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PERFORMANCE OF BULK QUEUE UNDER VACATION AND INTERRUPTION

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Abstract

This paper deals with bulk arrival and bulk service under vacation and interruption. To analyze the steady-state behavior as well as determine the different performance size distributions are obtained by using supplementary variable technique. Various performance indices such as expected probability, when the server is busy and on vacation and mean number of customers in the queue and system with utilization factor are carried out in order to validate the analytical result by numerical illustration. Objective of this paper is to analyze the performance of bulk queues with a single server. The sensitivity analysis is also performed to explore the effect of different parameters.

1. Introduction

In the recent era, the technology is growing very fast; performance evaluation is more concerned issue for design, development, configuration and modification of any system. Queueing model has more potential to on counter the routine life as well as industrial problems such as manufacturing and production systems, computer and transport systems, telecommunications and distribution systems etc. congestion situations are always arisen in real time system that can be solved by queueing model. In queueing model we deal with vacations which interrupt the ideal time for the server. Due to some technical problem the server could not perform the job such situation is known as interruption. Bulk queue models are more realistic

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in many queueing systems with the assumption that arrival occurs in batches for example an elevator that brings several people to another floor, a collection of several packets that is transmitted through the network, etc. Generally such queueing model drawn the attention of researchers to analyze the performance of service systems. Such queueing models provide huge applications in our daily life. Neuts [17] studied the waiting time distribution for a bulk queue using a steady-state condition by Downton [7] and Medhi [15]. Fakinos [8] derived the relation between limiting queue size distributions at arrival and departure epochs. Briere and Chaudhry [3] and Kambo and Chaudhry [13] used numerical approaches to get the performance indices. Chaudhry and Templeton [5] gave a more extensive study on batch arrival/service queues. Miller [16] gave the batch arrival, batch service queue. Jaiswal [12] established batch service queues with random service size. Shinde and Patankar [20] investigated the bulk service queue with balking, reneging and multiple vacations under the transient state. Sikdar et al. [19] analyzed the batch arrival single-server queue with renewal input and multiple exponential vacations. Chen et al. [4] Markovian bulk-arrival and bulk service queue incorporating state-dependent control and obtain the behavior of queue length regarding hitting and busy period are also explored. Ghimire et al. [9] formulated a mathematical model to examine the fixed batch size service rate. Kumar and Shinde [14] evaluated the performance measure such as average number of customers in the queue, average number of customers in the system, average waiting time of customers in queue, average waiting time of customers in the system, response time and efficiency of the server corresponding to number of customers with bulk arrival and bulk service queueing model. The steady-state analysis of $GI^{x}/M^{b}\ 1/L$ queue with multiple working vacations and partial batch rejection was considered by Yu, et al. [22] finite buffer bulk arrival, bulk service queueing system with multiple working vacations and partial batch rejection in which the inter-arrival and service times are respectively, arbitrarily and exponentially distributed. The supplementary variable and the embedded Markov chain techniques are used. The batch arrival $M^{x}/G^{y}\ 1/N$ queue with finite buffer under service's vacation has been studied by Gupta and Sikdar [10]. Jain and Jain [11] considered the queueing model with vacation where the server work of a different rate instead of being computably idle during the vacation in which the arrival rate varies according to the server status. Singh et al. [18] analyzed single server queueing model with bulk arrival and state-dependent rates. General distribution is considered for repair, delay to repair also for the service process. Ayyappan and Deepa [1] analyzed a queueing model with multiple vacation, closedown, essential and optional repair (first or second) or if there is no breakdown with probability, the server resumes closedown if less than $'a'$ customers are waiting. Otherwise, the server starts the service under the general bulk service rule. Using supplementary variable technique, the probability generating function of the queue size at an arbitrary time is obtained for the steady-state case. Also 2 some performance measures and cost model are derived. Ayyappan and Viji [2] studied the behavior of a non-Markovian bulk service queueing model with unreliable server, stand-by server, loss and feedback, N-policy and Bernoulli schedule multiple vacation. The PGF of queue size and some important performance measures are derived

The present paper is organized as follows: notation is given in section 2. In section 3, we described the mathematical model. Performance measures have been provided in section 4. In section 5, sensitivity analysis is mentioned. The conclusion is given in section 6.

2. Notation

The following notations have been used in the article.

n = number of customers in the system

N = Number of fixed customers in the system

λ = Arrival rate

α = Customers in group or batch (as different size) for arrival rate

μ = Service rate

l = Vacation length

β = Customers in group or Batch (as different size) for service rate

ρ = Utilization factor $(\rho = \alpha\lambda / \beta\mu < 1)$

$P_{n,1}$ = Expected probability of customers in the queue, when the server is on vacation

$P_{n,2}$ = Expected prob. of customers in the queue, when the server is interruption

L_q = Mean number of customers in the queue

L_s = Mean number of customers in the system

Rt_q = Response time of customers in the queue

Rt_s = Response time of customers in the system

3. Model Description

We consider a single-server queueing model with bulk arrival and bulk service under the vacation when the server is unavailable to provide the service for a certain period. The server vacations are valuable for the system in particular when the server wishes to consume his idle time for other ancillary work or server take rest. In interruption server is not working or under repair. Arrival pattern follows the Poisson process and service rate is exponentially distributed. Assume that the systems initially contain N customers, when the customers enter the system than the server starts the service in batch. After completion of the service if he finds more than N customers in the queue then the first N customers will be taken for service and serve them in a batch size of α and β.

Let $\langle N(t), C(t)\rangle$ be a random system where in $N(t)$ be the random variable which represents the variety of clients in a queue at time t and $C(t)$ be the random variable which represents the server popularity (vacation/interruption) at time t.

Here, $P_{n,1}(t)$ is the probability of the server is on vacation if there are n clients with in the queue at time t, and $P_{n,2}(t)$ is the probability of the server is an interruption, if there are n customers with in the queue at time t.

The Chapman-Kolmogorov equations is employed

$$P'_{n,1}(t) = -(\alpha\lambda p + \beta\mu(1-q))p_{0,1}(t) + \beta\mu(1-q)P_{N,1}(t) + aP_{N,2}(t) \tag{1}$$

$$P'_{n,1}(t) = -(\alpha\lambda p + \beta\mu(1-q))P_{n,1}(t) + \alpha\lambda pP_{n-1,1}(t) + \beta\mu(1-q)P_{n+N,1}(t) + aP_{n+N,2}(t)$$

$$\text{for } n = 1, 2, 3 \ldots \tag{2}$$

$$P'_{0,2}(t) = -\alpha\lambda pP_{0,2}(t) + \beta\mu(1-q)P_{0,1}(t) \tag{3}$$

$$P'_{0,2}(t) = -\alpha\lambda pP_{n,2}(t) + \alpha\lambda pP_{n-1,2}(t) + \beta\mu(1-q)P_{n,1}(t)$$

$$\text{for } n = 1, 2, 3, \ldots, N-1 \tag{4}$$

$$P'_{0,2}(t) = -(\alpha\lambda p + a)P_{n,2}(t) + \alpha\lambda pP_{n-1,2}(t) \text{ for } n > N. \tag{5}$$

Apply steady-state condition from (1) to (5) we have

$$(\alpha\lambda p + \beta\mu(1-q))P_{0,1}(t) = \beta\mu(1-q)P_{N,1}(t) + aP_{N,2}(t) \tag{6}$$

$$(\alpha\lambda p + \beta\mu(1-q))P_{n,1}(t) = \alpha\lambda pP_{n-1,1}(t) + \beta\mu(1-q)P_{n+N,1}(t) + aP_{n+N,2}(t)$$

$$\text{for } n = 1, 2, 3 \ldots \tag{7}$$

$$\alpha\lambda pP_{0,2}(t) = \beta\mu(1-q)P_{0,1}(t) \tag{8}$$

$$\alpha\lambda pP_{n,2}(t) = \alpha\lambda pP_{n-1,2}(t) + \beta\mu(1-q)P_{n,1}(t) \text{ for } n = 1, 2, 3, \ldots, N-1 \tag{9}$$

$$(\alpha\lambda p + a)P_{n,2}(t) = \alpha\lambda pP_{n-1,2}(t) \text{ for } n \geq N. \tag{10}$$

Use generating function to summarize the above equations

$$G(z) = \sum_{n=0}^{\infty} P_{n,1}(t)z^n \text{ and } H(z) = \sum_{n=0}^{\infty} P_{n,2}(t)z^n.$$

Using (1) and (7) in (6) with multiplication of z^n on both side and $n = 0$ to ∞, we get

$$G(z) = [\alpha\lambda pz^{N+1} - (\alpha\lambda p + \beta\mu(1-q))z^N + \beta\mu] + aH(z)$$

$$= \beta\mu(1-q)\sum_{n=0}^{N-1} P_{n,1}(t)z^n + a\sum_{n=0}^{N-1} P_{n,2}(t)z^n. \tag{11}$$

Adding (8), (9) and (10) after multiply by z^n we have

$$H(z)[a + a\lambda p(1 - z)] = \beta\mu(1 - q)\sum_{n=0}^{N-1} P_{n,1}(t)z^n + a\sum_{n=0}^{N-1} P_{n,1}(t)z^n. \tag{12}$$

From (11) and (12), we get

$$G(z) = \frac{H(z)[a + \alpha\lambda p(1 - z)]}{\alpha\lambda pz^{N+1} - (\alpha\lambda p + \beta\mu(1 - q))z^N + \beta\mu(1 - q)}. \tag{13}$$

From (12), we have

$$H(z) = \frac{\beta\mu(1 - q)\sum_{n=0}^{N-1} P_{n,1}(t)z^n + a\sum_{n=0}^{N-1} P_{n,2}(t)z^n}{a + \alpha\lambda p(1 - z)}. \tag{14}$$

Equation (14) is representing generating function for the quantity of clients inside the queue while the server is on vacation.

Now, from (13) and (14), we have

$$G(z) = \frac{[(\alpha\lambda p(1 - z))\beta\mu(1 - q)]\sum_{n=0}^{N-1} P_{n,1}(t)z^n + a\sum_{n=0}^{N-1} P_{n,2}(t)z^n}{\alpha\lambda pz^{k+1} - (\alpha\lambda p + \beta\mu(1 - q))z^N + \beta\mu(1 - q)}. \tag{15}$$

Equation (15) is representing the probability generating function for the number of customers in the queue when the server is an interruption.

We put $z = 1$ in (13), we have

$$G(1) = \frac{\alpha\lambda p}{N\beta\mu(1 - q) - \alpha\lambda p} \times H(1).$$

Using the normalized condition $G(1) + H(1) = 1.$ (17)

Using the (1) in (17), we get

$$H(1) = \frac{N\beta\mu(1 - q) - \alpha\lambda p}{N\beta\mu(1 - q)}. \tag{18}$$

From the (16) and (18), we have.

The steady-state probability that the sever is on vacation $G(1) = \frac{\alpha\lambda p}{N\beta\mu(1 - q)}$.

The steady-state probability that the sever is interruption $H(1) = \frac{N\beta\mu(1-q) - \alpha\lambda p}{N\beta\mu(1-q)}$.

The generating function $G(z)$ has the property that it must converge inside the unit circle. We notice that the denominator of $G(z), \alpha\lambda p z^{N+1} - (\alpha\lambda p + \beta\mu(1-q))z^N + \beta\mu(1-q)$ has $N+1$ zero's Applying Rouche's theorem, we notice that N zeros of this expression lies inside the $|z| < 1$ and must coincide with N zeros of numerator of $G(z)$ and zero lies outside the circle $|z| < 1$. Let z_0 be a zero which lies outside the circle $|z| < 1$. As $G(z)$ converges, k zeros of numerator and denominator will be cancelled, we obtain the generating function

$$G(z) = \frac{A}{(a + \alpha\lambda p(1-z)) + \alpha\lambda(z - z_0)}. \tag{19}$$

Putting $z = 1$ in (19), we get

$$G(1) = \frac{A}{\alpha\lambda(z - z_0)a}. \tag{20}$$

Using (16) and (18) in (20), we obtain

$$A = \frac{\alpha\lambda^2 p}{N\beta\mu(1-q)} \times (z - z_0)a. \tag{21}$$

From (19) and (21), we get

$$G(z) = \frac{\alpha\lambda p \times (1 - z_0)(a)}{(N\beta\mu(1-q) - \alpha\lambda p)(z - z_0)(a + \alpha\lambda p - \alpha\lambda p z)}. \tag{22}$$

By apply partial fractions, we obtain

$$G(z) = \frac{a}{(N\beta\mu(1-q) - \alpha\lambda p)} \frac{\alpha\lambda p \times (1 - z_0)}{(a + \alpha\lambda p - \alpha\lambda p z_0)}$$

$$\left[\sum_{n=0}^{\infty}\left(\frac{\alpha\lambda p}{a + \alpha\lambda p}\right)^{n+1} z^n - \sum_{n=0}^{\infty} \frac{z^n}{z_0^{n+1}}\right]. \tag{23}$$

Now, compare the coefficient of z^n on both sides of the (23)

$$P_{n,1} = \frac{\alpha}{(N\beta\mu(1-q)-\lambda p)} \times \frac{(1-r)^{s}}{(s-r)} \times (S^{n+1} + r^{n+1})\ n = 0, 1, 2, \ldots \tag{24}$$

Where $r = 1/z$ and $S = \dfrac{\alpha\lambda p}{\alpha\lambda p + \alpha}$.

We use the recursive relation in (24) in (8) and (9) for $n = 1, 2, 3, \ldots, N-1$,

$$P_{n,2} = \begin{cases} P_{n,2} = \dfrac{\beta\mu(1-q)}{\alpha\lambda p} \times \displaystyle\sum_{t=0}^{n} P_{t,1} \text{ for } n = 0, 1, 2, \ldots, N-1. \\ P_{n,2} = \dfrac{\alpha\lambda p^{n-N+1}}{(\alpha\lambda p + \alpha)} P_{N-1,2} \text{ for } n \geq N. \end{cases} \tag{25}$$

Form (24) and (25) are representing, the steady-state probability, when the server is on vacation and interruption mode.

4. Performance Measures

In this section, we define expected probability with vacation and interruption time and also obtained the average number of customers in the system, the average number of customers in the queue, response time in the system and queue with utilization factor. These measures are used to carry out the qualitative behavior of the queueing model with a single server.

- Utilization factor: $\rho = \dfrac{\alpha\lambda p}{N\beta\mu(1-q)} < 1$
- Expected probability, when the server is busy:

$$P_{n,1} = \frac{\alpha}{(N\beta\mu(1-q)-\lambda p)} \times \frac{(1-r)s}{(s-r)} \times (S^{n+1} + r^{n+1})\ n = 0, 1, 2, \ldots$$

where $r = 1/z$ and $s = \dfrac{\alpha\lambda p}{\alpha\lambda p + \alpha}$

- Expected probability, when the server is vacation:

$$P_{n,2} = \begin{cases} P_{n,2} = \dfrac{\beta\mu(1-q)}{\alpha\lambda p} \times \displaystyle\sum_{t=0}^{n} P_{t,1} \text{ for } n = 0, 1, 2, \ldots, N-1. \\ P_{n,2} = \dfrac{\alpha\lambda p}{(\alpha\lambda p + \alpha)} P_{N-1,2} \text{ for } n \geq N. \end{cases}$$

- Mean number of customers in the queue: $L_q = \sum_{n=0}^{\infty} n(P_{n,1} + P_{n,2})$

- Mean number of customers in the system: $L_s = \sum_{n=0}^{\infty} (n+N)(P_{n,1} + nP_{n,2})$

- Mean Response time in the system:

$$R_s = \frac{\sum_{n=0}^{\infty} (n+N)(P_{n,1} + nP_{n,2})}{Arrval\ rate\ with\ batch\ size\ of\ the\ customers}$$

- Mean Response time in the queue:

$$R_s = \frac{\sum_{n=0}^{\infty} n(P_{n,1} + nP_{n,2})}{Arrval\ rate\ with\ batch\ size\ of\ the\ customers}.$$

5. Sensitivity Analysis

In this section, we obtain some performance measures of the system with queueing model with various parameters as λ, α, β, μ, p, l, q, and N are chose so that they satisfy with utilization factor. Also, we study of expected probabilities corresponding to the server is on vacation and interruption. Here, arising some cases as given below:

Tables 1, 4, 7, 10 and 15 represent the steady-state probability distribution when the server is on vacation mode for various values of λ and also, tables 2, 5, 8, 11 and 14 are present the steady-state probability distribution, when the server is interruption mode for various values of λ.

Tables 3, 6, 9, 12 and 15 show the performance measures the Mean number of customers in the system, the Mean number of customers in the queue, response time in the system and queue with utilization factor and also, obtain the mean from the probability distribution when the server is on vacation and interruption mode.

Case I: Server is on vacation with $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 1. Steady-state probabilities corresponding Case I.

λ	pb_{01}	pb_{11}	pb_{21}	pb_{31}	pb_{41}	pb_{51}	pb_{61}	pb_{71}	pb_{81}	pb_{91}	pb_{101}
1	0.0247721	0.0323176	0.034046	0.0343996	0.0344675	00	00	00	00	00	00
2	0.0450938	0.0635413	0.06926	0.0708626	0.0712844	0.07139	0.07141	0.07142	0.07142	0.07142	0.0714285
3	0.0625626	0.0932802	0.10498	0.109065	0.110438	0.110891	0.11103	0.11108	0.11110	0.11110	0.11111
4	0.0781739	0.12171	0.140871	0.14871	0.15183	0.153057	0.153538	0.153726	0.153799	0.153828	0.153839
5	0.0925926	0.149177	0.176898	0.189647	0.195384	0.197946	0.199087	0.199594	0.19982	0.19992	0.199964
6	0.106293	0.17607	0.213199	0.231876	0.241107	0.245641	0.247865	0.248954	0.249488	0.249749	0.249877
7	0.119634	0.202775	0.250022	0.275537	0.289108	0.296294	0.300092	0.3021	0.30316	0.30372	0.304016
8	0.132908	0.229674	0.287691	0.320877	0.339611	0.350145	0.356062	0.359384	0.361249	0.362296	0.362884
9	0.14637	0.257147	0.32659	0.36825	0.392949	0.407545	0.416162	0.421248	0.424249	0.426021	0.427066
10	0.160256	0.285585	0.367162	0.418105	0.449578	0.468967	0.480902	0.488247	0.492768	0.495549	0.497261

Case II: Server is interruption and $N = 2$ with $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$.

Table 2. Steady-state probabilities corresponding Case II.

λ	pb02	pb12	pb22	pb32	pb42	pb52	pb62	pb72	pb82	pb92	pb102
1	0.371581	0.113183	0.070441	0.0098167	0.0013567	0.0013567	00	00	00	00	00
2	0.338203	0.138356	0.125938	0.0312342	0.007617	0.0018493	0.000448	0.000108	00	00	00
3	0.312813	0.153588	0.170239	0.0573606	0.0188376	0.006134	0.001992	0.000646	00	00	00
4	0.293152	0.163259	0.206152	0.0849267	0.0338374	0.013311	0.005211	0.002036	0.000794	0.000310	0.000121094
5	0.277778	0.169753	0.235863	0.112383	0.0514592	0.023170	0.010357	0.004615	0.002053	0.000913	0.000405911
6	0.265731	0.174443	0.26106	0.139068	0.0708264	0.035342	0.017467	0.008593	0.004217	0.002068	0.00101345
7	0.256358	0.178161	0.283044	0.164792	0.0913483	0.049458	0.02646	0.014074	0.007461	0.003949	0.00208848
8	0.249203	0.181437	0.302833	0.189623	0.11267	0.065215	0.037230	0.021096	0.011905	0.006702	0.00376916
9	0.24395	0.184629	0.321236	0.213765	0.134618	0.082397	0.049656	0.029663	0.017631	0.010448	0.00618155
10	0.240385	0.187993	0.187993	0.237504	0.157158	0.100884	0.063662	0.039775	0.024703	0.015288	0.00944051

Table 3. Performance of Lq, Ls, Rq, Rs with utilization factor.

Arrival Rate (λ)	Mean	Utilization	L_q	L_s	Rq	Rs
1	0.0345	0.0333	0.3025	1.5046	0.1512	0.7523
2	0.0714	0.0667	0.5645	1.9950	0.1411	0.4987
3	0.1111	0.1000	0.8667	2.5327	0.1445	0.4221
4	0.1540	0.1333	1.2173	3.1312	0.1522	0.3914
5	0.2004	0.1667	1.6231	3.8005	0.1623	0.3800
6	0.2509	0.2000	2.0898	4.5492	0.1741	0.3791
7	0.3061	0.2333	2.6225	5.3849	0.1873	0.3846
8	0.3667	0.2667	3.2260	6.3151	0.2016	0.3947
9	0.4332	0.3000	3.9055	7.3480	0.2170	0.4082
10	0.5067	0.3333	4.6669	8.4928	0.2333	0.4246

Case III: Server is on vacation with $\lambda = 1$ to 10, $\alpha = 3$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 4. Steady-state probabilities corresponding Case III.

λ	pb01	pb11	pb21	pb31	pb41	pb51	pb61	pb71	pb81	pb91	pb101
1	0.0353707	0.0481117	0.051560	0.0523915	0.0525796	0.05262	0.05262	0.05263	0.05263	0.05263	0.0526316
2	0.0625626	0.0932802	0.104981	0.109065	0.110438	0.11089	0.11103	0.11108	0.11110	0.11110	0.11111
3	0.0854993	0.13554	0.158862	0.16902	0.173339	0.17515	0.17592	0.17624	0.17637	0.17643	0.176454
4	0.106293	0.17607	0.213199	0.231876	0.241107	0.24564	0.24786	0.24895	0.24948	0.24974	0.249877
5	0.126263	0.216177	0.268728	0.297977	0.314029	0.3228	0.32758	0.33019	0.33162	0.33240	0.332825
6	0.14637	0.257147	0.32659	0.36825	0.392949	0.40754	0.41616	0.42124	0.42424	0.42602	0.427066
7	0.167432	0.300295	0.388233	0.444133	0.479305	0.50137	0.51521	0.52388	0.52932	0.53273	0.534871
8	0.190259	0.34707	0.455464	0.527618	0.575208	0.60652	0.62712	0.64066	0.64956	0.65542	0.659274
9	0.215765	0.399211	0.530598	0.621406	0.683643	0.72621	0.75531	0.77521	0.78880	0.79810	0.804458
10	0.245098	0.458958	0.616726	0.729227	0.808828	0.86504	0.90473	0.93275	0.95253	0.96649	0.976349

Case IV: Server is interruption with $\lambda = 1$ to 10, $\alpha = 3$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 5. Steady-state probabilities corresponding Case IV.

λ	pb02	pb12	pb22	pb32	pb42	pb52	p62	pb72	pb82	pb92	pb102
1	0.353707	0.127411	0.0997941	0.0196264	0.0038122 9	0.00073 8438	0.00014 2947	00	00	00	00
2	0.312813	0.153588	0.170239	0.0573606	0.0188376	0.00613	0.00199	0.00064	0.00020	00	00
3	0.284998	0.166801	0.221667	0.0987247	0.0423825	0.01792	0.00753	0.0031	0.00132	0.00055	0.0002321
4	0.265731	0.174443	0.26106	0.139068	0.0708264	0.03534	0.01746	0.00859	0.00421	0.00206	0.0010134
5	0.252525	0.179829	0.293158	0.177309	0.101924	0.05714	0.03163	0.01739	0.00952	0.00520	0.0028449
6	0.24395	0.184629	0.321236	0.213765	0.134618	0.08239	0.04965	0.02966	0.01763	0.01044	0.0061815
7	0.239189	0.189804	0.347671	0.249324	0.16867	0.11060	0.07124	0.04541	0.02876	0.01814	0.0114215
8	0.237823	0.196014	0.374354	0.285145	0.204404	0.14172	0.09635	0.06472	0.04314	0.02862	0.018934
9	0.239739	0.203829	0.402986	0.322601	0.242598	0.17615	0.12523	0.08785	0.06110	0.0422	0.0291179
10	245098	0.21386	0.435336	0.363352	0.284481	0.21476	0.15855	0.11538	0.08317	0.05957	0.0424811

Table 6. Performance of Lq, Ls, Rq, Rs with utilization.

Arrival Rate (λ)	Mean	Utilization	L_q	L_s	Rq	Rs
1	0.0526	0.0500	0.4291	1.7449	0.1430	0.5816
2	0.1111	0.1000	0.8667	2.5327	0.1445	0.4221
3	0.1767	0.1500	1.4129	3.4564	0.1570	0.3640
4	0.2509	0.2000	2.0898	4.5492	0.1741	0.3791
5	0.3357	0.2500	2.9151	5.8377	0.1943	0.3892
6	0.4332	0.3000	3.9055	7.3480	0.2170	0.4082
7	0.5463	0.3500	5.0806	9.1109	0.2419	0.4339
8	0.6782	0.4000	6.4675	11.1686	0.2695	0.4654
9	0.8336	0.4500	8.1059	13.5817	0.3002	0.5030
10	1.0188	0.5000	10.0549	16.4397	0.3352	0.5480

Case V: Server is on vacation with $\lambda = 1$ to 10, $\alpha = 4$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 7. Steady-state probability corresponding Case V.

λ	pb01	pb11	pb21	pb31	pb41	pb51	pb61	pb71	pb81	pb91	pb101
1	0.0450938	0.0635413	0.069266	0.0708626	0.0712844	0.07139	0.07141	0.07142	0.07142	0.07142	0.0714285
2	0.0781739	0.12171	0.140871	0.14871	0.15183	0.15305	0.15353	0.15372	0.15379	0.15382	0.153839
3	0.106293	0.17607	0.213199	0.231876	0.241107	0.24564	0.24786	0.24895	0.24948	0.24974	0.249877
4	0.132908	0.229674	0.287691	0.320877	0.339611	0.35014	0.35606	0.35938	0.36124	0.36229	0.362884
5	0.160256	0.285585	0.367162	0.418105	0.449578	0.46896	0.48090	0.48824	0.49276	0.49554	0.497261
6	0.190259	0.34707	0.455464	0.527618	0.575208	0.60652	0.62712	0.64066	0.64956	0.65542	0.659274
7	0.225051	0.418151	0.557904	0.655564	0.723257	0.77008	0.80246	0.82485	0.84033	0.85103	0.858429
8	0.267523	0.504486	0.682317	0.811435	0.90449	0.97144	1.01959	1.05422	1.07911	1.09702	1.1099
9	0.322165	0.614992	0.841297	1.01077	1.13681	1.23041	1.29989	1.35146	1.38975	1.41816	1.43925
10	0.396825	0.765306	1.05708	1.28121	1.45229	1.58269	1.68205	1.75775	1.81543	1.85937	1.89286

Case VI: Server is interruption with $\lambda = 1$ to 10, $\alpha = 4$, $\beta = 3$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 8. Steady-state probabilities corresponding Case VI.

λ	pb02	pb12	pb22	pb32	pb42	pb52	pb62	pb72	pb82	pb92	pb102
1	0.338203	0.138356	0.125938	0.0312342	0.007617	0.00184	00	00	00	00	00
2	0.293152	0.163259	0.206152	0.0849267	0.0338374	0.01331	0.00521	0.00203	0.00079	0.00031	0.0001210
3	0.265731	0.174443	0.26106	0.139068	0.0708264	0.03534	0.01746	0.00859	0.00421	0.00206	0.0010134
4	0.249203	0.181437	0.302833	0.189623	0.11267	0.06521	0.03723	0.02109	0.01190	0.00670	0.0037691
5	0.240385	0.187993	0.338919	0.237504	0.157158	0.10088	0.06366	0.039775	0.02470	0.01528	0.0094405
6	0.237823	0.196014	0.374354	0.285145	0.204404	0.14172	0.09635	0.06472	0.04314	0.02862	0.018934
7	0.241127	0.206893	0.413262	0.335726	0.256074	0.18850	0.13580	0.09650	0.06797	0.04759	0.033189
8	0.250803	0.222153	0.459989	0.393373	0.315315	0.24352	0.1838	0.13666	0.10059	0.07353	0.0535011
9	0.268471	0.244022	0.52039	0.464079	0.387426	0.31125	0.24407	0.18835	0.14377	0.10890	0.0820357
10	0.297619	0.276361	0.604044	0.557808	0.481745	0.4	0.32389	0.25788	0.20293	0.15835	0.122825

Table 9. Performance of Lq, Ls, Rq, Rs with utilization factor.

Arrival Rate (λ)	Mean	Utilization	L_q	L_s	Rq	Rs
1	0.0714	0.0667	0.5645	1.9950	0.1411	0.4987
2	0.1540	0.1333	1.2173	3.1312	0.1522	0.3914
3	0.2509	0.2000	2.0898	4.5492	0.1741	0.3791
4	0.3667	0.2667	3.2260	6.3151	0.2016	0.3947
5	0.5067	0.3333	4.6669	8.4928	0.2333	0.4246
6	0.6782	0.4000	6.4675	11.1686	0.2695	0.4654
7	0.8916	0.4667	8.7173	14.4795	0.3113	0.5171
8	1.1634	0.5333	11.5695	18.6558	0.3615	0.5830
9	1.5213	0.6000	15.2935	24.0976	0.4248	0.6694
10	2.0157	0.6667	20.3888	31.5415	0.5097	0.7885

Case VII: Server is on vacation with of $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 4$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 10. Steady-state probabilities corresponding case VII.

λ	pb01	pb11	pb21	pb31	pb41	pb51	Pb61	pb71	pb81	pb91	pb101
1	0.0184203	0.0240311	0.025316	0.0255792	0.0256297	0.025639	0.02564	0.02564	0.02564	0.02564	0.025641
2	0.033227	0.0468199	0.051038	0.0522145	0.0525253	0.05260	0.05262	0.05263	0.05263	0.05263	0.0526316
3	0.0456538	0.0680693	0.076607	0.0795879	0.0805898	0.08092	0.08102	0.08106	0.08107	0.08107	0.0810805
4	0.0564589	0.0879015	0.10174	0.107402	0.109655	0.11054	0.11088	0.11102	0.11107	0.11109	0.111106
5	0.0661376	0.106555	0.126355	0.135462	0.13956	0.14139	0.14220	0.14256	0.14272	0.1428	0.142832
6	0.07503	0.124284	0.150493	0.163677	0.170193	0.17339	0.17496	0.17573	0.17610	0.17629	0.176384
7	0.083381	0.141328	0.174258	0.174258	0.2015	0.20650	0.20915	0.21055	0.21129	0.21168	0.21189
8	0.0913743	0.157901	0.197788	0.220603	0.233482	0.24072	0.24479	0.24707	0.24835	0.24907	0.249483
9	0.0991539	0.174197	0.221238	0.24946	0.266192	0.27607	0.28191	0.28536	0.28739	0.28859	0.289303
10	0.106838	0.19039	0.244775	0.278737	0.299719	0.31264	0.32060	0.32549	0.32851	0.33036	0.331507

Case VIII: Server is interruption with $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 4$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 11. Steady-state probabilities corresponding case VIII.

λ	pb02	pb12	pb22	pb32	pb42	pb52	pb62	pb72	pb82	pb92	pb102
1	0.368406	0.112215	0.069839	0.0097328	0.0013451	0.00018	00	00	00	00	00
2	0.33227	0.135929	0.123729	0.0306862	0.0074833	0.00181	0.000440	0.00010	00	00	00
3	0.304358	0.149437	0.165638	0.0558104	0.0183285	0.00596	0.001938	0.00062	0.00020	00	00
4	0.282294	0.157213	0.198517	0.0817813	0.0325841	0.01281	0.005018	0.00196	0.000765	0.00029	0.0001166
5	0.26455	0.16167	0.224632	0.107032	0.0490088	0.02206	0.00986	0.00439	0.001955	0.00086	0.0003865
6	0.2501	0.164181	0.245703	0.130887	0.0666601	0.03326	0.01644	0.00808	0.003969	0.00194	0.0009538
7	0.238231	0.165563	0.26303	0.15314	0.0848893	0.04596	0.0245	0.01307	0.006934	0.00367	0.0019408
8	0.228436	0.166317	0.277597	0.173821	0.103281	0.05978	0.034128	0.01933	0.010913	0.00614	0.0034550
9	0.220342	0.166762	0.290149	0.193078	0.121591	0.07442	0.044850	0.02679	0.01592	0.00943	0.005583
10	0.213675	0.167105	0.301261	0.211114	0.139696	0.08967	0.05658	0.03535	0.02195	0.01358	0.008391

Table 12. Performance of Lq, Ls, Rq, Rs with Utilization factor.

Arrival Rate (λ)	Mean	Utilization	L_q	L_s	Rq	Rs
1	0.0256	0.0250	0.2968	1.4716	0.1484	0.7358
2	0.0526	0.0500	0.5455	1.9158	0.1364	0.4789
3	0.0811	0.0750	0.8249	2.3919	0.1375	0.3986
4	0.1112	0.1000	1.1411	2.9101	0.1426	0.3638
5	0.1432	0.1250	1.4982	3.4768	0.1498	0.3477
6	0.1773	0.1500	1.8989	4.0961	0.1582	0.3413
7	0.2138	0.1750	2.3446	4.7705	0.1675	0.3407
8	0.2529	0.2000	2.8359	5.5013	0.1772	0.3438
9	0.2949	0.2250	3.3730	6.2895	0.1874	0.3494
10	0.3399	0.2500	3.9560	7.1359	0.1978	0.3568

Case IX: Server is on vacation with $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 5$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 13. Steady-state probabilities corresponding case XI.

λ	pb1	pb11	pb21	pb31	pb41	pb51	pb61	pb71	pb81	pb91	pb101
1	0.014661	0.0191268	0.02015	0.020359	0.0203991	0.02040	0.02040	0.02040	0.02040	0.02040	0.0204082
2	0.0263047	0.0370657	0.040405	0.0413365	0.0415826	0.04164	0.04166	0.04166	0.04166	0.04166	0.0416666
3	0.0359402	0.0535865	0.060308	0.0626543	0.063443	0.06370	0.06378	0.06381	0.06382	0.06382	0.0638293
4	0.0441852	0.0687924	0.079622	0.0840536	0.0858169	0.08651	0.08678	0.08688	0.08693	0.08694	0.0869525
5	0.0514403	0.0828761	0.098276	0.105359	0.108547	0.10997	0.11060	0.11088	0.111011	0.11106	0.111091
6	0.0579777	0.096038	0.11629	0.126478	0.131513	0.13398	0.13519	0.13579	0.13608	0.13622	0.136297
7	0.0639901	0.108461	0.133733	0.14738	0.154639	0.15848	0.16051	0.16158	0.16215	0.16245	0.162613
8	0.0696185	0.120306	0.150695	0.168079	0.177891	0.18340	0.18650	0.18824	0.18922	0.18977	0.190082
9	0.07497	0.13171	0.167278	0.188616	0.201267	0.20874	0.21315	0.21576	0.21729	0.21820	0.218741
10	0.0801282	0.142793	0.183581	0.209053	0.224789	0.23448	0.24045	0.24412	0.24638	0.24777	0.248631

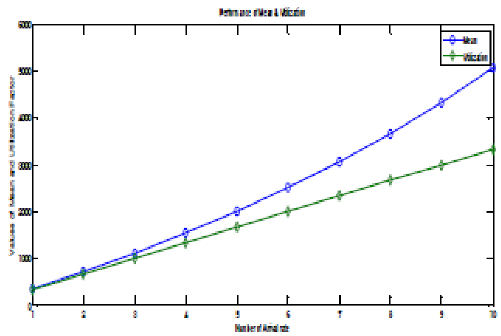
Case X: Server is interruption with $\lambda = 1$ to 10, $\alpha = 2$, $\beta = 5$, $\mu = 10$, $p = .8$, $l = 10$, $q = .2$, $z = 6$ and $N = 2$.

Table 14. Steady-state probabilities corresponding case X.

λ	pb2	pb12	pb22	pb32	pb42	pb52	pb62	pb72	pb82	pb92	pb102
1	0.366526	0.111643	0.069482	0.0096832	0.0013382	0.00018	00	00	00	00	00
2	0.328809	0.134513	0.12244	0.0303666	0.0074054	0.00179	0.00043	0.00010	00	00	00
3	0.299502	0.147053	0.147053	0.0549198	0.018036	0.00587	0.00190	0.00061	0.00020	00	00
4	0.276158	0.153795	0.194202	0.0800034	0.0318758	0.01253	0.00490	0.00191	0.00074	0.00029	0.0001140
5	0.257202	0.157179	0.218392	0.104058	0.0476474	0.02145	0.00959	0.00427	0.00190	0.00084	0.000375
6	0.241574	0.158584	0.237327	0.126425	0.0643876	0.03212	0.01587	0.00781	0.00383	0.00188	0.0009213
7	0.228536	0.158825	0.252326	0.146908	0.0814345	0.04409	0.02359	0.01254	0.00665	0.00352	0.0018618
8	0.217558	0.158397	0.264378	0.165544	0.0983627	0.05693	0.03250	0.01841	0.01039	0.00585	0.0032905
9	0.20825	0.15761	0.274226	0.182482	0.114918	0.07033	0.04238	0.02532	0.01505	0.00891	0.0052769
10	0.200321	0.156661	0.282432	0.19792	0.130965	0.08406	0.05305	0.03314	0.02058	0.01274	0.0078670

Table 15. Performance of Lq, Ls, Rq, Rs with utilization factor.

Arrival Rate (λ)	Mean	Utilization	L_q	L_s	Rq	Rs
1	0.0204	0.0200	0.2935	1.4521	0.1467	0.7260
2	0.0417	0.0400	0.5344	1.8695	0.1336	0.4674
3	0.0639	0.0600	0.8009	2.3109	0.1335	0.3852
4	0.0871	0.0800	1.0981	2.7851	0.1373	0.3481
5	0.1115	0.1000	1.4289	3.2969	0.1429	0.3297
6	0.1372	0.1200	1.7949	3.8490	0.1496	0.3207
7	0.1645	0.1400	2.1960	4.4418	0.1569	0.3173
8	0.1934	0.1600	2.6316	5.0751	0.1645	0.3172
9	0.2240	0.1800	3.1003	5.7474	0.1722	0.3193
10	0.2565	0.2000	3.6006	6.4574	0.1800	0.3229



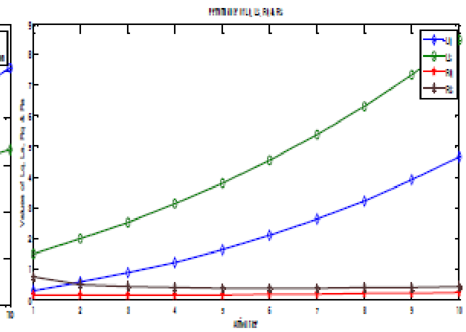
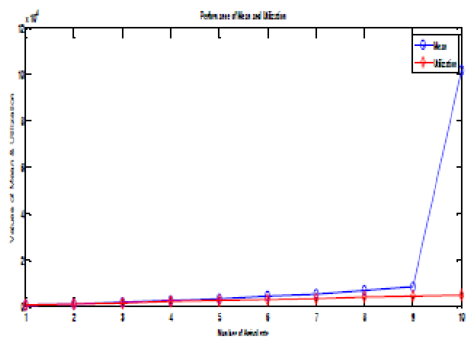


Fig. 1. Probability (Mean) and Utilization vs. Arrival rate. **Fig. 2.** Performance of Lq, Ls, Rq and Rs vs. Arrival rate.



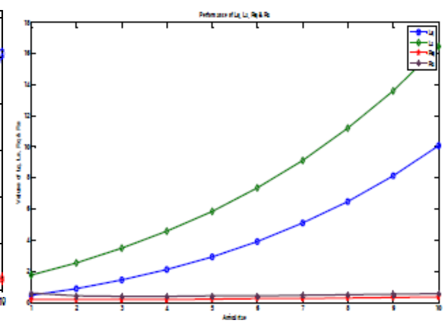
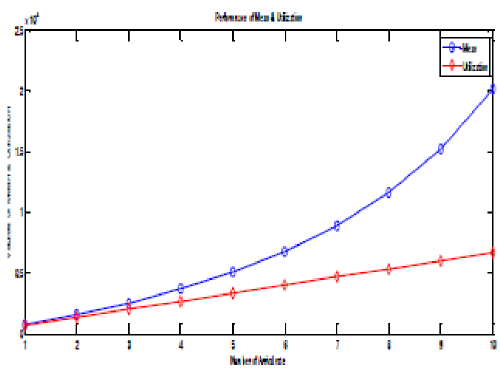


Fig. 3. Probability (Mean) and Utilization vs. Arrival rate. **Fig. 4.** Performance of Lq, Ls, Rq, Rs and Rs vs. Arrival rate.



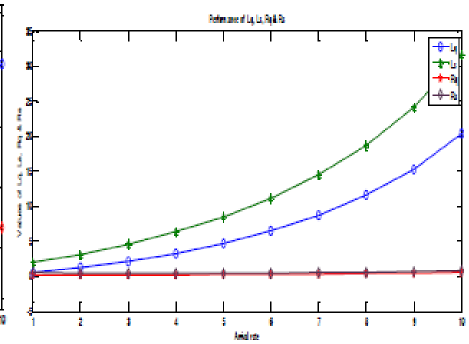
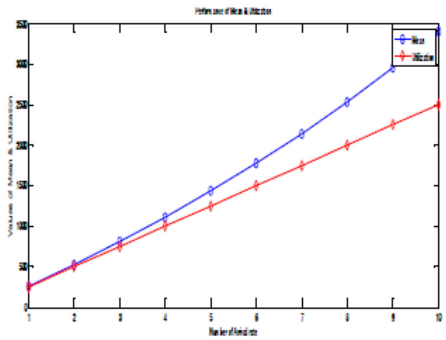


Fig. 5. Probability (Mean) and Utilization vs. Arrival rate. **Fig. 6.** Performance of Lq, Ls, Rq, Rs and Rs vs. Arrival rate.



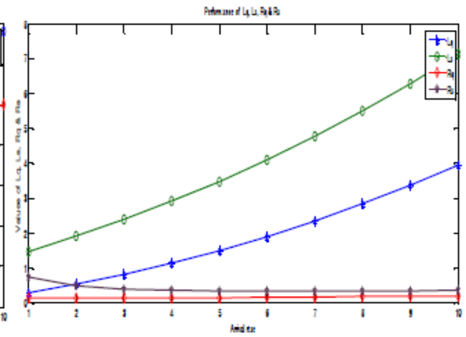
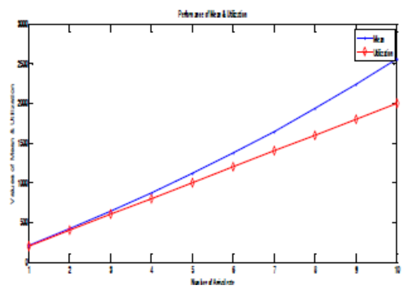


Fig. 7. Probability (Mean) and Utilization vs. Arrival rate. **Fig. 8.** Performance of Lq, Ls, Rq, Rs vs. Arrival rate.



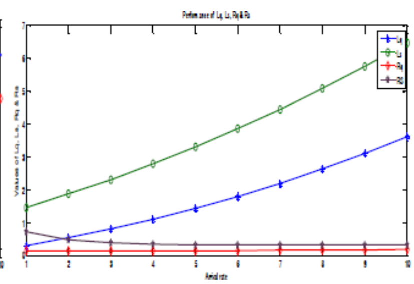


Fig. 9. Probability (Mean) and Utilization vs. Arrival rate. **Fig. 10.** Performance of Lq, Ls, Rq, Rs and Rs vs. Arrival rate.

Figures 1, 3, 5, 7 and 9 are depicted steady-state probability (mean) and utilization vs. number of arrival. Steady-state probability and utilization are going to increase corresponding to growing arrival rate. This indicates the better performance of the proposed model. However, performance of mean number of customers in the system, mean number of customers in the queue, response time in the system and queue vs. arrival rate are illustrated in figures 2, 4, 6, 8 and 10. The response time of the system under vacation and interruption corresponding to the increased arrival rate becomes goes down constantly.

6. Conclusion

The model studied here can be applied in many real-time systems such as information transmission systems, transportation, flexible manufacturing systems, etc. the considered model is more adaptable and represents more robust physical systems of bulk discipline. Steady-state probability generation functions are obtained various systems characteristic. We also examine the performance measures to analyze the effect of the parameters over the system. Our study may be useful to the system designers to achieve a better grade of service. The sensitivity analysis of the model reveals the bulk size distribution parameters, have significant influence on the system performance indices by varying bulk size parameters (arrival and service) the congestion in queues and mean delay in service, can be reduced.

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