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PROPERTIES OF EDGE DEGREE SEQUENCE OF INTUITIONISTIC FUZZY GRAPHS UNDER ISOMORPHISMS

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Abstract

In this paper, edge degree sequence of intuitionistic fuzzy graph (IFG) is introduced and some of its properties are studied. If two intuitionistic fuzzy graphs are isomorphic or co-weak isomorphic, then they must have same edge degree sequence. But weak isomorphism need not preserve edge degree sequence.

1. Introduction

Rosenfeld introduced the concept of fuzzy graphs in 1975 [10]. Bhattacharya [2] gave some remarks on fuzzy graphs. K. R. Bhutani also introduced the concepts of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs [2]. A. Nagoor Gani and J. Malarvizhi studied isomorphism on fuzzy graphs [6]. K. Radha and A. Rosemine introduced degree sequence of fuzzy graph [9]. K. T. Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic Fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry, economics and washing machine. R. Parvathi and M. G. Karunambigai discussed some concepts in intuitionistic fuzzy graphs [5]. R. Parvathi and M.

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G. Karunambigaiand and R. Buvaneswari introduced constant intuitionistic fuzzy graphs [4]. In this paper, we introduce edge degree sequence of intuitionistic fuzzy graph discuss about the edge degree sequence of isomorphic, co-weak isomorphic and weak isomorphic intuitionistic fuzzy graphs.

Definition 1.1 [5]. $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ is an intuitionistic fuzzy graph (IFG) on $G^* = (V, E)$ where

(i) $V = \{v_1, v_2, v_3, \ldots, v_n\}$, $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership value and the degree of non-membership value of the elements $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V$, $i = 1, 2, 3, \ldots, n$.

(ii) $E \subseteq V \times V$, $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j)$ and $\gamma_2(v_i v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j)$ and $0 \le \mu_2(v_i v_j) + \gamma_2(v_i v_j) \le 1$ for every $v_i v_j \in E$, $i, j = 1, 2, 3, \ldots, n$. [5]

Definition 1.2 [8]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. Then degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d\mu_1(v_i), d\gamma_1(v_i))$ where $d\mu_1(v_i) = \sum_{\mu_2}(v_i v_j)$ and $d\gamma_1(v_i) = \sum_{\gamma_2}(v_i v_j)$, where the summation runs over all $v_i v_j \in E$.

Definition 1.3 [8]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. Then degree of an edge $uv \in E$ is defined by $d(uv) = (d\mu_2(uv), d\gamma_2(uv))$ where $d\mu_2(uv) = d\mu_1(u) + d\mu_1(v) - 2\mu_2(uv)$ and $d\gamma_2(uv) = d\gamma_1(u) + d\gamma_1(v) - 2\gamma_2(uv)$.

Definition 1.4 [9]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G : (\mu_1', \gamma_1'; \mu_2', \gamma_2')$ be two IFGs on $G^* = (V, E)$ and $G^* = (V', E')$ respectively. A homomorphism of IFG $h : G \to G'$ is a map $h : V \to V'$ such that $\mu_1(x) \le \mu_1(h(x)) \forall x \in V$, $\gamma_1(x) \le \gamma_1(h(x)) \forall x \in V$ and $\mu_2(xy) \le \mu_2(h(x)h(y))$ $\forall x, y \in V$, $\gamma_2(xy) \le \gamma_2(h(x)h(y))$, $\forall x, y \in V$.

Definition 1.5 [9]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G' : (\mu_1', \gamma_1'; \mu_2', \gamma_2')$ be two IFGs on $G^* = (V, E)$ and $G^* = (V', E')$ respectively. A weak isomorphism of IFGs $h : G \to G'$ is map $h : V \to V'$ which is a bijective homomorphism that satisfies

$$\mu_1(x) = \mu_1(h(x)), \forall x \in V, \gamma_1(x) = \gamma_1'(h(x)), \forall x \in V$$

and

$$\mu_2(xy) \leq \mu_2'(h(x)h(y)), \forall x, y \in V, \gamma_2(xy) \leq \gamma_2'(h(x)h(y)), \forall x, y \in V.$$

Definition 1.6 [9]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G' : (\mu_1', \gamma_1'; \mu_2', \gamma_2')$ be two IFGs on $G^* = (V, E)$ and $G'^* = (V', E')$ respectively. A co-weak isomorphism of IFGs $h : G \to G'$ is map $h : V \to V'$ which is a bijective homomorphism that satisfies

$$\mu_1(x) = \mu_1(h(x)), \forall x \in V, \gamma_1(x) \leq \gamma_1'(h(x)), \forall x \in V$$

and

$$\mu_2(xy) = \mu_2'(h(x)h(y)), \forall x, y \in V, \gamma_2(xy) = \gamma_2'(h(x)h(y)), \forall x, y \in V.$$

Definition 1.7 [9]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G' : (\mu_1', \gamma_1'; \mu_2', \gamma_2')$ be two IFGs on $G^* = (V, E)$ and $G'^* = (V', E')$ respectively. An isomorphism of IFGs $h : G \to G'$ is map $h : V \to V'$ which is a bijective homomorphism that satisfies

$$\mu_1(x) = \mu_1'(h(x)), \forall x \in V, \gamma_1(x) = \gamma_1'(h(x)), \forall x \in V$$

and

$$\mu_2(xy) = \mu_2'(h(x)h(y)), \forall x, y \in V, \gamma_2(xy) = \gamma_2'(h(x)h(y)), \forall x, y \in V.$$

2. Degree Sequence

Definition 2.1. We use the following order relation to compare two ordered pairs of real numbers:

(i) $(u, v) = (x, y)$ if and only if $u = x$ and $v = y$.

(ii) $(u, v) > (x, y)$ if and only if either $u > x$ or $u = x$ and $v > y$.

Definition 2.2 [7]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. A sequence of ordered pairs of real numbers $(d_1, d_2, d_3, \ldots, d_n)$ with $d_1 \geq d_2 \geq d_3 \geq, \ldots, \geq d_n$, where $d_i = d(v_i) = (d\mu_2(v_i), d\gamma_2(v_i))$, is the degree sequence of the IFG *G*.

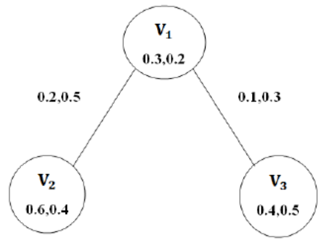

Figure 2.1

Here $d(v_1) = (0.3, 0.8)$, $d(v_2) = (0.2, 0.5)$ and $d(v_3) = (0.1, 0.3)$. Hence the degree sequence of *G* is ((0.3, 0.8), (0.2, 0.5), (0.1, 0.3)).

Definition 2.3 [7]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. A sequence $\xi = (d_1, d_2, d_3, \ldots, d_n) = ((d_{11}, d_{12}), (d_{21}, d_{22}), \ldots, (d_{n1}, d_{n2}))$ of ordered pairs of real number is said to be a graphic sequence of IFG if there exists an IFG *G* whose vertices have degrees $d_1, d_2; d_3, \ldots, d_n$ and *G* is called realization of ξ.

3. Edge Degree Sequence of Isomorphic IFGs

Definition 3.1. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. A sequence of ordered pairs of positive real numbers $(d_1, d_2, d_3, \ldots, d_n)$ with $d_1 \geq d_2 \geq d_3 \geq, \ldots, \geq d_n$, where $d_i = d(e_i) = (d\mu_2(e_i), d\gamma_2(e_i))$, is the edge degree sequence of the IFG *G*.

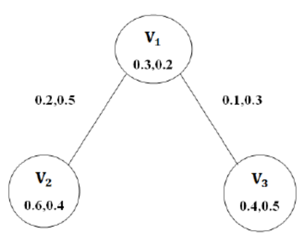

Figure 3.1.

Here $d(v_1v_2) = (0.1, 0.3)$ and $d(v_1v_3) = (0.2, 0.5)$. Hence the edge degree sequence of G is ((0.2, 0.5), (0.1, 0.3)).

Definition 3.2. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. A sequence $\xi = (d_1, d_2, d_3, \ldots, d_n) = ((d_{11}, d_{12}), (d_{21}, d_{22}), \ldots, (d_{n1}, d_{n2}))$ of ordered pairs of positive real number is said to be an edge-graphic sequence of IFG if there exists an IFG G whose edges have degrees $(d_{11}, d_{12}), (d_{21}, d_{22}), \ldots, (d_{n1}, d_{n2})$ and G is called realization of ξ.

For example, G is an edge graphic realization of ((0.2, 0.5), (0.1, 0.3)).

Theorem 3.1. *If G and G' are isomorphic IFGs, then the edge degree sequence of G and G' are same.*

Proof. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G' : (\mu_1', \gamma_1'; \mu_2', \gamma_2')$ be two isomorphic IFGs on $G^* = (V, E)$ and $G'^* = (V', E')$ respectively. Then there exists an isomorphism $h : G \to G'$ such that

$$\mu_1(x) = \mu_1'(h(x)), \forall x \in V, \gamma_1(x) = \gamma_1'(h(x)), \forall x \in V$$

and

$$\mu_2(xy) = \mu_2'(h(x)h(y)), \forall x, y \in V, \gamma_2(xy) = \gamma_2'(h(x)h(y)), \forall x, y \in V.$$

Let be any edge of G. Then there exists an edge uv of G' such that $h(u) = z$ and $h(v) = x$ with $h(u)h(v) = zx$. We have to prove that $d_G(uv) = d_{G'}(h(u)h(v))$.

Case (i) $d_G(uv) = (0, 0)$.

Then no edge of G is adjacent to uv. Therefore $d_G(u) = d_G(v) = (\mu_2(uv), \gamma_2(uv))$. Since G and G' are isomorphic intuitionistic fuzzy graphs, no edge of G' is adjacent to zx.

Hence $d_{G'}(z) = (d_{\mu_1'}(z), d_{\gamma_1'}(z)) = (\mu_2'(z), d_{\gamma_2'}(zx))$ and

$$d_{G'}(x) = (d_{\mu_1'}(x), d_{\gamma_1'}(x)) = (\mu_2'(x), \gamma_2'(zx)).$$

Therefore

$$d_{G'}(h(u)h(v)) = d_{G'}(xz)$$

$$= (d_{\mu'_1}(x) + d_{\mu'_1}(z) - 2\mu'_2(zx),\ d_{\gamma'_1}(x) + d_{\gamma'_1}(x) +_{\gamma'_1}(z) - 2\gamma'_2(zx))$$

$$= (\mu'_2(zx) + \mu'_2(zx) - 2\mu'_2 xd,\ \gamma'_2(zx) + \gamma'_2(zx) - 2\gamma'_2(zx))$$

$$= (0,\ 0)$$

$$= d_G(uv).$$

Case (ii) $d_G(uv) \neq (0,\ 0)$.

Since G and G' are isomorphic to each other and isomorphism preserves degree of the vertices, we have

$$d_{\mu_2}(uv) = d\mu_1(u) + d\mu_1(v) - 2\mu_2(uv)$$

$$= d\mu_1(h(u)) + d\mu_1(h(v)) - 2\mu'_2(h(u)h(v))$$

$$= d\mu_2(h(u)h(v)).$$

and

$$d\gamma_2(uv) = d\gamma_1(u) + d\gamma_1(v) - 2\gamma_2(uv)$$

$$= d\gamma_1(h(u)) + d\gamma_1(h(v)) - 2\gamma'_2(h(u)h(v))$$

$$= d\gamma'_2(h(u)h(v)).$$

Therefore

$$d_G(uv) = (d\mu_2(uv),\ d\gamma_2(uv))$$

$$= (d\mu'_2(h(u)h(v)),\ d\gamma'_2(h(u)h(v)))$$

$$= d_{G'}(h(u)h(v)) \forall uv \in E.$$

Since $uv \in E$ is arbitrarily chosen, the edge degree sequences of G and G' are same.

Remark 3.2. Two IFGs with same edge degree sequence need not be isomorphic. Consider the IFG in the following Figure 3.2.

The edge degree sequence of both G and G' is {(0.7, 0.7), (0.7, 0.7), (0.3, 0.9), (0.3, 0.9)}. But there is no bijective map from the vertex set of G to the vertex set of G' which carries v_1 with the same membership values. Hence G and G' cannot be isomorphic.

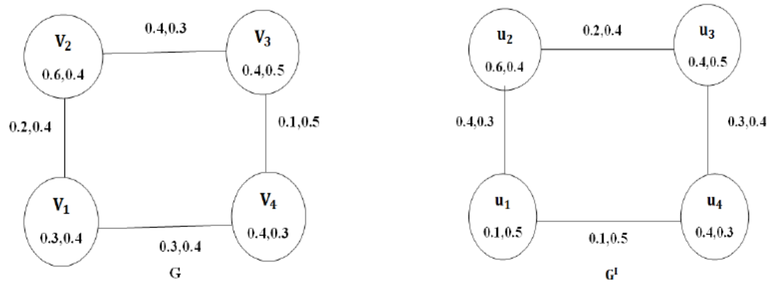

Figure 3.2.

Theorem 3.3. *Co-weak isomorphism of IFGs preserves edge degree sequence.*

Proof. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ and $G' : (\mu'_1, \gamma'_1; \mu'_2, \gamma'_2)$ be two isomorphic IFGs on $G^* : (V, E)$ and $G'^* : (V', E')$ respectively. Then there exists a bijective map $h : V \to V'$ which satisfies

$$\mu_1(x) = \mu'_1(h(x)), \forall x \in V, \gamma_1(x) = \gamma'_1(h(x)), \forall x \in V$$

and

$$\mu_2(xy) = \mu'_2(h(x)h(y)), \forall x, y \in V, \gamma_2(xy) = \gamma'_2(h(x)h(y)), \forall x, y \in V.$$

Then proceeding as in the proof of theorem 3.1, $d_G(uv) = d_{G'}(h(u)h(v)), \forall uv \in E$. Hence G and G' have identical edge degree sequence.

Remark 3.4. Two IFGs with same edge degree sequence need not be co-weak isomorphic. In the following figure 3.3, the edge degree sequence of both G and G' is {(0.7, 1), (0.6, 0.8), (0.5, 0.8)}. Since $\mu_1(v_1) = \mu'_1(u_1)$ and $\gamma_1(v_3) = \gamma'_1(u_3)$, under any bijection $h : V \to V'$, at most two vertices satisfy the inequality $\mu_1(u) \le \mu'_1(h(u))$. Hence G and G' cannot be co-weak isomorphic.

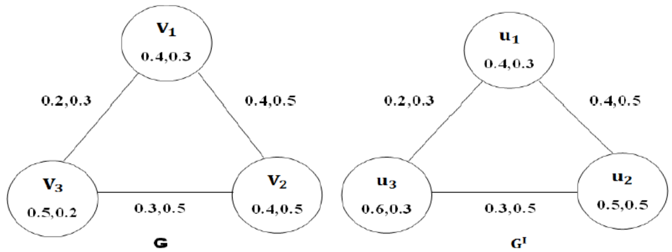

Figure 3.3.

Remark 3.5. Weak isomorphic IFGs need not preserve the edge degree sequence. For example consider the following figure 3.4. Let the bijective map $h : V \to V'$ be defined by $\mu_1(v_1) = u_1$, $h(v_2) = u_5$, $h(v_3) = u_4$, $h(v_4) = u_3$, $h(v_5) = u_2$, $\forall u, v \in V$.

It satisfies $\mu_1(v_i) = u_1'(h(v_i)) \forall v_i \in V$, $\gamma_1(v_i) = \gamma_1'(h(v_i))$, $\forall v_i \in V$,

$$\mu_2(u, v) = u_2'(h(u)h(v)) \forall u, v \in V, \ \gamma_2(uv) \leq \gamma_2(h(u)h(v)) \forall u, v \in V.$$

Hence G is weak isomorphic with G'.

But the edge degree sequence of G is ((0.9, 0.6), (0.8, 0.8), (0.6, 1.0), (0.6, 0.8), (0.5, 1.0)) and the edge degree sequence of G' is ((0.9, 0.9), (0.7, 1.0), (0.7, 1.0), (0.6, 1.1), (0.6, 1.1)) which are not identical.

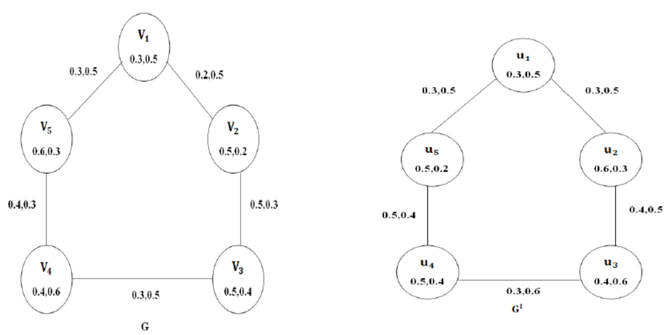

Figure 3.4

4. Properties of Edge Degree Sequence

Definition 4.1. If $d(uv) = (d\mu_2(uv), d\gamma_2(uv))$, then $\sum d(uv)$ is defined by

$$\sum d(uv) = \left(\sum d\mu_2(uv), \sum d\gamma_2(uv)\right).$$

That is, here we consider the coordinate wise addition of order pairs of real numbers.

Theorem 4.2. *Let* $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ *be a connected IFG on* $G^* = (V, E)$ *such that* $\mu_1(v) = c_1, \forall v \in V$ *and* $\gamma_1(v) = c_2 \forall v \in V$. *Then G is an effective IFG if and only if*

$$d_G(e) = (d\mu_2(e), d\gamma_2(e)) = (c_1 d_{G^*}(e), c_2 d_{G^*}(e)), \forall e \in E.$$

Proof. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$ such that, $\mu_1(v) = c, \forall v \in V$ and $\gamma_1(v) = c \forall v \in V$.

Assume that $d_G(e) = d_{G^*}(e)c \forall e \in E$. Suppose that G is not an effective IFG. Then there is an edge uv such that $\mu_2(uv) < \mu_1(u) \wedge \mu_1(v) = c_1$ and $\gamma_2(uv) < \gamma_1(u) \vee \gamma_1(v) = c_2$. Since G is connected, uv is adjacent to at least one other edge, say uw

Then

$$\begin{aligned} d_{\mu_2}(uw) &= \sum_{x \neq w} \mu_2(ux) + \sum_{y \neq u} \mu_2(yx) \\ &< \sum_{x \neq w} c_1 + \sum_{y \neq u} c_1 \\ &= c_1 \cdot d_{G^*}(uw) \end{aligned}$$

and

$$\begin{aligned} d_{\gamma_2}(uw) &= \sum_{x \neq w} \gamma_2(ux) + \sum_{y \neq u} \gamma_2(yx) \\ &< \sum_{x \neq w} c_2 + \sum_{y \neq u} c_2 \\ &= c_1 \cdot d_{G^*}(uw). \end{aligned}$$

Therefore, $d_G(uw) = (d\mu_2(uw), d\gamma_2(uw)) < (c_1 d_{G^*}(uw), c_2 d_{G^*}(uw))$ which is a contradiction. Hence G is effective.

Conversely, assume that G is an effective IFG.

Then $\mu_2(uv) = \mu_1(u) \wedge \mu_1(v) = c_1$ and $\gamma_2(u, v) = \gamma_1(u) \vee \gamma_1(v) = c_2 \, \forall \, uv \in E$.

Then $d_{\mu_2}(uv) = d_{G^*}(uv)c_1 \forall \, uv \in E$ and $d_{\gamma_2}(uv) = d_{G^*}(uv)c_2 \forall \, uv \in E$.

Theorem 4.3. *Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$ such that μ_2 and γ_2 are constant function with constant value c_1 and c_2 respectively. Then*

$$d_G(e) = (d\mu_2(e),\ d\gamma_2(e)) = (c_1 d_{G^*}(e),\ c_2 d_{G^*}(e)),\ \forall \, e \in E.$$

Proof. Since $\mu_2(e) = c_1$ and $\gamma_2(e) = c_2$

$$d_{\mu_2}(e) = \sum_{uv \in E} c_1 = d_{G^*}(e)c_1 \forall \, e \in E \text{ and}$$

$$d_{\gamma_2}(e) = \sum_{uv \in E} c_2 = d_{G^*}(e)c_2 \forall \, e \in E.$$

5. Conclusion

In this paper, Edge degree sequence of an IFG is introduced and some of its properties are studied. The Edge Degree sequence in isomorphic, weak isomorphic and co-weak isomorphic intuitionistic fuzzy graphs are discussed.

References

[1] T. K. Atanassov, Intuitionistic Fuzzy Sets, Theory, Application, Studies in Fuzziness and Soft Computing, Heidelberg, New York, Physica-Verlag, 1999.

[2] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Lett. 6 (1987), 297-302.

[3] K. R. Bhutani, On automorphism of fuzzy graphs, Pattern Recognition Letter 9 (1989), 159-162.

[4] M. G. Karunambigai, R. Parvathi and R. Buvaneswari, Constant intuitionistic fuzzy graphs, NIFS 17(1) (2011), 37-47.

[5] M. G. Karunambigai and R. Parvathi, Intuitionistic fuzzy graphs, Journal of Computational Intelligence, Theory and Application (2006), 139-150.

[6] A. Nagoor Gani and J. Malarvizhi, Isomorphism on fuzzy graphs, International Journal Computational and Mathematical Science 2(4) (2008).

[7] A. Nagoor Gani and K. Radha, Some sequence in fuzzy graphs, Far East Journal of Applied Mathematics, Volume 31, Number 3, 2008, 321-335.

[8] K. Radha and P. Pandian, Properties of degree sequence of intuitionistic fuzzy graphs under isomorphisms, Our Heritage, ISSN 0474-9030, Vol. 68, Issue 4, Jan. 2020, 367-378.

[9] K. Radha and A. Rosemine, Degree sequence of a fuzzy graph, Journal Academic Research Journal, 635-638.

[10] A. Rosenfeld, Fuzzy graphs, fuzzy sets and their application to cognitive and decision processes, Proc. U.S.-Japan Sem., Univ. Calif., Berkeley, Calif., 1974, Academic Press, New York (1995), 77-95.