, 13,14,15)	(1,3,5,7,9,11,13, 15,17,19,21,23, 25,27,29,31)	(1,3,4,5,7,9,10, 12,14,15,16,18, 20,21,22,24)
S_2	(2,4,6,8,10,12, 14,16,18,20,22, 24,26,28,30,32)	(2,3,5,7,9,11,13, 17,19,23,29,31, 35,37,41,43)	(1,4,7,10,13,16, 19,22,25,28,31, 34,37,40,43,46)	(0,4,6,9,11,12, 13,14,15,16,19, 21,23,25,27,29)
S_3	(1,2,3,4,7,10,13, 15,16,17,22,26, 30,34,35,36)	(2,4,6,8,9,13,15, 16,18,20,21,25, 27,28,30,31)	(1,2,3,4,5,7,9, 11,13,17,21,25, 27,31,32,34)	(2,4,8,9,11,13, 16,19,20,22,24, 25,27,29,30,31)
Demand	(1,2,3,4,8,9,10, 12,13,15,17,18, 20,22,23,24)	(0,2,3,5,6,7,9, 10,13,15,16,18, 21,24,29, 34)	(3,4,7,10,11,13, 15,16,18,21,24, 28,30,34,36,44)	

Ranking of Hexadecagonal fuzzy number

In order to find the optimum value of the given Hexadecagonal fuzzy cost given in table 1, first we convert the fuzzy cost into the crisp cost using the proposed ranking method (4.1) as shown in table 2.

Take the values of $k_1 = 0.3, k_2 = 0.5, k_3 = 0.8.$ The ranking of fuzzy numbers is done by using proposed ranking method (4.1).

The crisp assignment problem of the corresponding Hexadecagonal fuzzy assignment problem is given in table 2.

	D_1	D_2	D_3	Supply
S_1	8.5	7.5	16	12.6
S_2	17	20.5	23.5	15.4
S_3	17	15.3	20.6	18

Demand	12.8	13.4	19.8	

Step 1. The given transportation problem is balanced transportation problem.

Step 2. Elect the candidates according to each row and column. By using BCM, we determine the best combination that will produce the lowest total weight of the costs. From the selected candidates, we elect the best candidates and the result from the best candidate method is as follows:

	D_1	D_2	D_3	Supply
S_1	8.5	7.5	16	12.6
S_2	17	20.5	23.5	15.4
S_3	17	15.3	20.6	18
Demand	12.8	13.4	19.8	

Step 3.

	D_1	D_2	D_3	Supply
S_1	8.5	12.6 7.5	16	12.6
S_2	12.8 17	20.5	2.6 23.5	15.4
S_3	17	0.8 15.3	17.2 20.6	18
Demand	12.8	13.4	19.8	

The optimal transportation cost $12.6 \times 7.5 + 12.8 \times 17 + 2.6 \times 23.5 + 0.8 \times 15.3 + 17.2 \times 20.6 = 739.7$

Result and Discussion

The solution of the given transportation problem is obtained by various methods.

Method	Cost
North-West corner Rule	798.3
Least Cost Method	739.7
Vogel's Approximation method	767.5
Best Candidate method	739.7
MODI method	739.7

7. Conclusion

In this paper, a new ranking method is proposed for ranking the Hexadecagonal fuzzy numbers. Fuzzy transportation problem transformed into crisp transportation problem and solved by the best candidate method. Numerical example is presented and the optimal solution is obtained.

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