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ANTIMAGIC LABELING ON CERTAIN CLASSES OF SUBDIVIDED SHELL GRAPHS

J. JEBAJESINTHA and N. K. VINODHINI

PG Department of Mathematics
Women's Christian College
University of Madras, Chennai
E-mail: jjesintha_75@yahoo.com

Department of Mathematics
Anna Adarsh College for Women
University of Madras, Chennai
and
Part- Time Research Scholar
PG Department of Mathematics
Women's Christian College
University of Madras, Chennai
E-mail: vinookaran@yahoo.co.in

Abstract

Antimagic labeling was introduced by Hartsfield and Ringel [7]. Antimagic labeling of a graph $G = (V, E)$ with p vertices and q edges is an edge labeling which maps E to $\{1, 2, \ldots, q\}$ such that the resulting vertex labels are distinct, where the vertex label is defined to be sum of the labels of the edges incident with the vertex. In this paper we obtain the anitmagic labeling for a few classes of subdivided shell graphs.

1. Introduction

Alexander Rosa [10] pioneered the use of labeling techniques as tools to decompose the complete graph into isomorphic subgraphs. In particular β-labelings evolved as a means to attack Ringel Conjecture that K_{2n+1} can be decomposed into $2n+1$ subgraphs that are all isomorphic to a given tree

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with n edges. The notion of antimagic labeling was introduced by Hartsfield and Ringel [7]. It was conjectured by Hartsfield and Ringel that every connected graph other than K_2 has an antimagic labeling. An antimagic labeling of a graph with q edges and p vertices is a bijection from the set of edges to the set of integers $\{1, 2, \ldots, q\}$ where in all vertex labels are pairwise distinct. Here we note that a vertex label is the sum of labels of all edges incident with the vertex. A graph is antimagic if it has an antimagic labeling. Ahmad et al. [1] have obtained the antimagic labelings of disjoint union of plane graphs. Arumugam et al. [2] have proved that generalised pyramid graphs are antimagic. Baca et al. [3] have discussed about the antimagic labelings of certain families of connected graphs, circulant graphs and regular graphs. Barrus [4] has discussed antimagic labeling and canonical decomposition of graphs. Graphs that admit antimagic labeling techniques are widely discussed in Gallian [6]. Antimagic labelings finds its applications in surveillance security systems, designing printed circuit boards, coding theory, completely separating systems and in psychology.

2. Preliminaries

In this section we discuss few definitions used in this paper.

Definition 1. Let $G_1, G_2, \ldots, G_n$ be $n(\geq 2)$ copies of a graph G. We denote by $G(n)$ the graph obtained by adding an edge to G_i and $G_{i+1}, i = 1, 2, \ldots, (n-1)$ and we call $G(n)$ the path union [11] of n copies of the graph.

Definition 2. Shell graph was introduced by Deb and Limaye [5]. Shell graph is a cycle C_n with $(n-3)$ chords, sharing a common end point called the apex. In other words shell graph is the join of complete graph K_1 and path P_n with n vertices. See Figure 1.

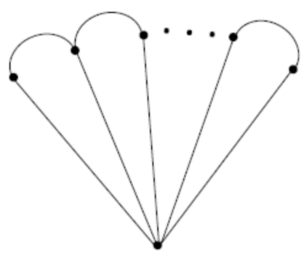

Figure 1. Shell Graph.

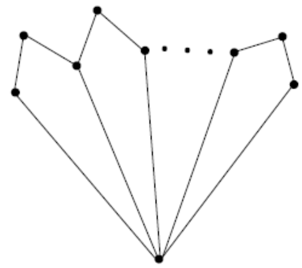

Figure 2. Subdivided Shell Graph.

Definition 3. A subdivided shell graph [8] is obtained from the shell graph $G = K_1 \vee P_n$ by subdividing the edges in the path P_n of the shell graph. See Figure 2.

Definition 4. Let $G_1, G_2, \ldots, G_m$ be connected graphs. Then the cycle of graphs $C(G_1, G_2, \ldots, G_m)$ [9] is obtained by adding an edge joining G_i to G_{i+1} for $i = 1, 2, \ldots, m-1$ and an edge joining G_m to G_1. When the m graphs are isomorphic to G. It is denoted as $C(m.G)$.

3. Main Section

In this section we prove that few classes of subdivided shell graphs are antimagic.

Theorem 1. *One point union of Subdivided Shell graph is antimagic.*

Proof. Let $G^{(m)}$ be the graph formed by taking one point union of 'm' copies of subdivided shells each with exactly 'n' subdivisions. Denote the apex vertex as u_o. The vertices introduced for the subdivision of the path of shell are denoted as $v_1, v_2, \ldots, v_{mn}$ in the anticlockwise direction.

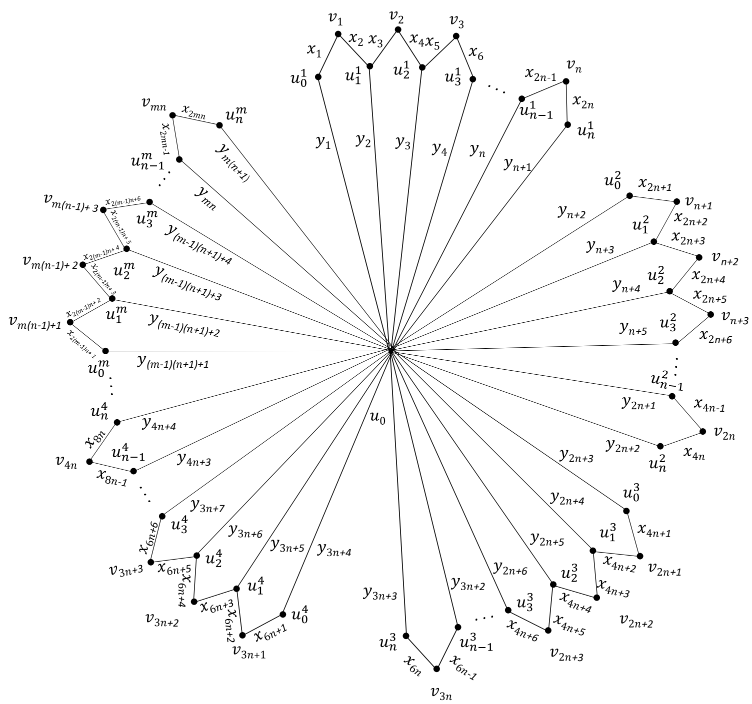

Figure 3. One Point Union of Subdivided Shell Graph.

The vertices in the first copy of the shell adjacent to the apex vertex are denoted as $u_0^1, u_1^1, \ldots, u_n^1$. The vertices in the second copy of the shell adjacent to the apex vertex are denoted as $u_0^2, u_1^2, \ldots, u_n^2$. The vertices in the m^{th} copy of the shell adjacent to the apex vertex are denoted as $u_0^m, u_1^m, \ldots, u_n^m$. Note that G has $2mn + m + 1$ vertices and $3mn + m$ edges. The edges in the subdivided shell non adjacent to the apex, are denoted as $x_1, x_2, \ldots, x_{2mn}$ in the anticlockwise direction. The edges in the subdivided shell adjacent to the apex, are denoted as $y_1, y_2, \ldots, y_{m(n+1)}$ in the anticlockwise direction. See Figure 3. The edge labelings $\zeta : E(G^{(m)}) \to \{1, 2, \ldots, (3mn + n)\}$ are as follows;

$$\zeta(x_\lambda) = \begin{cases} \frac{(\lambda+1)}{2}, & \lambda = 1, 3, \ldots, 2mn-1 \\ mn + \frac{\lambda}{2}, & \lambda = 2, 4, \ldots, 2mn \end{cases}$$

$$\zeta(y_\lambda) = 3mn + m + \lambda - 1, \lambda = 1, 2, \ldots, m(n+1)$$

We observe that the edge labeling function defined above generate distinct edge labels for all $3mn + m$ edges satisyfying the condition for antimagic labeling. The induced vertex labels are as follows;

$$\xi(v_\lambda) = mn + 2\lambda, \lambda = 1, 2, \ldots, mn$$

$$\xi(u_\lambda^\mu) = 4mn - n + m + \lambda + (n-1)\mu + 2, \lambda = 1, 2, \ldots, n-1, \mu = 1, 2, \ldots, m$$

$$\xi(u_0^\mu) = 3mn + m - \mu + 2, \mu = 1, 2, \ldots, m$$

$$\xi(u_n^\mu) = 4mn + m - \mu + 1, \mu = 1, 2, \ldots, m$$

$$\xi(u_0) = \frac{1}{2} m[(n+1)(5mn + m + 1)]$$

Thus entire $3mn + m$ vertices are labeled. We observe that the vertex labels are distinct. Thus one point union of subdivided shell graphs are antimagic.

Corollary. *One point union of two copies of subdivided shell graph is called a subdivided uniform shell bow graph. Thus in the above theorem if we take* $m = 2$, *we obtain that subdivided uniform shell bow graph is also antimagic.*

Theorem 2. *Subdivided shell flower graph is antimagic.*

Proof. Let G be the graph formed by taking one point union of 'm' copies subdivided shells each with exactly 'n' subdivisions. Denote the apex vertex as u_0. The vertices introduced for the subdivision of the path of shell are denoted as $v_1, v_2, \ldots, v_{mn}$ in the anticlockwise direction. The vertices in the first copy of the shell adjacent to the apex vertex are denoted as $u_0^1, u_1^1, \ldots, u_n^1$. The vertices in the second copy of the shell adjacent to the apex vertex are denoted as $u_0^2, u_1^2, \ldots, u_n^2$. The vertices in the m^{th} copy of the shell adjacent to the apex vertex are denoted as $u_0^m, u_1^m, \ldots, u_n^m$. Denote the pendant vertices of the flower as $w_1, w_2, \ldots, w_m$. The edges in the subdivided shell non adjacent to the apex, are denoted as $x_1, x_2, \ldots, x_{2mn}$ in the anticlockwise direction. The edges in the subdivided shell adjacent to the apex, are denoted as $y_1, y_2, \ldots, y_{mn+1}$ in the anticlockwise direction. The edges adjacent to pendant vertices are labeled as $z_1, z_2, \ldots, z_m$. Note that G has $2mn + 2m + 1$ vertices and $3mn + 2m$ edges. See Figure 4.

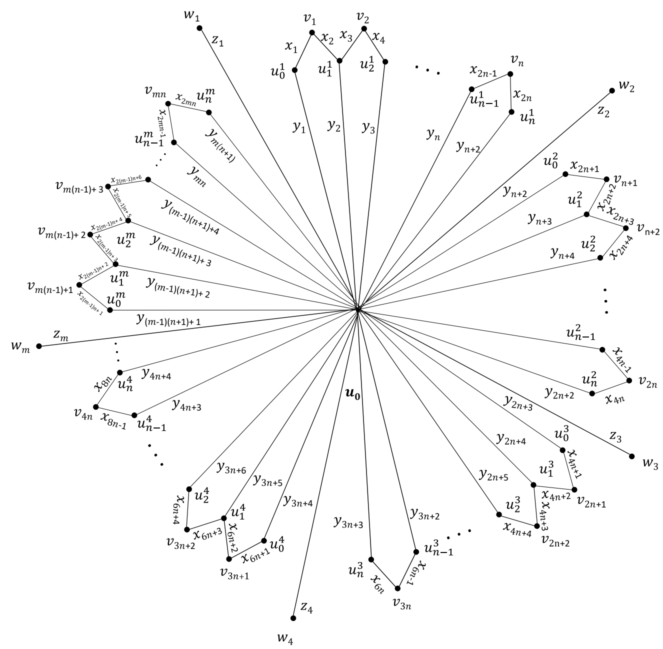

Figure 4. Subdivided Shell Flower Graph.

The edge labelings $\zeta : E(G) \to \{1, 2, \ldots, (3mn + n)\}$ are as follows.

$$\zeta(x_\lambda) = \begin{cases} \dfrac{(\lambda + 1)}{2}, & \lambda = 1, 3, \ldots, 2mn - 1 \\ mn + m + \dfrac{\lambda}{2}, & \lambda = 2, 4, \ldots, 2mn \end{cases}$$

$$\zeta(y_\lambda) = 3mn + 2m - \lambda + 1, \; \lambda = 1, 2, \ldots, m(n + 1)$$

$$\zeta(z_\lambda) = \lambda, \; \lambda = 1, 2, \ldots, m$$

We observe that the edge labeling function defined above generate distinct edge labels for all $3mn + 2m$ edges satisfying the condition for antimagic labeling. The induced vertex labels are as follows;

$$\xi(v_\lambda) = mn + 2m + 2\lambda,\ \lambda = 1, 2, \ldots, mn$$

$$\xi(u_\lambda^\mu) = 4mn - n + 4m + \lambda + (n-1)\mu + 2,\ \lambda = 1, 2, \ldots, n-1,\ \mu = 1, 2, \ldots, m$$

$$\xi(u_0^\mu) = 3mn + 3m - j + 2,\ \mu = 1, 2, \ldots, m$$

$$\xi(u_n^\mu) = 4mn + 3m - j + 1,\ \mu = 1, 2, \ldots, m$$

$$\xi(u_0) = \frac{1}{2} m[(n+1)\,(5mn + m + 1) + (m+1)]$$

$$\zeta(w_\lambda) = \lambda,\ \lambda = 1, 2, \ldots, m$$

Thus entire $2mn + 2m + 1$ vertices are labeled. We observe that the vertex labels are distinct. Thus subdivided shellflower graphs are antimagic.

Theorem 3. *Path of subdivided shell graph admits antimagic labeling*.

Proof. Let $G(m)$ be the path union of 'm' copies subdivided shells each with exactly 'n' subdivision. The vertices introduced for the subdivision of the path of shell are denoted as $v_1, v_2, \ldots, v_{mn}$ in the anticlockwise direction. The vertices on the path are denoted as $w_1, w_2, \ldots, w_m$. The vertices in the first copy of the subdivided shell adjacent to the vertex w_1 on the path are denoted as $u_0^1, u_1^1, \ldots, u_n^1$. The vertices in the second copy of the subdivided shell adjacent to the vertex w_2 on the path are denoted $u_0^2, u_1^2, \ldots, u_n^2$. The vertices in the m^{th} copy of the shell adjacent to the vertex w_m on the path are denoted as $u_0^m, u_1^m, \ldots, u_n^m$. The edges of the subdivided shell non adjacent to the apex are denoted as $x_1, x_2, \ldots, x_{2mn}$ in the anticlockwise direction. The edges in the subdivided shell adjacent to the path, are denoted as $y_1, y_2, \ldots, y_{m(n+1)}$ in the anticlockwise direction. The edges on the path be denoted as $z_1, z_2, \ldots, z_{m-1}$. Note that G has $2mn + 2m$ vertices and $3mn + 2m - 1$ edges. See Figure 5.

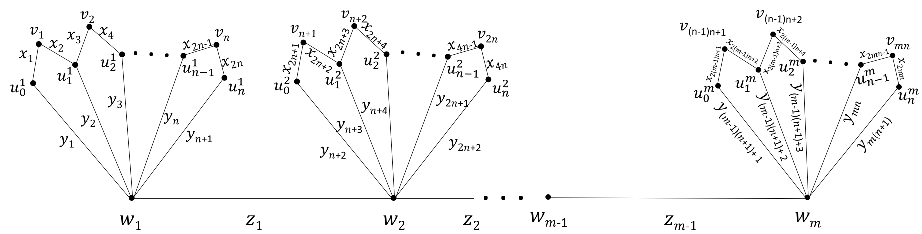

Figure 5. Path Union of 'm' copies of Subdivided Shell Graph.

The edge labelings $\zeta : E(G) \to \{1, 2, \ldots, (3mn + n)\}$ are as follows;

$$\zeta(x_\lambda) = \begin{cases} \frac{(\lambda + 1)}{2}, & \lambda = 1, 3, \ldots, 2mn - 1 \\ mn + \frac{\lambda}{2}, & \lambda = 2, 4, \ldots, 2mn \end{cases}$$

$$\zeta(y_\lambda) = 3mn + 2m + \lambda - 1, \lambda = 1, 2, \ldots, m(n + 1)$$

$$\zeta(z_\lambda) = 3mn + m + \lambda, \lambda = 1, 2, \ldots, m - 1.$$

We observe that the edge labeling function defined above generate distinct edge labels for all $3mn + m$ edges satisfying the condition for antimagic labeling. The induced vertex labels are as follows;

$$\xi(v_\lambda) = mn + 2\lambda, \lambda = 1, 2, \ldots, mn$$

$$\xi(u_\lambda^\mu) = 4mn - n + m + \lambda + (n - 1)\mu + 2, \lambda = 1, 2, \ldots, n - 1, \mu = 1, 2, \ldots, m$$

$$\xi(u_0^\mu) = 3mn + m - \mu + 2, \mu = 1, 2, \ldots, m$$

$$\xi(u_n^\mu) = 4mn + m - \mu + 1, \mu = 1, 2, \ldots, m$$

$$\zeta(w_\lambda) = (n + 3)(3nm + m) + m + 1 - \frac{n(n + 1)}{2}$$

$$\zeta(w_\mu) = (n + 1)[3nm + m + \mu(n + 1) - 1] - \frac{n(n + 1)}{2}, \mu = 2, 3, \ldots, m$$

Thus entire $2mn + 2m$ vertices are labeled. We observe that the vertex labels are distinct. Thus path union of subdivided shell graphs are antimagic.

Theorem 4. *Cycle of subdivided shell graph admits antimagic labeling.*

Proof. Let $C(m \cdot G)$ be the cycle of subdivided shell graph formed by attaching 'm' copies subdivided shells each with exactly 'n' subdivisionson each vertex of the cycle C_m. The vertices introduced for the subdivision of the path of shell are denoted as $v_1, v_2, \ldots, v_{mn}$ in the anticlockwise direction. The vertices on the cycle are denoted as $w_1, w_2, \ldots, w_m$.

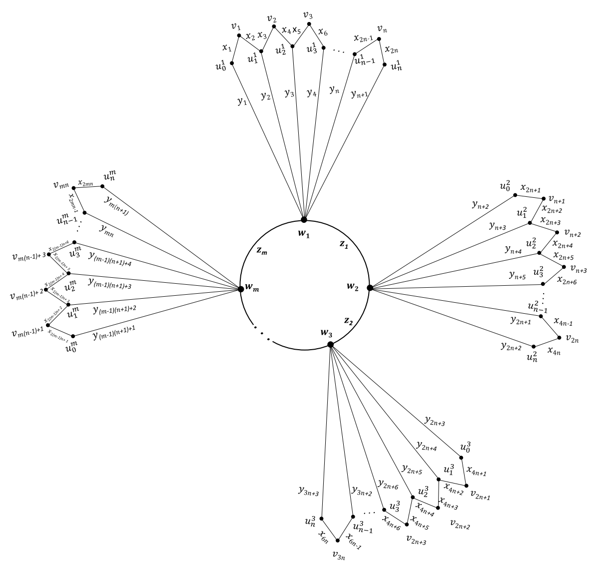

Figure 6. Cycle of Subdivided Shell Graph.

The vertices in the first copy of the subdivided shell adjacent to the vertex w_1 on the cycle are denoted as $u_0^1, u_1^1, \ldots, u_n^1$. The vertices in the second copy of the subdivided shell adjacent to the vertex w_2 on the cycle are denoted as $u_0^2, u_1^2, \ldots, u_n^2$. The vertices in the m^{th} copy of the shell adjacent to the vertex w_m on the cycle are denoted as $u_0^m, u_1^m, \ldots, u_n^m$. Note that G has $2mn + 2m$ vertices and $3mn + 2m$ edges. The edges of the subdivided shell non adjacent to the apex are denoted as $x_1, x_2, \ldots, x_{2mn}$ in the anticlockwise direction. The edges in the subdivided shell adjacent to the cycle, are denoted as $y_1, y_2, \ldots, y_{m(n+1)}$ in the anticlockwise direction. See Figure 6. The edges on the cycle be denoted as $z_1, z_2, \ldots, z_m$.

The edge labelings $\zeta : E(G) \to \{1, 2, \ldots, (3mn + n)\}$ are as follows

$$\zeta(x_\lambda) = \begin{cases} \frac{(\lambda+1)}{2}, & \lambda = 1, 3, \ldots, 2mn-1 \\ mn + \frac{\lambda}{2}, & \lambda = 2, 4, \ldots, 2mn \end{cases}$$

$$\zeta(y_\lambda) = 3mn + m + \lambda - 1, \lambda = 1, 2, \ldots, m(n+1)$$

$$\zeta(z_\lambda) = 3mn + 2m - \lambda + 1, \lambda = 1, 2, \ldots, m$$

We observe that the edge labeling function defined above generate distinct edge labels for all $3mn + 2m$ edges satisfying the condition for antimagic labeling. The induced vertex labels are as follows;

$$\xi(v_\lambda) = mn + 2\lambda, \lambda = 1, 2, \ldots, mn$$

$$\xi(u_\lambda^\mu) = 4mn - n + m + \lambda + (n-1)\mu + 2, \lambda = 1, 2, \ldots, n-1, \mu = 1, 2, \ldots, m$$

$$\xi(u_0^\mu) = 3mn + m - \mu + 2, \mu = 1, 2, \ldots, m$$

$$\xi(u_n^\mu) = 4mn + m - \mu + 1, \mu = 1, 2, \ldots, m$$

$$\zeta(w_1) = (n+3)(3nm+m) + m + 1 - \frac{n(n+1)}{2}$$

$$\zeta(w_\mu) = (n+1)[3nm + m + \mu(n+1) - 1] - \frac{n(n+1)}{2}, \mu = 2, 3, \ldots, m$$

Thus entire $2mn + 2m$ vertices are labeled. We observe that the vertex labels are distinct. Thus cycle of subdivided shell graphs are antimagic.

4. Conclusion

In this paper we have obtained the antimagic labeling of certain classes of subdivided shell graphs.

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