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ANALYSE THE AFFECT OF PARTICLE SIZE ON THE ROTATING DISK

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Abstract

The secondary stage deformation in rotating disk is investigated in current study by taking different particle size. It is investigated that the radial stress is maximum at central part in comparison to inner and outer radii and tangential stress is higher at internal part radii then decreases at outer radii. In case of radial strain, the radial strain increases with increases in the particle size at inner radii and decreases at outer part of disk. Tangential strain is maximum at inner radii then decreases at outer radii. Mathematical modeling is used to calculate the results and graphs are drawn to show the results.

Introduction

Composite material is made from two or more materials combined together. There are several applications of composite material in different fields automobile area: combustion chamber, engine, cylinder linear, brake rotors, flywheels, racing car brakes, pulleys etc. In commercial and industry area: computer hard disk drive, needles for carpet weaving machine, laptop cases, electric motors, MRI scanner cryonics tubes, drilling motor shift, artificial ligaments, crane components, X-ray tubes, heart values, helmets. In

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aerospace equipment and structures space structure: rocket nozzle, heat exchanges panels, wind tunnel blades, engine parts, solar panels, turbine motor, turbine wheels, space shuttle. In aircraft and missile structures are wings, rotating launcher, engines wings, drive shaft, and helicopter components. In nuclear reactor are storage casks for spent fuel rods from nuclear reactors, Also in fields of sports: tennis rackets, golf shafts, racing bicycles frame, fishing rod and pools cues. Rotating disk provide an area of investigate in finite use in revolving machinery, flywheels, turbo generator, automotive baking system and computer disk drives.

Gupta et al. (2003) analysis stress distribution along the radial and tangential in a rotating disk for combinations of material parameter including size and particle content at temperature 623K. It has been found that tangential stress increases as move from internal to external part. The radial stress does not vary possibly for different combinations of material parameters for rotating disk. The tangential as well as radial strain rate in the disk decreases significantly with increasing particle content as well as decreasing particle size.

Gupta et al. (2004) investigate Sherby's constituted creep model give better results than Norton's to describe the creep behavior of composite material in a isotropic rotating disk at the steady state. The radial/tangential stress and strain has been found for different combination of material parameter such as size and content of SiCP and temperature. They compare the experiment result of Whal's with present study. It has been found that radial along with tangential strain rates reduces with reduces particle size, increasing particle content and decreasing temperature.

Singh et al. (2017) analysis thermal gradient on stable state creep response in rotating disk. The disk is assumed to operate at uniform temperature 625K, with thermal gradient of 52K i.e. 658K at inner and 588K at the outer part of the disk and with thermal gradient of 110K i.e. 623K inner and 563K outer radii with volume 10%. They observed that in isotropic disks tangential stresses is little higher near the internal radii and slightly decreases near the external radii in presence of thermal gradient as compare to disk without temperature gradient.

Singh et al. (2018) compared different properties of composite material and determined that Al-SiC have excessive tensile, correct fatigue, high melting factor, excessive thermal and electric conductivity by comparing different composite and monolithic material. It has been found that Al- SiC is the best and cheap material for used.

Gupta et al. (2019) investigate the value of stress and strain in the rotating disk at various angular speed $\omega = 13000$ and $\omega = 15000$ at constant temperature $T = 623\text{K}$ and $V = 10\%$. They observed that by increasing the angular speed, radial and tangential stress and strain is increased over entire radii in the isotropic disk operate at the different angular speed.

In this paper the disk is operate under uniform temperature $T = 625\text{K}$ with volume $V = 20\%$ and particle size $1.7\mu m$, $14.5\mu m$ and $42.5\mu m$. The internal radii is 32.35mm and external radii is 154.4mm with thickness 26.4mm of composite disk.

2. Estimation of Creep Parameters

The secondary stage deformation Al-SiCP composite of different composition is described in terms of Sherby threshold stress based model given by

$$\dot{\varepsilon} = [M(\bar{\sigma} - \sigma_0)]^8 \tag{1}$$

where $M = \frac{1}{E}\left[\frac{AD_L\lambda^3}{|\vec{b_r}|^5}\right]^{1/8}$,

where, σ effective stress, "M" material creep constant, D_L lattice diffusivity, "A" constant, $|\vec{b_r}|$ magnitude of Burger's vector, "E" young's modulus, λ sub grain size, σ_0 threshold stress.

The standards creep parameters M, σ_0 obtain from the creep results of Al-SiCP composite disk given by (Pandey et. al, 1992)

$$InM = -34.91 + 0.2112InP + 4.89InT - 0.591InV \tag{2}$$

$$\sigma_0 = -0.02050P + 0.01378T + 1.033V - 4.9695 \tag{3}$$

Where P is particle size, T is temperature and V is volume content.

3. Mathematical Formulation

Taking $AlSiC_p$ particulate composite disk with thickness "h" having inner radii "a" and outer radii "b" resp.

From symmetry consideration, principal stresses are in radial, tangential and axial directions. The following assumption are made for study

- Secondary stage of stress is estimate.
- Elastic deformations can be ignored as very less in comparison to creep deformation.
- The thickness is very less in comparison to diameter and can be ignored.

For biaxial state of stress, the general constitutive equations. Used for deformation in a isotropic composite obtain in following type as suggestion frame is use as along principal directions r, θ and z

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[2\sigma_r - \sigma_\theta]$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[2\sigma_\theta - \sigma_r]$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[-(2\sigma_r - \sigma_\theta)]. \tag{4}$$

Where, ε_r, ε_θ, $\dot{\varepsilon}_z$ and σ_r, σ_θ, σ_z are strain rates and stresses rates resp. also within the direction r, θ, z as indicate by subscripts.

Effective stress depend on Von Misses Criterion (1913), used for biaxial condition of stress and effective stress, $\bar{\sigma}$ be given as,

$$\bar{\sigma} = \frac{1}{\sqrt{2}} = [\sigma_r^2 + \sigma_\theta^2 + (\sigma_r - \sigma_\theta)^2]^{1/2} \tag{5}$$

Using equations (1) and (5) in equation (4), we gets,

$$\varepsilon_r = \frac{d\mu_r}{dr} = \frac{[M(\bar{\sigma} - \sigma_0)]^8 (2y - 1)}{2[y^2 - y + 1]^{1/2}} \tag{6}$$

$$\varepsilon_\theta = \frac{\mu_r}{r} = \frac{[M(\bar{\sigma} - \sigma_0)]^8 (2 - y)}{2[y^2 - y + 1]^{1/2}} \tag{7}$$

$$\dot{\varepsilon}_z == \frac{-[M(\bar{\sigma} - \sigma_0)]^8 (y + 1)}{2[y^2 - y + 1]^{1/2}} \tag{8}$$

Wherever $Y = \sigma_r / \sigma_\theta$ is ratio of radial with tangential stress on equations (6), (7) can be used to find σ_θ which is written below,

$$\sigma_\theta = \frac{(\dot{u}_a)1/8}{M} \psi_1 + \psi_2. \tag{9}$$

Where,

$$\dot{u}_a^{1/8} = \frac{\int_a^b M\sigma_\theta dr - \int_a^b M\psi_2 dr}{\int_a^b \psi_1 dr} \tag{10}$$

$$\psi_2 = \frac{\psi}{[y^2 - y \_ 1]^{1/2}} \tag{11}$$

$$\psi_2 = \frac{\sigma_0}{[y^2 - y + 1]^{1/2}} \tag{12}$$

$$\psi = \left[\frac{2[y^2 - y + 1]}{r(2 - y)} \exp \int_a^r \frac{\phi}{r} dr \right]^{\frac{1}{8}} \tag{13}$$

also

$$\phi = \frac{2y - 1}{2 - y}. \tag{14}$$

The equilibrium of forces inside the radial direction are,

$$\frac{d}{dr}[r\sigma_r] - \sigma_\theta + \rho\omega^2 r^2 = 0. \tag{15}$$

Integration equation (15) taking limits of 'r' from "a" to "b" also by taking boundary condition $\sigma_r = 0$ at $r = a$ also at $r = b$ we get,

$$\int_a^b \sigma_\theta dr = \rho\omega^2(b^3 - a^3)/3. \tag{16}$$

In the first iteration, $\sigma_\theta = \sigma_{\theta_{avg}}$, where $\sigma_{\theta_{avg}}$ average tangential stress on cross segment of disk, so equations (10) write in the form given below,

$$\dot{u}_a^{1/8} = \frac{\sigma_{\theta_{avg}} \int_a^b M dr - \int_a^b M\psi_2 dr}{\int_a^b \psi_1 dr}. \tag{17}$$

The value of σ_r can be calculating by taking integration of equations (15) By using limits a to b w.r.t. 'r' are given below,

$$\sigma_r = (1/r)\int_a^b (\sigma_\theta) dr - \frac{\rho\omega^2(r^3 - a^3)}{3r}. \tag{18}$$

Finding the values of σ_θ from equation (9), σ_r is radial stress find by equation (18) at some point in composite disk also ε_r and ε_θ strain rate can be calculated using equations (6) and (7) resp.

4. **Numerical Results**

The stress and strain distribution of rotating Al - SiC_p disk rotating motion at uniform temperature with 20% volume of SiC_p and particle size $P = 1.7\mu m$, $14.5\mu m$ and $14.5\mu m$ are calculated from above mathematical modeling.

	Radial/tangential stresses and strains		
Radii	$P = 1.7\mu m$	$P = 14.5\mu m$	$P = 42.5\mu m$
$r = 40$	$\sigma_r = 1.04643501E + 15MPa$	$\sigma_r = 1.04643707E + 15MPa$	$\sigma_r = 1.04643707E + 15$
	$\sigma_\theta = 6.30274049E + 15MPa$	$\sigma_\theta = 6.30274080E + 15MPa$	$\sigma_\theta = 6.30274081E + 15$
	$\dot{\varepsilon}_r = -2.96832839E + 120s^{-1}$	$\dot{\varepsilon}_r = -3.50753379E + 120s^{-1}$	$\dot{\varepsilon}_r = -1.36631932E + 120s^{-1}$
	$\dot{\varepsilon}_\theta = 8.14997142E + 120s^{-1}$	$\dot{\varepsilon}_\theta = 9.63064744E + 120s^{-1}$	$\dot{\varepsilon}_\theta = 9.7510737E + 120s^{-1}$
$r = 60$	$\sigma_r = 2.26418195E + 15MPa$	$\sigma_r = 2.26418231E + 15MPa$	$\sigma_r = 2.26418231E + 15MPa$
	$\sigma_\theta = 6.3027399E + 15MPa$	$\sigma_\theta = 6.302740E + 15MPa$	$\sigma_\theta = 6.30274079E + 15MPa$
	$\dot{\varepsilon}_r = -8.43721254E + 118s^{-1}$	$\dot{\varepsilon}_r = -623090738E + 118s^{-1}$	$\dot{\varepsilon}_r = 3.88348526E + 118s^{-1}$
	$\dot{\varepsilon}_r = 9.9170728E + 118s^{-1}$	$\dot{\varepsilon}_\theta = 3.63145532E + 119s^{-1}$	$\dot{\varepsilon}_\theta = 7.26334674E + 120s^{-1}$
$r = 80$	$\sigma_r = 2.49337862E + 15MPa$	$\sigma_r = 2.49338093E + 15MPa$	$\sigma_r = 2.54146310E + 15MPa$
	$\sigma_\theta = 6.30273991E + 15MPa$	$\sigma_\theta = 6.302711E + 15MPa$	$\sigma_\theta = 6.30274076E + 15MPa$
	$\dot{\varepsilon}_r = 6.00732791E + 118s^{-1}$	$\dot{\varepsilon}_r = -4.4658028E + 118s^{-1}$	$\dot{\varepsilon}_r = -2.73427150E + 118s^{-1}$
	$\dot{\varepsilon}_\theta = 4.61597552E + 117s^{-1}$	$\dot{\varepsilon}_\theta = 3.40910824E + 119s^{-1}$	$\dot{\varepsilon}_\theta = 2.70103888E + 120s^{-1}$
$r = 100$	$\sigma_r = 2.22591470E + 15MPa$	$\sigma_r = 2.22591451E + 15MPa$	$\sigma_r = 2.22591450E + 15MPa$
	$\sigma_\theta = 6.30273979 + 15MPa$	$\sigma_\theta = 6.30273980E + 15MPa$	$\sigma_\theta = 6.30273999E + 15MPa$
	$\dot{\varepsilon}_r = 5.69085747E + 117s^{-1}$	$\dot{\varepsilon}_r = 6.5514179E + 118s^{-1}$	$\dot{\varepsilon}_r = -2.782447080E + 117s^{-1}$
	$\dot{\varepsilon}_\theta = 4.87367033E + 117s^{-1}$	$\dot{\varepsilon}_\theta = 3.67391265E + 119s^{-1}$	$\dot{\varepsilon}_\theta = 2.2890860E + 120s^{-1}$
$r = 120$	$\sigma_r = 1.62574355E + 15MPa$	$\sigma_r = 1.625744356E + 15MPa$	$\sigma_r = 1.625749357E + 15MPa$
	$\sigma_\theta = 6.30273891 + 15MPa$	$\sigma_\theta = 6.30273863E + 15MPa$	$\sigma_\theta = 6.30273880E + 15MPa$
	$\dot{\varepsilon}_r = 1.68816203E + 117s^{-1}$	$\dot{\varepsilon}_r = -1.27258400E + 119s^{-1}$	$\dot{\varepsilon}_r = 7.93152356E + 118s^{-1}$
	$\dot{\varepsilon}_\theta = 6.07474133E + 117s^{-1}$	$\dot{\varepsilon}_\theta = 4.57931081E + 119s^{-1}$	$\dot{\varepsilon}_\theta = 2.8510864E + 119s^{-1}$
$r = 140$	$\sigma_r = 147574355E + 15MPa$	$\sigma_r = 1.5874356E + 15MPa$	$\sigma_r = 1.59574356E + 15$
	$\sigma_\theta = 6.30273888E + 15MPa$	$\sigma_\theta = 6.30273769E + 15MPa$	$\sigma_\theta = 6.30273797E + 15MPa$
	$\dot{\varepsilon}_r = -7.49872011E + 117s^{-1}$	$\dot{\varepsilon}_r = -1.27258400E + 119s^{-1}$	$\dot{\varepsilon}_r = -1.76156981E + 118s^{-1}$
	$\dot{\varepsilon}_\theta = 1.85916551E + 117s^{-1}$	$\dot{\varepsilon}_\theta = 4.57931081E + 117s^{-1}$	$\dot{\varepsilon}_\theta = 2.19319696E + 117s^{-1}$

5. Discussion and Graphical Representation

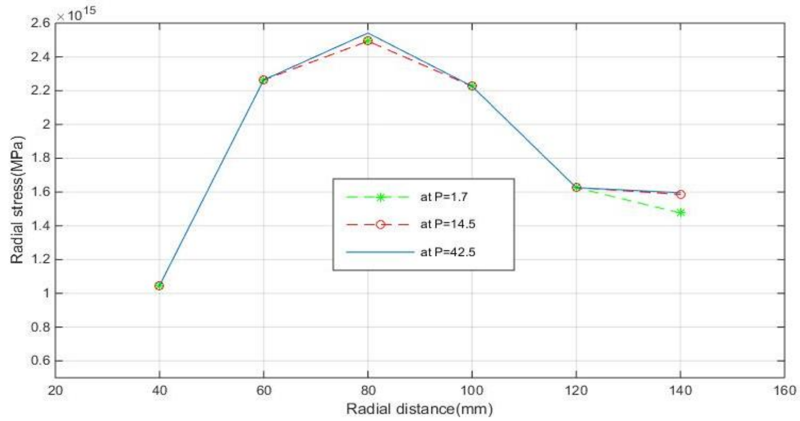

Figure 1. Radial stress in the disk for different particle size.

In figure 1, radial stress is plotted at $P = 1.7\mu m$, $14.5\mu m$ and $42.5\mu m$, it's found that radial stress is high at center of disk as compared inner and outer radii. At the inner radii the size of the particle does not affect the radial stress slightly increases at the center with the increases in the particle size and at outer radii the radial stress decrease.

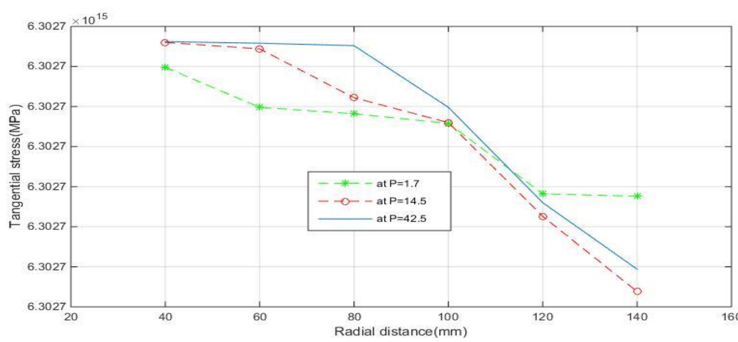

Figure 2. Tangential stress in the disk for different particle size.

In figure 2, tangential stress is plotted at $P = 1.7\mu m$, $42.5\mu m$ and $42.5\mu m$ It has been observe that tangential stress is higher at inner radii and starts decreases at the outer part of the disk. The increases in the particle size increases tangential stress. But at the outer part of the disk tangential stress decreases with increases in size.

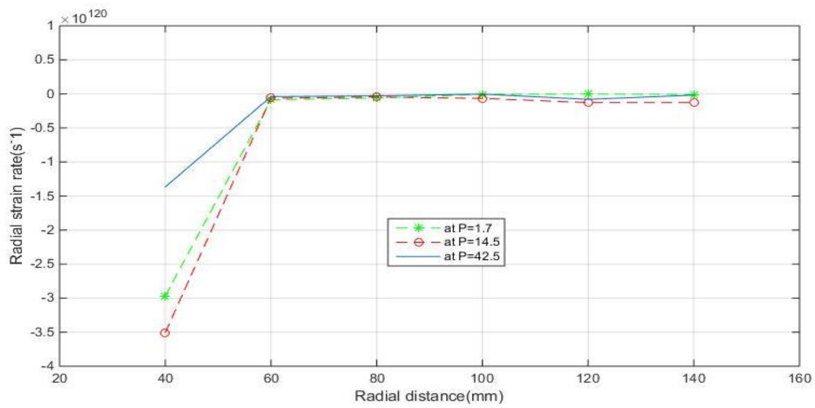

Figure 3. Radial strain in the disk for different particle size.

In figure 3, the radial strain is plotted at $P = 1.7\mu m$, $14.5\mu m$ and $42.5\mu m$. It has been observed radial strain is higher at center part and outer part of disk and lesser at inner radius. With the increases in the particle size the radial strain increases at the inner part and variation in the particle size does not affect at the middle and outer part of the disk.

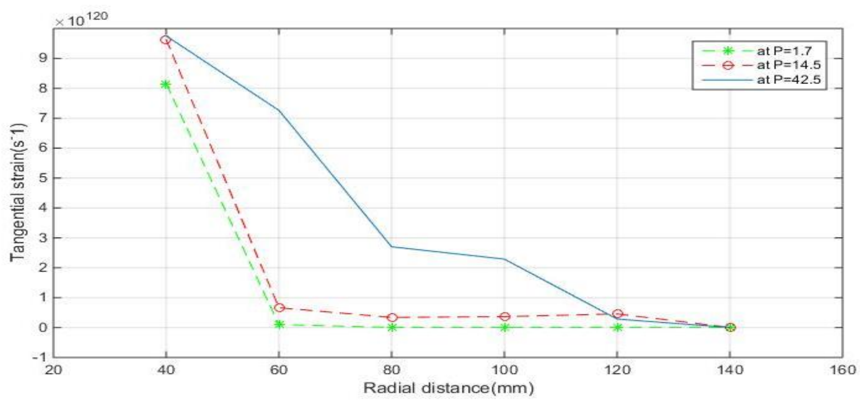

Figure 4. Tangential strain in the disk for different particle size.

In figure 4, the tangential strain is plotted at $P = 1.7\mu m$, $14.5\mu m$ and $14.5\mu m$. It is observed that tangential strain is higher at inner radii and immediately decreases. The tangential strain is almost same for the middle and outer part of the disk. The increases in the particle size increase the tangential strain.

6. Conclusions

The study reported following conclusions based on the table and graphical representation.

1. The radial stress does not vary significantly for different particle size. At the center of the disk the radial stress is almost same for particle size 1.7μm and 14.5μm and less as compared to particle size 42.5μm. At the outer part the radial stress for particle size 14.5μm and 42.5μm is almost same and radial stress decreases for particle size 1.7μm as compared to 14.5μm and 42.5μm. The radial stress increase as move from inner to the outer radii and reaches maximum at middle before decreasing near outer radii.

2. The tangential stress increase with increasing particle size for particle size 1.7μm the tangential stress is less at the inner part of the disk as compared to particle size 14.5μm and 42.5μm. The tangential stress for particle size 14.5μm and 42.5μm is almost same at the inner part. At the center part of the radii the tangential stress is higher for particle size 42.5μm as compared to particle size 14.5μm and 1.7μm. At the outer part of disk tangential stress is higher for the particle size 1.7μm in comparison to particle size 14.5μm and 42.5μm.

3. The radial strain increase as move from inner to outer radii. At the outer radii of rotating disk the radial strain is less for particle size $P = 14.5\mu m$ as compared to radial strain for other particle size. At the middle of the disk the radial strain is almost same for various particle size P = 1.7μm, 14.5μm and 42.5μm. The radial strain is higher at the inner part of the disk for particle size 42.5μm is less the radial strain is almost same for 1.7μm and 14.2μm as compared to 42.5μm.

4. The tangential strain for the particle size 42.5μm is higher at inner and middle part of the disk as compared to the particle size 1.7μm and 14.5μm. The tangential strain is almost same for the particle size 1.7μm and 14.5μm in comparison to particle size 42.5μm. At the external part of the disk the tangential strain is almost same of various particle sizes. We concluded that particle size 1.7μm and 14.5μm almost give same affects as compared to particle size 42.5μm. So in order to reduce distortion the particle size can be taken between 1.7μm to 14.5μm.

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