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VERTEX PRIME AND EDGE TERNION SUM LABELING OF CERTAIN GRAPHS

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Abstract

A graph $\mathbb{G}(\mathbb{V}, \mathbb{E})$ is observed to admit vertex prime and edge ternion sum labeling when the vertices of the graph are labeled with unique integral values from $[1, |\mathbb{V}|]$ in a way that for each edge uv, the end vertices u and v are designated labels that share no common positive factors except 1 and the edges of the graph are assigned labels with distinct integral values from $[1, |\mathbb{V}|+|\mathbb{E}|]$ such that the ternion sum of the designated labels of the vertices u, v and the edge uv is an unique constant. In this research article we have instigated a new category of labeling named as vertex prime and edge ternion sum labeling and we investigate that the path, odd cycle, star, comb, coconut tree and crown graphs admit vertex prime and edge ternion sum labeling and we also establish and deduce the ternion sum of the above stated graphs.

1. Introduction

In this research article the graphs examined are non-oriented, non-disconnected, finite with no multiple edges and loops. We symbolize the vertex set by $V(G)$ with cardinality $|\mathbb{V}(\mathbb{G})|$ and the edge set by $\mathbb{E}(\mathbb{G})$ with cardinality $|\mathbb{E}(\mathbb{G})|$. Graph labeling is one of the important branches of graph

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theory in which the vertices or edges or both are assigned integral values with some conditions. There are various applications of graph labeling such as astronomy, management of database, theory of coding, communication network addressing, x-ray crystallography, circuit design and much more. The notion of Prime labeling was introduced by R. Entringer. In this research article we have instigated a new category of labeling named as vertex prime and edge ternion sum labeling and we investigate that the path, odd cycle, star, comb, coconut tree and crown graphs admit vertex prime and edge ternion sum labeling. We also establish and deduce the ternion sum of the above stated graphs.

2. Preliminaries

Definition 2.1 [3]. A star graph $K_{1,n}, n \geq 3$ is constructed by considering a single vertex with n pendent edges from the single vertex.

Definition 2.2 [6]. The path graph $P_n, n \geq 2$ is constructed by forming a route from one vertex to another by virtue of placing edges linking successive vertices of the path.

Definition 2.3 [6]. The cycle graph $C_n, n \geq 3$ is constructed by forming a route from one vertex to another by virtue of placing edges linking successive vertices where the last edge connects the last vertex with the starting vertex.

Definition 2.4 [1]. The comb graph $P_n \odot K_1$ is constructed by appending absolutely one pendent edge at every vertex of the path P_n for $n \geq 2$.

Definition 2.5 [4]. The coconut tree, $CT(m, n)$ is constructed by connecting path P_m with star $K_{1,n}$ by introducing an edge between end vertex of path P_m to the apex vertex of star $K_{1,n}$ whenever $m \geq 2$ and $n \geq 3$.

Definition 2.6 [5]. The crown graph $C_n \odot K_1, n \geq 3$ is constructed by attaching exactly one pendent edge at every vertex of the cycle C_n for $n \geq 3$.

Definition 2.7 [2]. A graph $\mathbb{G}(\mathbb{V}, \mathbb{E})$ is observed to admit prime labeling when the vertices of the graph are labeled with unique integral values from $[1, |V|]$ in a way that for each edge uv, the end vertices u and v are designated labels that share no common positive factors except 1.

3. Main Results

Definition 3.1. A graph $\mathbb{G}(\mathbb{V}, \mathbb{E})$ is observed to admit vertex prime and edge ternion sum labeling when the vertices of the graph are labeled with unique integral values from $[1, |\mathbb{V}|]$ in a way that for each edge uv, the end vertices u and v are designated labels that share no common positive factors except 1 and the edges of the graph are assigned labels with distinct integral values from $[1, |\mathbb{V}| + |\mathbb{E}|]$ such that the ternion sum of the designated labels of the vertices u, v and the edge uv is an unique constant.

Definition 3.2. The ternion sum for a graph $\mathbb{G}$ is defined as the unique constant obtained by taking the sum of the designated labels of the vertices u, v and the edge uv of the graph $\mathbb{G}$. It is denoted by $\mathbb{T}_s(\mathbb{G})$.

4. Vertex prime and edge ternion sum labeling of path graph

Theorem 4.1. *The path graph P_n for $n \geq 2$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(P_n) = 2n$.*

Proof. Let P_n be the Path for $n \geq 2$.

Then $\mathbb{V}(P_n) = \{v_1, v_2, \ldots, v_n\}$. In general, $\mathbb{V}(P_n) = \{v_i / 1 \leq i \leq n\}$ and $|\mathbb{V}(P_n)| = n$

The Edge set $\mathbb{E}(P_n) = \{e_1, e_2, \ldots, e_{n-1}\}$.

In general, $\mathbb{E}(P_n) = \{e_i = v_i v_{i+1} / 1 \leq i \leq n-1\}$ and $|\mathbb{E}(P_n)| = n - 1$

Let us define the labeling of the vertices as a bijective mapping, $f : \mathbb{V}(P_n) \to \{1, 2, 3, \ldots, n\}$ by

$$f(v_i) = i, 1 \leq i \leq n$$

Let us define the labeling of the edges as an injective mapping, $g : \mathbb{E}(P_n) \to \{1, 2, 3, \ldots, 2n-1\}$ by $g(e_i) = 2n - 1 - 2i, 1 \le i \le n-1$

Then for the edges $v_i v_{i+1}$, we observe

$$gcf(f(v_i), f(v_{i+1})) = gcf(i, i+1) = 1, 1 \le i \le n-1$$

we also observe that for $1 \le i \le n-1$, the ternion sum, $\mathbb{T}_s(P_n)$ of the labels

$$f(v_i) + f(v_{i+1}) + g(e_i) = (i) + (i+1) + (2n - 1 - 2i)$$

$$= i + i + 1 + 2n - 1 - 2i = 2n, \text{ a unique constant.}$$

Therefore, path graph P_n admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(P_n) = 2n$.

The path graph and its vertex prime and edge ternion sum labeling are as in the following figures 4.1.1, 4.1.2 and 4.1.3 respectively.

Figure 4.1.1. Path graph P_n.

Illustration 4.1.

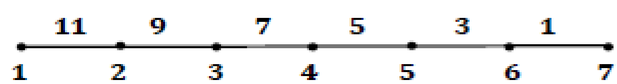

Figure 4.1.2. vertex prime and edge ternion sum labeling of path graph P_7 with $\mathbb{T}_s(P_7) = 14$.

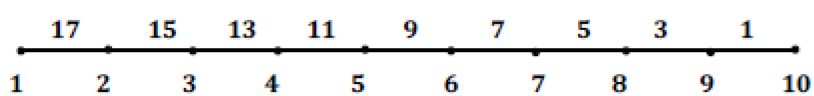

Figure 4.1.3. vertex prime and edge ternion sum labeling of path graph P_{10} with $\mathbb{T}_s(P_{10}) = 20$.

5. Vertex prime and edge ternion sum labeling of odd cycle graph

Theorem 5.1. *The odd cycle* C_n *for* $n \geq 3$ *and n is odd, admits vertex prime and edge ternion sum labeling with ternion sum* $\mathbb{T}_s(C_n) = 2n$.

Proof. Let C_n be the Cycle graph for $n \geq 3$ and n is odd.

Then $\mathbb{V}(C_n) = \{u_1, u_2, \ldots, u_n\}$. In general, $\mathbb{V}(C_n) = \{u_i / 1 \leq i \leq n\}$ and $|\mathbb{V}(C_n)| = n$

The Edge set $\mathbb{E}(C_n) = \{e_1, e_2, \ldots, e_n\}$.

In general, $\mathbb{E}(C_n) = \{e_i = u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{e_n = u_n u_1\}$ and $|\mathbb{E}(C_n)| = n$.

Let us define the labeling of the vertices as a bijective mapping, $f : \mathbb{V}(C_n) \to \{1, 2, 3, \ldots, n\}$ by, $f(u_i) = i, 1 \leq i \leq n$.

Let us define the labeling of the edges as an injective mapping, $g : \mathbb{E}(C_n) \to \{1, 2, 3, \ldots, 2n\}$ by $g(e_i) = 2n - 1 - 2i, 1 \leq i \leq n-1$

$g(e_n) = n - 1$

Then for the edges $u_i u_{i+1}$, we observe

$gcf(f(u_i), f(u_{i+1})) = gcf(i, i+1) = 1, 1 \leq i \leq n-1$ and

$gcf(f(u_i), f(u_1)) = gcf(n, 1) = 1$

we also observe that for $1 \leq i \leq n-1$, the ternion sum $\mathbb{T}_s(C_n)$ of the labels

$f(u_i) + f(u_{i+1}) + g(e_i) = (i) + (i+1) + (2n - 1 - 2i)$

$= i + i + 1 + 2n - 1 - 2i = 2n$, a unique constant

Also, $f(v_n) + f(v_1) + g(e_n) = (n) + (1) + (n-1)$

$= n + 1 + n - 1 = 2n$, a unique constant

Therefore, odd cycle C_n admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(C_n) = 2n$.

The cycle graph and its vertex prime and edge ternion sum labeling are as in the following figures 5.1.1 and 5.1.2 respectively.

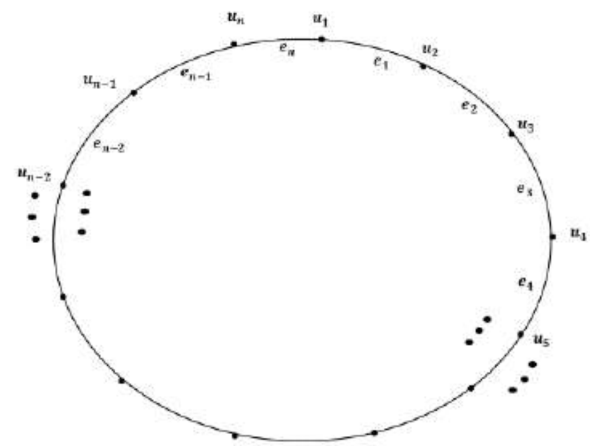

Figure 5.1.1. Cycle graph C_n.

Illustration 5.1.

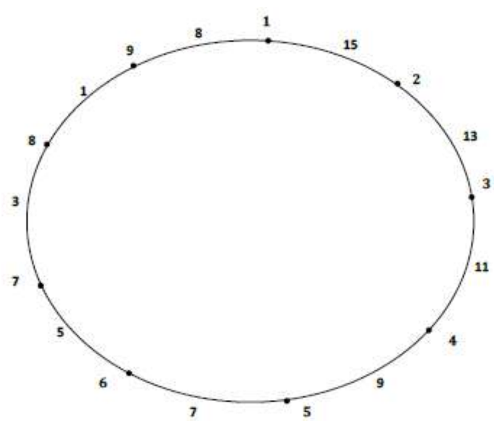

Figure 5.1.2. vertex prime and edge ternion sum labeling of cycle graph C_9 with $\mathbb{T}_s(C_9) = 18$.

6. Vertex prime and edge ternion sum labeling of star graph

Theorem 6.1. *The star graph* $K_{1,\,n}$ *for* $n \geq 3$ *admits vertex prime and edge ternion sum labeling with ternion sum* $\mathbb{T}_s(K_{1,\,n}) = n + 3$.

Proof. Let $K_{1, n}$ be the star graph for $n \geq 3$.

The vertex set $\mathbb{V}(K_{1, n}) = \{u, v_1, v_2, \ldots, v_n\}$. In general, $\mathbb{V}(K_{1, n}) = \{u, v_i / 1 \leq i \leq n\}$ and $|\mathbb{V}(K_{1, n})| = n + 1$

The Edge set $\mathbb{E}(K_{1, n}) = \{e_1, e_2, \ldots, e_n)$.

In general, $\mathbb{E}(K_{1, n}) = \{e_i = uv_i / 1 \leq i \leq n\}$ and $|\mathbb{E}(K_{1, n})| = n$

Let us define the labeling of the vertices as a bijective mapping,

$f : \mathbb{V}(K_{1, n}) \to \{1, 2, 3, \ldots, n + 1\}$

$f(u) = 1$

$f(v_i) = i + 1, 1 \leq i \leq n$

Let us define the labeling of the edges as an injective mapping, $g : \mathbb{E}(K_{1, n}) \to \{1, 2, 3, \ldots, 2n + 1\}$ by $g(e_i) = n - i + 1, 1 \leq i \leq n$

Then we observe that for edges uv_i,

$gcf(f(u), f(v_i)) = gcf(1, i + 1) = 1, 1 \leq i \leq n$ and

we also observe that for $1 \leq i \leq n$, the ternion sum $\mathbb{T}_s(K_{1, n})$ of the labels

$f(u) + f(v_i) + g(e_i) = (1) + (i + 1) + (n - i + 1)$

$= 1 + i + 1 + n - i + 1 = n + 3$, a unique constant

Therefore, the star graph $K_{1, n}$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(K_{1, n}) = n + 3$.

The star graph and its vertex prime and edge ternion sum labeling are as in the following figures 6.1.1, 6.1.2 and 6.1.3 respectively.

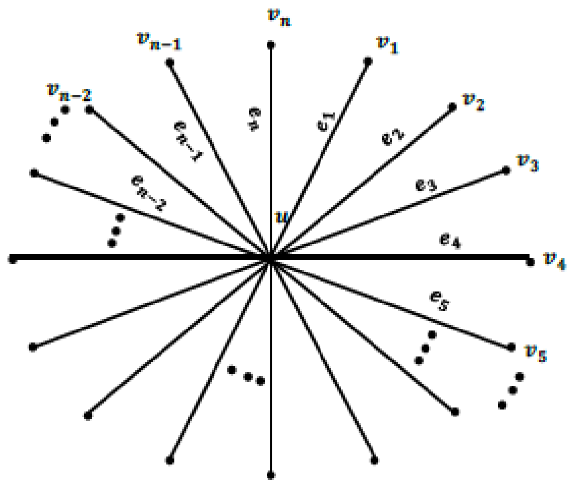

Figure 6.1.1. Star graph.

Illustration 6.1.

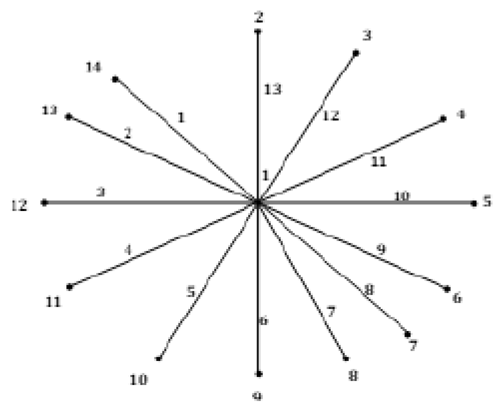

Figure 6.1.2. Vertex prime and edge ternion sum labeling of star graph $K_{1,\,13}$ with $T_s(K_{1,\,13}) = 16$.

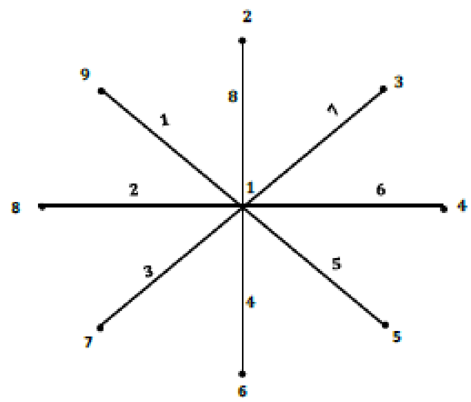

Figure 6.1.3. Vertex prime and edge ternion sum labeling of star graph $K_{1,\,8}$ with $T_s(K_{1,\,8}) = 11$.

7. Vertex prime and edge ternion sum labeling of comb graph

Theorem 7.1. *The comb graph* $P_n \odot K_1$ *for* $n \geq 2$ *admits vertex prime and edge ternion sum labeling with ternion sum* $\mathbb{T}_s(P_n \odot K_1) = 4n$.

Proof. Let $\mathbb{G} = P_n \odot K_1$ be the Comb graph for $n \geq 2$.

Then $\mathbb{V}(\mathbb{G}) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$. In general, $\mathbb{V}(\mathbb{G}) = \{u_i, v_i / 1 \leq i \leq n\}$ and $|\mathbb{V}(\mathbb{G})| = 2n$ and $\mathbb{E}(\mathbb{G}) = \{e_1, e_2, \ldots, e_{n-2}, e_{n-1}, x_1, x_2, \ldots, x_n)$.

In general, $\mathbb{E}(\mathbb{G}) = \{e_i = u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{x_i = u_i v_i / 1 \leq i \leq n\}$ and $|\mathbb{E}(\mathbb{G})| = 2n - 1$.

Let us define the labeling of the vertices as a bijective mapping, $f : \mathbb{V}(\mathbb{G}) \to \{1, 2, 3, \ldots, 2n\}$ $f(u_i) = 2i - 1,\ 1 \leq i \leq n$

$$f(v_i) = 2i,\ 1 \leq i \leq n$$

Let us define the labeling of the edges as an injective mapping, $g : \mathbb{E}(\mathbb{G}) \to \{1, 2, 3, \ldots, 2n - 1\}$ by

$$g(e_i) = 4n - 4i,\ 1 \leq i \leq n - 1$$

$$g(x_i) = 4n - 4i + 1,\ 1 \leq i \leq n$$

Then, we observe

For the edges $u_i u_{i+1}$, $gcf(f(u_i), f(u_{i+1}))$

$= gcf(2i-1, 2i+1) = 1, 1 \leq i \leq n-1$

for the edges $u_i v_i$, $gcf(f(u_i), f(u_i)) = gcf(2i-1, 2i) = 1, 1 \leq i \leq n$ and

we also observe that for $1 \leq i \leq n-1$, the ternion sum $\mathbb{T}_s(P_n \odot K_1)$ of the labels

$f(u_i) + f(u_{i+1}) + g(e_i) = (2i-1) + (2i+1) + (4n-4i)$

$= 2i - 1 + 2i + 1 + 4n - 4i = 4n$, a unique constant

Also, for $1 \leq i \leq n-1$,

$f(u_i) + f(u_i) + g(x_i) = (2i-1) + (2i) + (4n-4i+1)$

$= 2i - 1 + 2i + 4n - 4i + 1 = 4n$, a unique constant.

Therefore, the comb graph $P_n \odot K_1$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(P_n \odot K_1) = 4n$.

The comb graph and its vertex prime and edge ternion sum labeling are as in the following figures 7.1.1, 7.1.2 and 7.1.3 respectively.

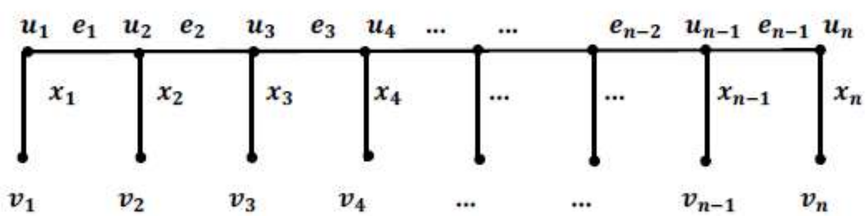

Figure 7.1.1. Comb graph.

Illustration 7.1.

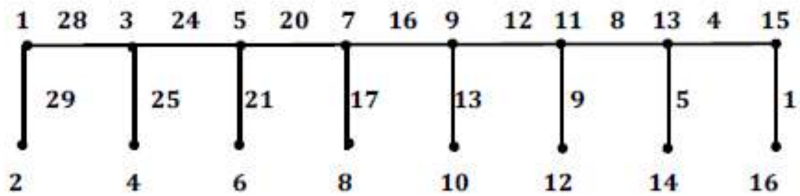

Figure 7.1.2. vertex prime and edge ternion sum labeling of comb graph $P_8 \odot K_1$ with $\mathbb{T}_s(P_8 \odot K_1) = 32$.

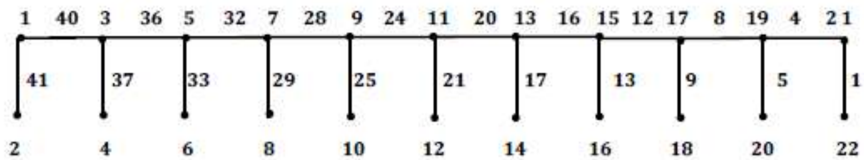

Figure 7.1.3. Vertex prime and edge ternion sum labeling of comb graph $P_{11} \odot K_1$ with $\mathbb{T}_s(P_{11} \odot K_1) = 44$.

8. Vertex prime and edge ternion sum labeling of coconut tree graph

Theorem 8.1. *The coconut tree graph $CT(m, n)$ for $m \geq 2, n \geq 3$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(CT(m, n)) = 2m + 2n + 2$.*

Proof. Let $CT(m, n) = P_m \odot K_{1, n}$ be the coconut tree graph for $m \geq 2, n \geq 3$.

The vertex set $\mathbb{V}(CT(m, n)) = \{u, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m\}$.

In general, $\mathbb{V}(CT(m, n)) = \{u, v_i, u_j / 1 \leq i \leq n, 1 \leq j \leq m\}$ and $|\mathbb{V}(CT(m, n))| = m + n + 1$.

The Edge set $\mathbb{E}(CT(m, n)) = \{e_1, e_2, \ldots, e_m, x_1, x_2, \ldots, x_n\}$.

In general, $\mathbb{E}(CT(m, n)) = \{e_j = u_j u_{j+1} / 1 \leq j \leq m - 1\} \cup \{em = u_m u\} \cup \{x_i = u_i v_i / 1 \leq i \leq n\}$ and $|\mathbb{E}(CT(m, n))| = m + n$.

Let us define a vertex labeling $f : \mathbb{V}(CT(m, n)) \to \{1, 2, 3, \ldots, m + n + 1\}$
$f(u) = 1$

$$f(v_i) = i + 1, 1 \le i \le n$$

$$f(u_j) = n + 1 + j, 1 \le j \le m$$

Let us define an edge labeling

$$g : \mathbb{E}(CT(m, n)) \to \{1, 2, 3, \ldots, 2m + 2n + 1\} \text{ by}$$

$$g(e_j) = 2m - 2j - 1, 1 \le j \le m - 1 \; g(e_m) = m + n$$

$$g(x_i) = 2m + 2n - i, 1 \le i \le n$$

Then, we observe

for the edges $u_j u_{j+1}$, $gcf(f(u_j), f(u_{j+1}))$

$= gcf(n + 1 + j, n + 2 + j) = 1, 1 \le j \le m - 1$

for the edges $u_m u$, $gcf(f(u_m), f(u)) = gcf(n + 1 + m, 1) = 1$ and for the edges uv_i, $gcf(f(u), f(v_i)) = gcf(1, i + 1) = 1, 1 \le i \le n$ and we also observe that for $1 \le j \le m - 1$, the ternion sum $\mathbb{T}_s(CT(m, n))$ of the labels $f(u_j) + f(u_{j+1}) + g(e_j) = (n + 1 + j) + (n + 1 + j + 1) + (2m - 2j - 1)$

$$= n + 1 + j + n + 1 + j + 1 + 2m - 2j - 1$$

$= 2m + 2n + 2,$ a unique constant

Also, for $1 \le i \le n - 1$,

$f(u_m) + f(u) + g(e_m) = (n + 1 + m) + (1) + (m + n)$

$$= n + 1 + m + 1 + m + n = 2m + 2n + 2$$

Also, for $1 \le i \le n$,

$f(u) + f(v_i) + g(x_i) = (1) + (i + 1) + (2m + 2n - i)$

$$= 1 + i + 1 + 2m + 2n - i$$

$= 2m + 2n + 2,$ a unique constant.

Therefore, the coconut tree graph $CT(m, n)$ for $m \ge 2, n \ge 3$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(CT(m, n)) = 2m + 2n + 2$.

The coconut tree graph and its vertex prime and edge ternion sum labeling are as in the following figures 8.1.1, 8.1.2 and 8.1.3 respectively.

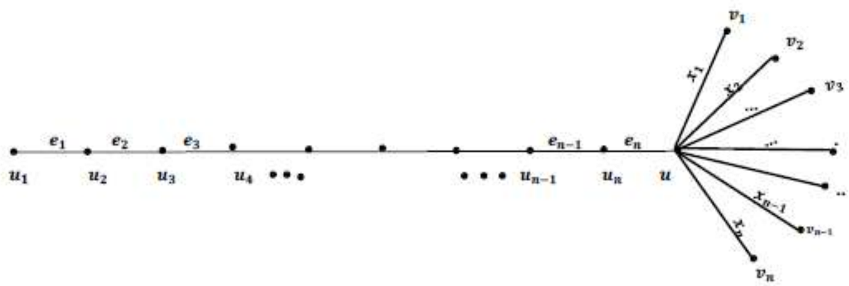

Figure 8.1.1. Coconut Tree graph.

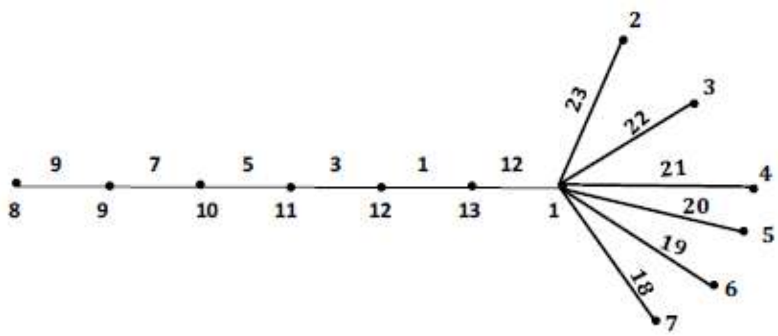

Figure 7.1.2. Vertex prime and edge ternion sum labeling of coconut tree graph $CT(6, 6)$ with $\mathbb{T}_s(CT(6, 6)) = 26$.

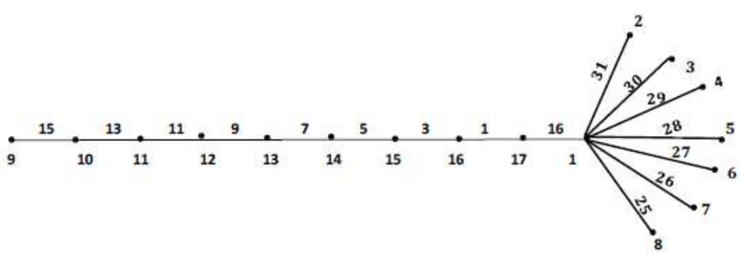

Figure 7.1.3. Vertex prime and edge ternion sum labeling of coconut tree graph $CT(9, 7)$ with $\mathbb{T}_s(CT(9, 7)) = 34$.

9. Vertex prime and edge ternion sum labeling of crown graph

Theorem 9.1. *The crown graph* $C_n \odot K_1$ *for* $n \geq 3$ *and n is odd admits vertex prime and edge ternion sum labeling with ternion sum* $\mathbb{T}_s(C_n \odot K_1) = 4n$.

Proof. Let $\mathbb{G} = C_n \odot K_1$ be the Crown graph.

The vertex set $\mathbb{V}(\mathbb{G}) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$. In general, $\mathbb{V}(\mathbb{G}) = \{u, v_i / 1 \leq i \leq n\}$ and $|\mathbb{V}(\mathbb{G})| = 2n$ and $\mathbb{E}(\mathbb{G}) = \{e_1, e_2, \ldots, e_{n-1}, e_n, x_1, x_2, \ldots, x_n\}$.

In general, $\mathbb{E}(\mathbb{G}) = \{e_i = u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{e_i = u_n u_1\} \cup \{x_i = u_i v_i / 1 \leq i \leq n\}$ and $|\mathbb{E}(\mathbb{G})| = 2n$.

Let us define a vertex labeling, $f : \mathbb{V}(\mathbb{G}) \to \{1, 2, 3, \ldots, 2n\}$

$f(u_i) = 2i - 1, 1 \leq i \leq n$

$f(v_i) = 2i, 1 \leq i \leq n$

Let us define an edge labeling $g : \mathbb{E}(\mathbb{G}) \to \{1, 2, 3, \ldots, 2n\}$ by

$g(e_n) = 2n$

$g(e_i) = 4n - 4i, 1 \leq i \leq n - 1$

$g(x_i) = 4n - 4i + 1, 1 \leq i \leq n$

Then, we observe

For the edges $u_i u_{i+1}$, $gcf(f(u_i), f(u_{i+1}))$

$= gcf(2i - 1, 2i + 1) = 1, 1 \leq i \leq n - 1$

for the edges $u_i v_i$, $gcf(f(u_i), f(v_i)) = gcf(2i - 1, 2i) = 1, 1 \leq i \leq n$ and

we also observe that for $1 \leq i \leq n$, the ternion sum $\mathbb{T}_s(C_n \odot K_1)$ of the labels $f(u_i) + f(u_{i+1}) + g(e_i) = (2i - 1) + (2i + 1) + (4n - 4i)$

$$= 2i - 1 + 2i + 1 + 4n - 4i$$

$= 4n,$ a unique constant

$f(u_n) + f(u_1) + g(e_n) = (2n - 1) + (1) + (2n)$

$$= 2n - n + 1 + 1 + 2n$$

$= 4n,$ a unique constant

Also, $f(u_i) + f(u_i) + g(x_i) = (2i - 1) + (2i) + (4n - 4i + 1)$

$$= 2i - 1 + 2i + 4n - 4i + 1$$

$= 4n,$ a unique constant.

Therefore, the crown graph $C_n \odot K_1$ admits vertex prime and edge ternion sum labeling with ternion sum $\mathbb{T}_s(C_n \odot K_1) = 4n.$

The crown graph and its vertex prime and edge ternion sum labeling are as in the following figures 9.1.1 and 9.1.2 respectively.

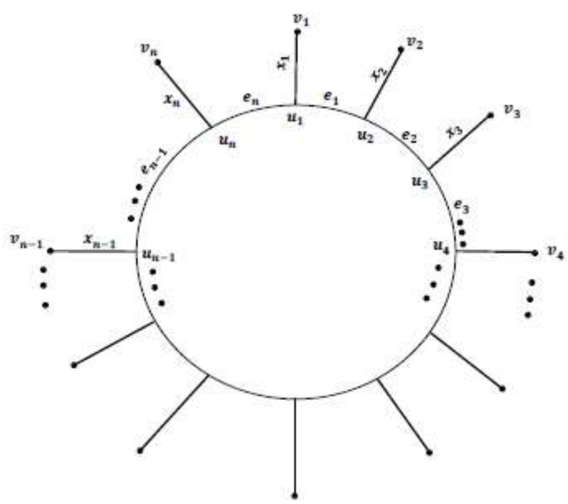

Figure 9. 1. 1. Crown graph.

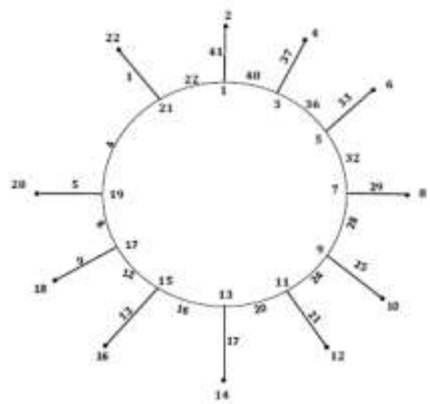

Figure 9.1.2. Vertex prime and edge ternion sum labeling of crown graph $C_{11} \odot K_1$ with $\mathbb{T}_s(C_{11} \odot K_1) = 44$.

10. Conclusion

In this research article we have instigated a new category of labeling named as vertex prime and edge ternion sum labeling and we have also investigated that the graphs such as the path, odd cycle, star, comb, coconut tree and crown graphs admit vertex prime and edge ternion sum labeling and we have also deduced the ternion sum of the each of the above stated graphs. To investigate graphs that admit vertex prime and edge ternion sum labeling is interesting and engrossing.

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