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β^{**} GENERALIZED HOMEOMORPHISMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract

In this paper the concepts of intuitionistic fuzzy β^{**} generalized homeomorphisms in intuitionistic fuzzy topological spaces is introduced. Suitable examples are given in intuitionistic fuzzy topological spaces for this concept. Also some characterizations of intuitionistic fuzzy β^{**} generalized homeomorphisms are provided.

1. Introduction

Zadeh [8] introduced the notion of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. Sudha and Jayanthi [4] introduced the concept of β^{**} generalized closed sets in intuitionistic fuzzy topological spaces. With the extension of the work, in this paper I have introduced the concepts of intuitionistic fuzzy β^{**} generalized homeomorphisms and studied some of their properties.

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2. Preliminaries

Definition 2.1 [1]. An *intuitionistic fuzzy set* (IFS) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

Definition 2.2 [1]. Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_{\sim} = \langle x, 0, 1\rangle$ and $1_{\sim} = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [2]. An *intuitionistic fuzzy topology* (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_{\sim}, 1_{\sim} \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called the *intuitionistic fuzzy topological space* (IFTS) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS) in *X*. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in *X*.

Definition 2.4 [4]. An IFS *A* of an IFTS (X, τ) is said to be an intuitionistic fuzzy β^{**} generalized closed set (IFβ^{**}GCS) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \cap \mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). The complement A^c of an IFβ^{**}GCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy* β^{**} *generalized open set* (IFβ^{**}GOS) in *X*.

Definition 2.5 [5]. An IFTS (X, τ) is an *intuitionistic fuzzy* $\beta^{**}pT_{1/2}$ (IF$\beta^{**}pT_{1/2}$) space if every IF β^{**}GCS is an IFPCS in *X*.

Definition 2.6 [5]. An IFTS (X, τ) is an *intuitionistic fuzzy* $\beta^{**}gT_{1/2}$ (IF$\beta^{**}gT_{1/2}$) space if every IFβ^{**}GCS is an IFGCS in *X*.

Definition 2.7 [5]. A mapping $f : (X, \tau) \to (Y, \sigma)$ is an *intuitionistic fuzzy* β^{**} *generalized irresolute* (IFβ^{**}G irresolute) *mapping* if $f^{-1}(V)$ is an IFβ^{**}GCS in (X, τ) for every IFβ^{**}GCS *V* of (Y, σ).

Definition 2.8 [6]. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy* β^{**} *generalized continuous* (IFβ^{**}G continuous) *mapping* if $f^{-1}(V)$ is an IFβ^{**}GCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.9 [7]. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy* β^{**} *generalized* (*IF*β^{**}G) *closed mapping* if $f(V)$ is an IFβ^{**}GCS in *Y* for every IFCS *V* of *X*.

Definition 2.10 [7]. A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy* β^{**} *generalized* (*IF*β^{**} *G*) *open mapping* if $f(V)$ is an IFβ^{**}GOS in *Y* for each IFOS *A* in *X*.

Definition 2.11 [3]**.** Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) *intuitionistic fuzzy* (IF)*homeomorphism* if both f and f^{-1} are IF continuous mappings.

(ii) *intuitionistic fuzzy* $\alpha(IF\alpha)$ *homeomorphism* if both f and f^{-1} are IFα continuous mappings.

(iii) *intuitionistic fuzzy semi*(IFS)*homeomorphism* if both f and f^{-1} are IFS continuous mappings.

(iv) *intuitionistic fuzzy generalized*(IFG)*homeomorphism* if both f and f^{-1} are IFG continuous mappings.

3.1 Intuitionistic Fuzzy β^{**} Generalized Homeomorphisms

In this section the ideas of intuitionistic fuzzy β^{**} generalized homeomorphisms reexamined with some of their properties.

Definition 3.1. A bijection mapping $f : (X, \tau) \to (Y, \sigma)$ is called on *intuitionistic fuzzy* β^{**} *generalized* (IFβ^{**}G) *homeomorphism* if f is both an IFβ^{**}G continuous mapping and an IFβ^{**}G closed mapping.

Example 3.2. Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFT_s on X and Y respectively. Then (X, τ) is an IFTS. Define a bijection mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is both an IFβ^{**}G continuous mapping and an IFβ^{**}G closed mapping. Therefore f is an IFβ^{**}G homeomorphism.

Theorem 3.3. *Every IF homeomorphism is an IFβ^{**}G homeomorphism but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF homeomorphism. Then f is both an IF continuous mapping and an IF closed mapping and hence f is both an IFβ^{**}G continuous mapping and an IFβ^{**}G closed mapping. Therefore the mapping f is an IFβ^{**}G homeomorphism.

Example 3.4. Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFT_s on X and Y respectively. Define a bijection mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is an IFβ^{**}G homeomorphism but not an IF homeomorphism, since f is not an IF continuous mapping as $G_2^c = \langle y, (0.2_u, 0.4_v), (0.8_u, 0.6_v)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFCS in X, since $\mathrm{cl}(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$.

Theorem 3.5. *Every IF semi homeomorphism is an IFβ^{**}G homeomorphism but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi homeomorphism. Then f is both an IF semi continuous mapping and an IF semi closed mapping. This implies f is both an IFβ^{**}G continuous mapping and an IFβ^{**}G closed mapping. Hence f is an IFβ^{**}G homeomorphism.

Example 3.6. Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFT_s on X and Y respectively. Define a bijection mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFβ^{**}G homeomorphism but not an IF semi homeomorphism, since f is not an IF semi continuous mapping, as $G_2^c = \langle y, (0.2_u, 0.4_v), (0.8_u, 0.6_v)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFSCS in X, since $\mathrm{int}(\mathrm{cl}(f^{-1}(G_2^c))) = G_1 \not\subseteq f^{-1}(G_2^c)$.

Theorem 3.7. *Every IFα homeomorphism is an IFβ^{**}G homeomorphism but not conversely in general.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be an IFα homeomorphism. Then f is both an IFα continuous mapping and an IFα closed mapping. As every IFα continuous mapping is an IFβ^{**} G continuous mapping and every IFα closed mapping is an IFβ^{**} G closed mapping, f is an IFβ^{**} G homeomorphism.

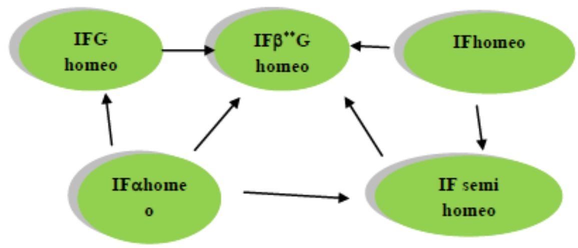
Example 3.8. Let $X=\{a,b\}$, $Y=\{u,v\}$, $G_1=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_2=\langle y,(0.8_u,0.6_v),(0.2_u,0.4_v)\rangle$. Then $\tau=\{0_\sim,G_1,1_\sim\}$ and $\sigma=\{0_\sim,G_2,1_\sim\}$ are IFT_s on X and Y respectively. Define a bijection mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is an IFβ^{**} G homeomorphism but not an IFβ^{**} homeomorphism, since f is not an IFβ^{**} continuous mapping, as $G_2^c=\langle y,(0.2_u,0.4_v),(0.8_u,0.6_v)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFαCS in X, since $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(G_2^c))))=G_1^c\nsubseteq f^{-1}(G_2^c)$.

Theorem 3.9. *Every IFG homeomorphism is an IFβ^{**} G homeomorphism but not conversely in general.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be an IFG homeomorphism. Then f is both an IFG continuous mapping and an IFG closed mapping. As every IFG continuous mapping is an IFβ^{**} G continuous mapping and every IFG closed mapping is an IFβ^{**} G closed mapping, f is an IFβ^{**} G homeomorphism.

Example 3.10. Let $X=\{a,b\}$, $Y=\{u,v\}$, $G_1=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_2=\langle y,(0.8_u,0.6_v),(0.2_u,0.4_v)\rangle$. Then $\tau=\{0_\sim,G_1,1_\sim\}$ and $\sigma=\{0_\sim,G_2,1_\sim\}$ are IFT_s on X and Y respectively. Define a bijection mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is an IFβ^{**} G homeomorphism but not an IFG homeomorphism, since f is not an IFG continuous mapping, as $G_2^c=\langle y,(0.2_u,0.4_v),(0.8_u,0.6_v)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFGCS in X, since $\mathrm{cl}(f^{-1}(G_2^c))=G_1^c\nsubseteq G_1$ whereas $f^{-1}(G_2^c)\subseteq G_1$.

Relation among various types of intuitionistic fuzzy homeomorphisms is given in the following diagram. In this diagram 'homeo' means homeomorphism.



The reverse implications are not true in general in the above diagram.

Theorem 3.11. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a bijective mapping. Then the following are equivalent*:

(i) *f is an IF*β^{**} *G closed mapping*

(ii) f^{-1} *is an IF*β^{**} *G continuous mapping*

(iii) *f is an IF*β^{**} *G open mapping*.

Proof. (i) $\Rightarrow$ (ii) Let B be an IFCS in X. Since f is an IFβ^{**} G closed mapping $f(B) = (f^{-1})^{-1}(B)$ is an IFβ^{**} GCS in Y. This implies f^{-1} is an IFβ^{**} G continuous mapping.

(ii) $\Rightarrow$ (iii) Let A be an IFOS in X. Then by hypothesis $(f^{-1})^{-1}(A) = f(A)$ is an IFβ^{**} GOS in Y. That is f is an IFβ^{**} G open mapping.

(iii) $\Rightarrow$ (i) Let f be an IFβ^{**} G open mapping. Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis $f(A^c) = f(A)^c$ is an IFβ^{**} GOS in Y as f is bijective. Therefore $f(A)$ is an IFβ^{**} GCS in Y. Hence f is an IFβ^{**} G closed mapping.

Theorem 3.12. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be an IF*β^{**} *G homeomorphism then f is an IFG homeomorphism if X and Y are IF*$\beta^{**}gT_{1/2}$ *space*.

Proof. Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFβ^{**}GCS in X, by hypothesis. Since X is an IF$\beta^{**}gT_{1/2}$ space, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping. By hypothesis $f^{-1} : (Y, \sigma) \to (X, \tau)$ is a IFβ^{**}G continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFβ^{**}GCS in Y, by hypothesis. Since Y is an IF$\beta^{**}gT_{1/2}$ space, $f(A)$ is an IFGCS in X. Hence f^{-1} is an IFG continuous mapping. Therefore the mapping f is an IFG homeomorphism.

Theorem 3.13 *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. If f is an IFβ^{**}G continuous mapping then the following are equivalent*:

(i) *f is an IFβ^{**}G closed mapping*

(ii) *f is an IFβ^{**}G open mapping*

(iii) *f is an IFβ^{**}G homeomorphism.*

Proof. The proof is obviously true from the Theorem 3.11.

Remark 3.14. The composition of two IFβ^{**}G homeomorphism need not be an IFβ^{**}G homeomorphism in general.

Example 3.15. Let $X = \{a, b\}, Y = \{u, v\}, z = \{p, q\}, G_1 = \langle x, (0.5_a, 0.6_b), (0.3_a, 0.3_b)\rangle$ and $G_2 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle, G_3 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ and $G_4 = \langle z, (0.2_p, 0.2_q), (0.5_p, 0.8_q)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}, \sigma = \{0_\sim, G_3, 1_\sim\}$ and $\delta = \{0_\sim, G_4, 1_\sim\}$ are IFT$_s$ on X, Y and Z respectively. Now define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$ and $g : (Y, \sigma) \to (Z, \delta)$ by $g(u) = p$ and $g(v) = q$. Then f and g are IFβ^{**}G homeomorphisms. But their composition $g \circ f : (X, \tau) \to (Z, \delta)$ defined by $g(f(a)) = p$ and $g(f(b)) = q$ is not an IFβ^{**}G homeomorphism, since $G_4^c = \langle z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)\rangle$ is an IFCS in Z but $f^{-1}(g^{-1}(G_4^c))$ $= \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle \nsubseteq G_2$ is not an IFβ^{**}GCS in X, as

$\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(g^{-1}(G_4^c))))) \cap \mathrm{int}(f^{-1}(g^{-1}(G_4^c))))) = 1_\sim \nsubseteq G_2$.

Theorem 3.16. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be an IF*β^{**} *G homeomorphism, then f is an IF pre homeomorphism if X and Y are IF*$\beta^{**} pT_{1/2}$ *space.*

Proof. Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFβ^{**} GCS in X, by hypothesis. Since X is an IF$\beta^{**} pT_{1/2}$ space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IF pre continuous mapping. Similarly f^{-1} is also an IF pre continuous mapping. Therefore the mapping f is an IF pre homeomorphism.

4.1 Intuitionistic Fuzzy M-β^{**} Generalized Homeomorphisms

In this section the concept of intuitionistic fuzzy M-β^{**} generalized homeomorphism are investigated with some of their properties.

Definition 4.1. A bijection mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy M-*β^{**} *generalized* (*IFM-*β^{**} *G*) *homeomorphism* if f is both an IFβ^{**} G irresolute mapping and an IFM-β^{**} G closed mapping.

Example 4.2. Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Then (X, τ) is an IFTS. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is both an IFβ^{**} G irresolute mapping and an IFM-β^{**} G closed mapping. Hence f is an IFM-β^{**} G homeomorphism.

Theorem 4.3. *Every IFM-*β^{**} *G homeomorphism is an IF*β^{**} *G homeomorphism but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFM-β^{**} G homeomorphism. Let B be an IFCS in Y. This implies B is an IFβ^{**} GCS in Y. By hypothesis $f^{-1}(B)$ is an IFβ^{**} GCS in X. Hence f is an IFβ^{**} G continuous mapping. Similarly f

is also an IF β^{**} G closed mapping. Hence f is an IF β^{**} G homeomorphism.

Example 4.4. Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is an IF β^{**} G homeomorphism but not an IFM- β^{**} G homeomorphism, since f is not an IF β^{**} G irresolute mapping, as the IFS $A = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.4_v)\rangle$ is an IF β^{**} GCS in Y but $f^{-1}(A)$ is not an IF β^{**} GCS in X, since $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(A)))) \cap \mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(A)))) = 0_\sim \not\subseteq G_1$. whereas $f^{-1}(A) \subseteq G_1$.

Theorem 4.5. *The composition of two IFM-* β^{**} *G homeomorphisms is an IFM-* β^{**} *G homeomorphism in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two IFM- β^{**} G homeomorphisms. Let $A \subseteq Z$ be an IF β^{**} GCS. Then by hypothesis, $g^{-1}(A)$ is an IF β^{**} GCS in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IF β^{**} GCS in X. Therefore $g \circ f$ is an IF β^{**} G irresolute mapping. Now let $B \subseteq X$ be an IF β^{**} GCS in X. Then by hypothesis, $f(B)$ is an IF β^{**} GCS in Y and also $g(f(B))$ is an IF β^{**} GCS in Z. This implies $g \circ f$ is an IFM- β^{**} G closed mapping. Hence $g \circ f$ is an IFM- β^{**} G homeomorphism.

Theorem 4.6. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a bijective mapping. If f is an IF* β^{**} *G irresolute mapping, then the following are equivalent*:

(i) *f is an IFM-* β^{**} *G closed mapping*

(ii) *f is an IFM-* β^{**} *G open mapping*

(iii) *f is an IFM-* β^{**} *G homeomorphism.*

Proof. Straight forward.

Theorem 4.7. *The set of all IFM-*β^{**}*G homeomorphisms in an IFTS* (X, τ) *is a group under the composition of maps.*

Proof. Define a binary operation $*$: IFM-β^{**}G homeomorphism (X) x IFM-β^{**}G homeomorphism (X) by $f * g = g \circ f$ for every $f, g \in$ IFM-β^{**}G homeomorphism (X) and $\circ$ is the usual operation of composition of maps. Since $g \in$ IFM-β^{**}G homeomorphism (X) and $g \in$ IFM-β^{**}G homeomorphism (X), by Theorem 4.5, $g \circ f \in$ IFM-β^{**}G homeomorphism (X). We know that the composition of maps is associative. The identity map $I : (X, \tau) \to (X, \tau)$ belonging to IFM-β^{**}G homeomorphism (X) is the identity element. If $f \in$ IFM-β^{**}G homeomorphism (X), then $f^{-1} \in I$FM-β^{**}G homeomorphism (X). Therefore $f \circ f^{-1} = f^{-1} \circ f = I$ and so the inverse exists for each element of IFM-β^{**}G homeomorphism (X). Hence (IFM-β^{**}G homeomorphism $(X), o)$ is a group under the composition of maps.

Theorem 4.8. *Let* $f : X \to Y$ *be an IFM-*β^{**}*G homeomorphism. Then f induces an isomorphism from the group IFM-*β^{**}*G homeomorphism* (X) *onto the group IFM-*β^{**}*G homeomorphism* (Y).

Proof. Using *f*, we define a map $\phi_f : h(X) \to h(Y)$ by $\phi_f(h) = f \circ h \circ f^{-1}$ for every $h \in$ IFM-β^{**}G homeomorphism (X). Then ϕ_f is a bijection. Also for all $h_1, h_2 \in$ IFM-β^{**}G homeomorphism (X), $\phi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1}$ $= (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \phi_f(h_1) \circ \phi_f(h_2)$. This implies ϕ_f is a homeomorphism and so ϕ_f is an isomorphism induced by *f*.

Theorem 4.9. *If* $f : (X, \tau) \to (Y, \sigma)$ *is an IFM-*β^{**}*G homeomorphism, where X and Y are IF*$\beta^{**}pT_{1/2}$ *spaces, then* $\text{pcl}(f^{-1}(B)) = f^{-1}(\text{pcl(B)})$ *for every IFS B in Y.*

Proof. Since f is an IFM-β^{**} G homeomorphism, f is an IFβ^{**} G irresolute mapping. Consider an IFS B in Y. Clearly $\mathrm{pcl(B)}$ is an IFβ^{**} GCS in Y. Since X is an IF$\beta^{**}pT_{1/2}$ space, $f^{-1}(\mathrm{pcl(B)})$ is an IFPCS in X. Now, $f^{-1}(B) \subseteq f^{-1}(\mathrm{pcl(B)})$. We have $\mathrm{pcl}(f^{-1}(B)) \subseteq \mathrm{pcl}(f^{-1}(\mathrm{pcl(B)})) = f^{-1}(\mathrm{pcl(B)})$. This implies $\mathrm{pcl}(f^{-1}(B) \subseteq f^{-1}(\mathrm{pcl(B)}) \ldots (*)$.

Again since f is an IFM-β^{**} G homeomorphism, f^{-1} is IFβ^{**} G irresolute mapping, since $\mathrm{pcl}(f^{-1}(B))$ is an IFβ^{**} GCS in X, $(f^{-1})^{-1}(\mathrm{pcl}(f^{-1}(B)))$ $= f(\mathrm{pcl}((f^{-1}(B)))$ is an IFβ^{**} GCS in Y. Now $B \subseteq (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}$ $(\mathrm{pcl}(f^{-1}(B))) = f(\mathrm{pcl}((f^{-1}(B)))$. Therefore $\mathrm{pcl(B)} \subseteq \mathrm{pcl}(f(\mathrm{pcl}(f^{-1}(B))))$ $= f(\mathrm{pcl}(f^{-1}(B)))$, since Y is an IF$\beta^{**}pT_{1/2}$ space.

Hence $f^{-1}(\mathrm{pcl(B)}) \subseteq f^{-1}(f(\mathrm{pcl}(f^{-1}(B)))) \subseteq \mathrm{pcl}(f^{-1}(B))$. That is $f^{-1}(\mathrm{pcl(B)}) \subseteq \mathrm{pcl}(f^{-1}(B)) \ldots (**)$

Thus from $(*)$ and $(**)$ we get $\mathrm{pcl}((f^{-1}(B)) = f^{-1}(\mathrm{pcl(B)})$ and hence the proof.

Corollary 4.10. *If $f : (X, \tau) \to (Y, \sigma)$ is an IFM-β^{**} G homeomorphism, where X and Y are IF$\beta^{**}pT_{1/2}$ spaces, then $pcl(f(B)) = f(pcl(B))$ for every IFS B in X.*

Proof. Since f is an IFM-β^{**} G homeomorphism, f^{-1} is also an IFM-β^{**} G homeomorphism. Therefore by Theorem 4.9, $pcl((f^{-1})^{-1}(B)) = (f^{-1})^{-1}$ $(pcl(B))$ for every $B \subseteq X$. That is $pcl(\mathrm{f(B)}) = f(pcl(B))$ for every IFS B in X.

Corollary 4.11. *If $f : (X, \tau) \to (Y, \sigma)$ is an IFM-β^{**} G homeomorphism, where X and Y are IF$\beta^{**}pT_{1/2}$ spaces, then $pint(\mathrm{f(B)}) = f(pint(B))$ for every IFS B in X.*

Proof. For any IFS $B \subseteq X$, $\mathrm{pint(B)} = (\mathrm{pcl}(B^c))^c$. By corollary 4.10,

$f(\text{pint(B)}) = (f(\text{pcl}(B^c)))^c = (f(\text{pcl}(B^c)))^c = (pcl(f(B^c)))^c = \text{pint}(f(B^c))^c$ $\text{pint}f(B)$.

Corollary 4.12. *If $f : (X, \tau) \to (Y, \sigma)$ is an IFM-β^{**} G homeomorphism, where X and Y are IF$\beta^{**}pT_{1/2}$ spaces, then $pint(f^{-1}(B)) = f^{-1}(pint(B))$ for every IFS B in Y.*

Proof. The proof is trivial.

Conclusion

Thus the ideas of intuitionistic fuzzy β^{**} generalized homeomorphisms and intuitionistic fuzzy M-β^{**} generalized homeomorphisms are studied with some of its properties. And some characterizations of intuitionistic fuzzy β^{**} generalized homeomorphisms are discussed.

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