Advances and Applications in Mathematical Sciences
Volume 21, Issue 3, January 2022, Pages 1463-1477

SOLVING FUZZY UNCONSTRAINED OPTIMIZATION PROBLEMS USING INTERVAL NEWTON'S METHOD

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Abstract

In this paper, unconstrained optimization problems with fuzzy valued functions are considered. Moreover single variable and multivariable fuzzy unconstrained optimization problems using Interval Newton's Method are discussed with illustrations.

1. Introduction

In recent days, we need to optimize the given systems, and to deal with linear and non-linear programming problems. The term optimization means to get the best solution. Interval analysis method was evolved in 1950-1960. On the advent of Computational Mathematics, Interval analysis is a means of representing uncertainty by replacing the single fixed point with an interval. Generally we solve the non-linear programming problems which includes only the crisp numbers in objective and its constrained coefficient. When we handle such natural problems, complications, due to uncertainties and inexactness tangled in the resulting parameters. Since occurrence of the uncertainty and inexactness, it is inadequate to apply the classic method to

2020 Mathematics Subject Classification: 49M15.
Keywords: Newton's method, interval analysis, interval numbers, triangular fuzzy number (tfn), interval arithmetic.
Received November 2, 2021; Accepted November 16, 2021

handle such problems. Fuzzy sets was first introduced by Zadeh in 1965 [15], it has many results of fuzzy sets and it plays an important role to reverse the natural world positions. Zadeh introduced Concept of a Linguistic Variable and its application to approximate reasoning in 1975. Bellman and Zadeh [1] have precisely defined the notation of decision making in a fuzzy environment. Basic theory for triangular fuzzy number is discussed and operations of triangular fuzzy numbers are put forwarded by Nagoorgani et al. [5]. Ganesan et al. introduced a comparative solution of fuzzy unconstrained optimization problems with triangular fuzzy number (2017) (2020). Throughout most of the recent decades, many researchers have designed optimization problems with fuzzy valued objective problems. [Ben et al. [2] Debski [3], Dennis and Shanable [4], Cheng and Kovalyov [8], Timothy [9] and Shi [17]. Moreover, the unconstrained problems are solved by differential calculus. But, here non linear fuzzy unconstrained problems are solved using Newton's Method by establishing interval analysis. Single variable unconstrained optimization techniques using Interval Analysis was introduced by Vereramalai et al. [12]. Sophia porchelvi et al. introduced the Bivariate unconstrained optimization problems using Interval Analysis [6]. Irene Hepzibah et al. introduces the a comparative study on Interval arithmetic operations with Intutionistic fuzzy numbers [14]. In this paper, the Newton Method is used for solving unconstrained optimization problem by interval analysis. This Paper is organized as follows. Section 2, introduces the fuzzy sets, triangular fuzzy number, their arithmetic operations and a ranking method for solving fuzzy unconstrained optimization problem. The interval arithmetic operations for solving the fuzzy unconstrained optimization problems are provided in section 2. Section 3 deals with the Newton's method for unconstrained optimization and Interval Newton's method. Illustrative examples are provided in section 4 and some concluding remarks are given in section 5.

2. Preliminaries

This section provides an introduction to fuzzy unconstrained optimization models and stressed the importance to consider the topics like linear and non linear optimization problems in fuzzy environment using arithmetic operations and provides certain definitions which are related to this research work.

2.1. Basic concepts of fuzzy sets and fuzzy numbers

Definition 1 [10]**.** Let $\Re$ be the set of real numbers and $\widetilde{A} : \Re \to [0, 1]$ be a fuzzy set then we say that $\widetilde{A}$ is a fuzzy number which satisfies the following properties:

(i) 0 is normal, i.e., there exist $x_0 \in \Re$ such that $\widetilde{A}(x_0) = 1$;

(ii) $\widetilde{A}$ is convex, i.e., $\widetilde{A}(tx + (1 - t)y \geq \min\{\widetilde{A}(x), \widetilde{A}(y)\},$ where $x, y \in \Re$ and $t \in [0, 1]$.

(iii) $\widetilde{A}(x)$ is upper semi-continuous on $\Re$, i.e., $\left\{\frac{x}{\widetilde{A}(x)} \geq \alpha\right\}$ is a closed subset of $\Re$ for each $\alpha \in [0, 1]$.

Definition 2 [10]**.** Suppose a fuzzy number $\widetilde{A}$ on $\Re$ is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\widetilde{A} : \Re \to [0, 1]$ has the following characteristics. It is a fuzzy number represents with three points as follows $\widetilde{A} = (a_1, a_2, a_3)$. This representative is interpreted as membership functions and holds the following conditions:

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function.

(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 \text{ for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} \text{ for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} \text{ for } a_2 \leq x \leq a_3 \\ 0 \text{ otherwise.} \end{cases}$$

The pictorial representation of triangular fuzzy number is shown below.

$\mu_{\widetilde{A}}(x)$

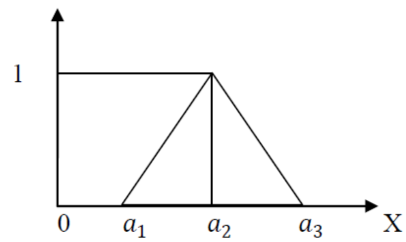

Figure 1.1. Triangular Fuzzy number.

Let $F(\Re)$ to denote the set of all TFNs. The α level set of $\tilde{A}$ is defined as $\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$.

Definition 3 [10]. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number, then the parametric form of the TFN is defined as $\tilde{A} = (a_0, a_*, a^*)$ where $a_* = a_0 - \underline{a}$ and $a^* = \overline{a} - a_0$, $\overline{a}(\alpha) = a_3 - (a_3 - a_2)\alpha]$, $\underline{a}(\alpha) = (a_2 - a_1)\alpha + a_1$, $a_0 = \frac{\underline{a}(\alpha) + \overline{a}(\alpha)}{2}$ here $\alpha = 1$, we get $a_0 = a_2$ where $r \in [0, 1]$.

Definition 4 [10]. Fuzzy numbers represent in terms of their location and fuzziness index functions are denoted as $\tilde{A} = (a_0, a_*, a^*)$. Arithmetic operations for the triangular fuzzy numbers are based upon both location index and fuzzy index functions. The location index numbers follows usual arithmetic and the fuzziness index function follows the lattice rule.

For arbitrary fuzzy numbers $\tilde{A} = (a_0, a_*, a^*)$ and $\tilde{B} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$ the arithmetic operations on

$$\tilde{A} * \tilde{B} = (a_0, a_*, a^*) * (b_0, b_*, b^*) = (a_0 * b_0, \max(a_* * b_*), \max(a^*, b^*))$$

In particular, fuzzy number $\tilde{A} = (a_0, a_*, a^*)$ $\tilde{B} = (b_0, b_*, b^*)$, such that we define

Addition.

$$\tilde{A} + \tilde{B} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max(a_* * b_*), \max(a^* * b^*))$$

Subtraction.

$$\widetilde{A} - \widetilde{B} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max(a_* * b_*), \max(a^* * b^*))$$

Multiplication.

$$\widetilde{A} \times \widetilde{B} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max(a_* * b_*), \max(a^* * b^*))$$

Division.

$$\widetilde{A} \div \widetilde{B} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \max(a_* * b_*), \max(a^* * b^*))$$

Definition 5 [9]**.** One of the ways for solving mathematical programming problems in fuzzy environment is to compare fuzzy numbers. The comparison between fuzzy numbers is done by using a ranking function. An appropriate approach for ordering the elements of $F(\Re)$ is defined by a ranking function $\widetilde{\Re} : F(\Re) \to \Re,$ which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{A} = (a_1, a_2, a_3) = \widetilde{A} = (a_0, a_*, a^*)$ be a triangular fuzzy numbers, then a special form of ranking function is

$$\Re(\widetilde{A}) = \frac{a_0 + a_* + a^*}{3}$$

2.2. Basic concepts of interval valued fuzzy numbers

Definition 6[13]**. Interval Valued Fuzzy Number**

A Interval valued fuzzy number $\tilde{u}$ is pair of $[\underline{u}, \overline{u}]$ of $\underline{u}(r), \overline{u}(r); 0 \le r \le 1$ which satisfies the following conditions:

Let $\tilde{v} = [\underline{v}(r), \overline{v}(r)]$

$x > 0; x = [x\underline{v}(r), x\overline{v}(r)]$ and $x < 0; x = [x\overline{v}(r), x\underline{v}(r)]$

$$\widetilde{V} + \widetilde{U} = [\underline{v}(r) + \underline{u}(r), \overline{v}(r) + \overline{u}(r)]$$

$$\widetilde{V} - \widetilde{U} = [\underline{v}(r) - \underline{u}(r), \overline{v}(r) - \overline{u}(r)]$$

Definition 7[13]**. Interval Valued Triangular Fuzzy Number.**

If $\widetilde{A} = [A^-, A^+], \widetilde{B} = [B^-, B^+]$ are two interval valued triangular fuzzy number and $\widetilde{A}^- = (a_1^-, a_2^-, a_3^-), \widetilde{A}^+ = (a_1^+, a_2^+, a_3^+); \widetilde{B}^- = (b_1^-, b_2^-, b_3^-),$

$\widetilde{B}^{+} = (b_1^{+}, b_2^{+}, b_3^{+})$ then for $\forall k \in R^{+}$

$$\widetilde{A} + \widetilde{B} = [A^{-} + B^{-}, A^{+} + B^{+}]$$

$$\widetilde{A} - \widetilde{B} = [A^{-} - B^{-}, A^{+} - B^{+}]$$

$$k\widetilde{A} = [kA^{-}, kA^{+}] \text{ where } A^{-} \pm B^{-} = (a_1^{-} \pm b_1^{-}, a_2^{-} \pm b_2^{-}, a_3^{-} \pm b_3^{-})$$

$$A^{+} \pm B^{+} = (a_1^{-} \pm b_1^{-}, a_2^{-} \pm b_2^{-}, a_3^{-} \pm b_3^{-})$$

$$kA^{-} = (ka_1^{-}, ka_2^{-}, ka_3^{-}) \text{ and } kA^{+} = (ka_1^{+}, ka_2^{+}, ka_3^{+})$$

Definition 8[6]**. Interval Arithmetic Operations.**

The Interval Arithmetic Operations are given below as explained in [5]. Let $\widetilde{x} = [x_1, x_2], \widetilde{y} = [y_1, y_2]$

Addition.

$$\widetilde{x} + \widetilde{y} = [x_1, x_2] + [y_1, y_2]$$

Subtraction.

$$\widetilde{x} - \widetilde{y} = [x_1, x_2] - [y_1, y_2]$$

Multiplication.

$$\widetilde{x} \cdot \widetilde{y} = [x_1, x_2]\,[y_1, y_2] = [\min(x_1y_1, x_1y_2, x_2y_1, x_2y_2), \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)]$$

Inverse. $\widetilde{x}^{-1} = \dfrac{1}{\widetilde{\widetilde{x}}} = \dfrac{1}{[x_1, x_2]}$.

3. Newton's Method for Unconstrained Fuzzy Optimization

Now we consider an unconstrained fuzzy optimization problem $\min_{x \in X} \widetilde{G}(x)$ where $\widetilde{G} : X \to F(\Re)$ is a fuzzy valued function defined on $X \subseteq \Re^{n}$. Newton method is an iterative method used to determine the root of a function $f(x)$ and an initial guess to the root x_0, improved guesses are given. Newton method for single variable optimization [12] for obtaining the root,

$$x_{k+1} = x_k - \left[\frac{f'_x(x_k)}{f''_x(x_k)}\right] until \mid x_{k+1} - x_k \mid < \in$$

Newton method for two variable optimization [6] for obtaining the root,

$$x_{k+1} = x_k - \left[\frac{f'_x(x_k,\ y_k)}{f''_x(x_k,\ y_k)}\right] until \mid x_{k+1} - x_k \mid < \in$$

$$y_{k+1} = y_k - \left[\frac{f'_y(x_k,\ y_k)}{f''_y(x_k,\ y_k)}\right] until \mid y_{k+1} - y_k \mid < \in$$

In order to apply Newton's method to find the critical points of a function, the function's first and second derivatives must exist. If the second order derivatives at an iteration x_{i+1} is zero, the method fails. The first derivatives of the Newton method only finding the roots of the function. The second derivatives of the Newton method find the maximum or minimum of the given function.

3.1 Interval Newton Method

Interval Newton method is similar to Bijection method it requires a bracketing interval to begin with each iteration generates smaller and smaller interval which are bounded by intersection with previous iterations, The formula [12] is as follows

$$x_{k+1} = \left[m(x_k) - \frac{f'(m(x_k))}{f''^{(x_k)}}\right] \cap (x_k) \tag{3.1.1}$$

where x's are the interval $m(x)$ is the midpoint of interval x, and f is unconstrained optimization function.

For an fuzzy unconstrained optimization problem, equations (3.1.1) takes the form

$$\widetilde{X}_{k+1} = \left[m(\widetilde{X}_k) - \frac{f''(m(\widetilde{X}_k))}{f''^{(\widetilde{X}_k)}}\right] \cap (\widetilde{X}_k) \tag{3.1.2}$$

For Bivariate, the formula [6]

$$X_{k+1} = N_k((X_k, Y_k)) \cap (X_k)$$

where $N_k((X_k, Y_k)) = \left[m(X_k) - \frac{f'_X(m(X_k), m(Y_k))}{f''_X(X_k, Y_k)} \right]$

$$X_{k+1} = \left[m(X_k) - \frac{f'_X(m(X_k), m(Y_k))}{f''_X(X_k, Y_k)} \right] \cap (X_k),\ f''_x(X_k, Y_k) \neq 0 \qquad (3.1.3)$$

and

$$Y_{i+1} = N_k((X_k, Y_k)) \cap (Y_k)$$

where $N_k((X_k, Y_k)) = \left[m(Y_k) - \frac{f'_Y(m(X_k), m(Y_k))}{f''_Y(X_k, Y_k)} \right]$

$$Y_{k+1} = \left[m(Y) - \frac{f'_Y(m(X_k), m(Y_k))}{f''_Y(X_k, Y_k)} \right] \cap (Y_k),\ f''_Y(X_k, Y_k) \neq 0 \qquad (3.1.4)$$

where x's are the interval $m(x)$ is the midpoint of interval x, and f is unconstrained optimization function.

For an fuzzy unconstrained optimization problem, equation (3.1.3) and (3.1.4) takes the form

$$\tilde{X}_{k+1} = N_k((\tilde{X}_k, \tilde{Y}_k)) \cap (\tilde{X}_k)$$

where $N_k((\tilde{X}_k, \tilde{Y}_k)) = \left[m(\tilde{X}_k) - \frac{f'_{\tilde{X}}(m(\tilde{X}_k), m(\tilde{Y}_k))}{f''_{\tilde{X}}(\tilde{X}_k, \tilde{Y}_k)} \right]$

$$\tilde{X}_{k+1} = \left[m(\tilde{X}_k) - \frac{f'_{\tilde{X}}(m(\tilde{X}_k), m(\tilde{Y}_k))}{f''_{\tilde{X}}(\tilde{X}_k, \tilde{Y}_k)} \right] \cap (\tilde{X}_k),\ f''_{\tilde{X}}(\tilde{X}_k, \tilde{Y}_k) \neq 0 \qquad (3.1.5)$$

and

$$\tilde{Y}_{i+1} = N_k((\tilde{X}_k, \tilde{Y}_k)) \cap (\tilde{Y}_k)$$

where $N_k((\tilde{X}_k, \tilde{Y}_k)) = \left[m(\tilde{Y}_k) - \frac{f'_{\tilde{Y}}(m(\tilde{X}_k), m(\tilde{Y}_k))}{f''_{\tilde{Y}}(\tilde{X}_k, \tilde{Y}_k)} \right]$

$$\widetilde{Y}_{k+1} = \left[m(\widetilde{Y}) - \frac{f'_{\widetilde{Y}}(m(\widetilde{X}_k), m(\widetilde{Y}_k))}{f''_{Y}(\widetilde{X}_k, \widetilde{Y}_k)} \right] \cap (\widetilde{Y}_k),\ f''_{Y}(\widetilde{X}_k, \widetilde{Y}_k) \neq 0 \tag{3.1.6}$$

3.2 Algorithms

3.2.1 Algorithm for Single Variable Unconstrained Fuzzy Optimization Problems by Interval Newton's Method

Step 1. Consider the unconstrained optimization problem with triangular fuzzy number coefficients.

Step 2. Convert the triangular fuzzy number coefficients into interval numbers and then to parametric form.

Step 3. Compute first and second derivative for the given function

Step 4. Choose any initial interval points.

Step 5. Solve the system with Fuzzy Interval Newton's Method

$$\widetilde{X}_{k+1} = N_k((\widetilde{X}_k, \widetilde{Y}_k)) \cap (\widetilde{X}_k)$$

Step 6. Continuing this process until $\widetilde{X}_{k+1} - (\widetilde{X}_k) < \in$ or return to Step 5.

3.2.2 Algorithm for Bivariate Unconstrained Fuzzy Optimization Problems by Interval Newton's Method:

Step 1. Consider the unconstrained optimization problem with triangular fuzzy number coefficients.

Step 2. Convert the triangular fuzzy number coefficients into interval numbers and then to parametric form.

Step 3. Compute first and second derivative for the given function.

Step 4. Choose any initial interval points.

Step 5. Solve the system with Fuzzy Interval Newton's Method

$$\widetilde{X}_{k+1} = N_k((\widetilde{X}_k, \widetilde{Y}_k)) \cap (\widetilde{X}_k)$$

$$\widetilde{Y}_{k+1} = N_k((\widetilde{X}_k, \widetilde{Y}_k)) \cap (\widetilde{Y}_k)$$

Step 6. Continuing this process until $\widetilde{X}_{k+1} - (\widetilde{X}_k) < \in$ and

$\tilde{Y}_{k+1} - (\tilde{Y}_k) < \in$ or return to Step 5.

4. Illustrative Examples

Example 4.1. Let us consider the following unconstrained fuzzy optimization problem

$$f(\tilde{X}) = \tilde{X}^3 - 6\tilde{X} + 3$$

$f(\tilde{X}) = (-1, 1, 3)X^3 - (5, 6, 7)X + (2, 3, 4)$ where $x \in \Re$. Using our proposed arithmetic operation first we transform all the triangular number in terms of their location index and fuzziness index functions then the parametric form of the triangular fuzzy number [7] for unconstrained optimization problem is written as

$$f(\tilde{X}) = (1, 2 - 2\alpha, 2 - 2\alpha)\tilde{X}^3 - (6, 1 - 1\alpha, 1 - 1\alpha)\tilde{X} + (3, 1 - 1\alpha, 1 - 1\alpha)$$

$$f(\tilde{X}) = 3(1, 2 - 2\alpha, 2 - 2\alpha)\tilde{X}^2 - (6, 1 - 1\alpha, 1 - 1\alpha)$$

$$f''(X) = 6(1, 2 - 2\alpha, 2 - 2\alpha)\tilde{X}.$$

The interval analysis of Newton method for unconstrained fuzzy optimization is $\tilde{X}_{i+1} = N(\tilde{X}) \cap (\tilde{X}_i)$

Here $N(\tilde{X}) = \left[m(\tilde{X}) - \dfrac{f'(m(\tilde{X}))}{f''(\tilde{X})} \right]$

Let us take the initial interval $\tilde{X}_0 = [([1, 1], [0\alpha, 0\alpha], [0\alpha, 0\alpha]), ([2, 2], [0\alpha, 0\alpha], [0\alpha, 0\alpha])]$ in triangular fuzzy number.

we get $\tilde{X}_1 = [([1.375, 1.375], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]), ([1.4375, 1.4375], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])]$

$\tilde{X}_2 = [([1.41406, 1.41406], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]), ([1.4141, 1.4141], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])]$

$\tilde{X}_3 = [([1.4142, 104142], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$

$([1.4142, 1.4142], [12-12\alpha, 12-12\alpha], [12-12\alpha, 12-12\alpha])]$

Here $\widetilde{X}_i = [([1.4142, 104142], [12-12\alpha, 12-12\alpha], [12-12\alpha, 12-12\alpha]),$ $([1.4142, 1.4142], [12-12\alpha, 12-12\alpha], [12-12\alpha, 12-12\alpha])]$ is the optimal solution for the unconstrained fuzzy optimization problem. When $\alpha = 1$ the optimal value of this problem is -2.6568.

Example 4.2. Let us consider the following unconstrained fuzzy optimization Problem

$$f(\widetilde{X}) = \widetilde{X}^3 - 3\widetilde{X}\widetilde{Y} + \widetilde{Y}^3$$

$f(\widetilde{X}) = (-1, 1, 3)\widetilde{X}^3 - (2, 3, 4)\widetilde{X}\widetilde{Y} + (-1, 1, 3)\widetilde{Y}^3$ where $x \in \Re$. Using our proposed arithmetic operation first we transform all the triangular number in terms of their location index and fuzziness index functions then the given unconstrained optimization problem is written as

$$f(\widetilde{X}, \widetilde{Y}) = (1, 2-2\alpha, 2-2\alpha)\widetilde{X}^3 - (3, 1-1\alpha, 1-1\alpha)\widetilde{X}\widetilde{Y} + (1, 2-2\alpha, 2-2\alpha)\widetilde{Y}^3$$

$$f'(\widetilde{X}) = 3(1, 2-2\alpha, 2-2\alpha)\widetilde{X}^2 - (3, 1-1\alpha, 1-1\alpha)\widetilde{Y}$$

$$f''(X) = 6(1, 2-2\alpha, 2-2\alpha)\widetilde{X}$$

$$f'(\widetilde{Y}) = 3(1, 2-2\alpha, 2-2\alpha)\widetilde{Y}^2 - (3, 1-1\alpha, 1-1\alpha)\widetilde{Y}$$

$$f''(\widetilde{Y}) = 6(1, 2-2\alpha, 2-2\alpha)\widetilde{Y}$$

The interval analysis of Newton method for unconstrained fuzzy optimization is

$$\widetilde{X}_{i+1} = N_k(\widetilde{X}) \cap (\widetilde{X}_i)$$

Here $N_k(\widetilde{X}) = \left[m(\widetilde{X}) - \frac{f'(m(\widetilde{X}))}{f''(\widetilde{X})} \right]$

The interval algorithm for an fuzzy unconstrained optimization problem is

$$\widetilde{X}_{i+1} = N_k((\widetilde{X}_k, \widetilde{Y}_k)) \cap (\widetilde{X}_k)$$

where $N_k((\widetilde{X}_k, \widetilde{Y}_k)) = \left[m(\widetilde{X}_k) - \dfrac{f'_{\widetilde{X}}(m(\widetilde{X}_k), m(\widetilde{Y}_k))}{f''_{\widetilde{X}}(\widetilde{X}_k, \widetilde{Y}_k)} \right]$

$$\widetilde{X}_{k+1} = \left[m(\widetilde{X}_k) - \frac{f'_{\widetilde{X}}(m(\widetilde{X}_k), m(\widetilde{Y}_k))}{f''_{\widetilde{X}}(\widetilde{X}_k, \widetilde{Y}_k)} \right] \cap (\widetilde{X}_k),\ f''_{\widetilde{X}}(\widetilde{X}_k, \widetilde{Y}_k) \neq 0$$

and

$$\widetilde{Y}_{k+1} = N_k((\widetilde{X}_k, \widetilde{Y}_k)) \cap (\widetilde{Y}_k)$$

where $N_k((\widetilde{X}_k, \widetilde{Y}_k)) = \left[m(\widetilde{Y}_k) - \dfrac{f'_{\widetilde{Y}}(m(\widetilde{X}_k), m(\widetilde{Y}_k))}{f''_{\widetilde{Y}}(\widetilde{X}_k, \widetilde{Y}_k)} \right]$

$$\widetilde{Y}_{k+1} = \left[m(\widetilde{Y}_k) - \frac{f'_{\widetilde{Y}}(m(\widetilde{X}_k), m(\widetilde{Y}_k))}{f''_{\widetilde{Y}}(\widetilde{X}_k, \widetilde{Y}_k)} \right] \cap (\widetilde{Y}_k),\ f''_{\widetilde{Y}}(\widetilde{X}_k, \widetilde{Y}_k) \neq 0$$

Let us take the interval $\widetilde{X}_0 = [([1, 1], [0\alpha, 0\alpha], [0\alpha, 0\alpha]), ([2, 2], [0\alpha, 0\alpha], [0\alpha, 0\alpha])]$ $\widetilde{Y}_0 = [([1, 1], [0\alpha, 0\alpha], [0\alpha, 0\alpha]), ([2, 2], [0\alpha, 0\alpha], [0\alpha, 0\alpha])]$ in triangular fuzzy number.

$\widetilde{X}_1 = [([1.1250, 1.1250], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$
$([1.3125, 1.312], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])];$

$\widetilde{Y}_1 = [([1.1250, 1.1250], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$
$([1.3125, 1.312], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])];$

$\widetilde{X}_2 = [([1.1003, 1.1003], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$
$([1.1172, 1.1172], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])];$

$\widetilde{Y}_2 = [([1.1003, 1.1003], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$
$([1.1172, 1.1172], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])];$

$\widetilde{X}_3 = [([1.0599, 1.0599], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$
$([1.0307, 1.0307], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha])];$

$\widetilde{Y}_3 = [([1.0599, 1.0599], [12 - 12\alpha, 12 - 12\alpha], [12 - 12\alpha, 12 - 12\alpha]),$

([1.0307, 1.0307], [12 – 12α, 12 – 12α], [12 – 12α, 12 – 12α])];

Here

$\widetilde{X}_3$ = [([1.0599, 1.0599], [12 – 12α, 12 – 12α], [12 – 12α, 12 – 12α]), ([1.0307, 1.0307], [12 – 12α, 12 – 12α], [12 – 12α, 12 – 12α])];

$\widetilde{Y}_3$ = [([1.0599, 1.0599], [12 – 12α, 12 – 12α], [12 – 12α, 12 – 12α]), ([1.0307, 1.0307], [12 – 12α, 12 – 12α], [12 – 12α, 12 – 12α])]; is the optimal solution for the unconstrained fuzzy optimization problem. When $\alpha = 1$ then the optimal value of this problem is -1

Example 4.3. Let us consider the consider the another numerical example $\widetilde{X} - \widetilde{Y} + 2\widetilde{X}^2 + 2\widetilde{X}\widetilde{Y} + \widetilde{Y}^2 f(\widetilde{X}, \widetilde{Y}) = (-1, 1, 3)\widetilde{X} - (-1, 1, 3)\widetilde{Y} + (1, 2, 3)\widetilde{X}\widetilde{Y}$ $+ (-1, 1, 3)\widetilde{Y}^2$.

Using our proposed arithmetic operation first we transform all the triangular number in terms of their location index and fuzziness index functions then the given unconstrained optimization problem is written as $f(\widetilde{X}, \widetilde{Y}) = (1, 2-2\alpha, 2-2\alpha)\widetilde{X} - (1, 2-2\alpha, 2-2\alpha)\widetilde{Y} + (1, 2-2\alpha, 2-2\alpha)\widetilde{X}^2$ $+ (1, 2-2\alpha, 2-2\alpha)\widetilde{X}\widetilde{Y} + (1, 1-1\alpha, 1-\alpha)\widetilde{Y}^2$

Let us take the interval $\widetilde{X}_0$ = [([0, 0], [0α, 0α], [0α, 0α]), ([0, 0], [0α, 0α], [0α, 0α])] $\widetilde{Y}_0$ = [([0, 0], [0α, 0α], [0α, 0α]), ([0, 0], [0α, 0α], [0α, 0α])] in triangular fuzzy number.

$\widetilde{X}_1$ = [([– 0.25, – 0.25], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α]), ([0.5, 0.5], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α])]

$\widetilde{Y}_1$ = [([– 0.25, – 0.25], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α]), ([0.5, 0.5], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α])]

Continuing this process until we get

$\widetilde{X}_i$ = [([– 1, – 1], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α]), ([1.5, 1.5], [2 – 2α, 2 – 2α], [2 – 2α, 2 – 2α])].

$$\widetilde{Y}_i = [(-1, -1], [2-2\alpha, 2-2\alpha], [2-2\alpha, 2-2\alpha]),$$
$$([1.5, 1.5], [2-2\alpha, 2-2\alpha], [2-2\alpha, 2-2\alpha])].$$

This is the optimal solution of this problem. When $\alpha = 1$ then the optimum solution of this problem is -5

5. Conclusion

This paper, proposed a new method to find the solutions of interval valued fuzzy unconstrained optimization problems. Here triangular interval valued fuzzy number and interval Newton's method is used for solving interval valued fuzzy unconstrained optimization problems and the validity of the proposed method is examined with numerical examples.

Acknowledgement

The authors thank the unanimous referees for their useful suggestions to improve the quality of the research paper.

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