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STEPHEN'S ALGORITHM FOR SOLVING ASSIGNMENT PROBLEMS

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Abstract

The aim of this paper is to present the efficient algorithm named as Stephen's Algorithm to find the optimal solution for assignment problems. This algorithm provides less number of iterations to reach the optimality. To validate this algorithm, a numerical example is solved and the results are compared with Hungarian method.

1. Introduction

An Assignment Problem is a subclass of transportation problem in which the ultimate aim is to assign a number of origins to equal number of destinations at a minimum cost or maximum profit. An Assignment problem is a variation of transportation problem with two characteristics (i) the cost matrix should be a square matrix and (ii) the optimum solution for the problem has only one assignment in a given row or column of the cost matrix.

Dutta and Pal [2] have developed modified version of Hungarian method. Stephen Dinagar and K. Palanivel [3] solved transportation problem in fuzzy environment. Sambasiva Rao and Maruthi Srinivas [4] proposed an effective algorithm to find the optimal solution of an assignment problem aiming to reduce computational cost. Sudha and Vanisri [5] have introduced an

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improved zero suffix method for optimal solution with least iterations of an assignment problem. Thiruppathi and Iranian [6] introduced an innovative method named as TVAM to find an optimal solution for any assignment problem. Bhavika M. Patel and Mitali J. Doshi [1] developed a modified approach of zero suffix method which is based on suffix value of zeroes which is obtained by row and column reduction in cost matrix. Only few of them have yielded optimum solutions.

Hungarian method is the most convenient and always produces optimal solution for A. P. In recent years, there is more number of algorithms and innovative methods available for assignment problem. Even though there are several alternative methods for A. P., very few of them yield optimal solution. The purpose of this paper is to present an effective algorithm named as Stephen's algorithm to find optimal solution for assignment problems with less computational cost.

The section 2 gives the tabulated form of assignment problem and its mathematical formulation of A. P. In section 3, Stephen's Algorithm is summarized. A relevant illustration is provided in section 4. A brief conclusion is given in section 5.

2. Assignment Problem

2.1. Assignment problem in the Tabulated form

Workers/Job	1	2	3	…	n
1	C_{11}	C_{12}	C_{13}	…	C_{1n}
2	C_{21}	C_{22}	C_{23}	…	C_{2n}
3	C_{31}	C_{32}	C_{33}	…	C_{3n}
⋮	⋮	⋮	⋮	⋮	⋮
n	C_{n1}	C_{n2}	C_{n3}	…	C_{nn}

2.2. The Linear programming model of Assignment problem is

$$Min\, z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}.$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, 3, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ j = 1, 2, 3, \dots, n,$$

$$x_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j,$$

where

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned to } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$$

C_{ij} = the cost associated with assigning i^{th} resource to j^{th} activity.

3. Stephen's Algorithm

In this section, Stephen's algorithm for solving the assignment problem is summarized below:

Step 1. Construct the assignment problem.

Step 2. Examine the smallest assignment cost in each column and then subtract the same cost from each cost in the corresponding column.

Step 3. If each column and each row has only one zero, the solution is optimal. Make the assignment. Otherwise go to step 4.

Step 4. If any row has more than one zeros, find the penalty i.e. determine the difference between the smallest and the next-to-smallest. Display them alongside of the assignment table by enclosing them in parenthesis against the respective rows. Similarly compute the penalty for each column and display them below the assignment table by enclosing them in parenthesis against the respective column.

Step 5. Identify the row or column with largest penalty among all rows and columns. If a tie occurs, find the next penalty i.e. difference between second smallest cost and next to the second smallest. Repeat the process until break the tie.

Step 6. From the identified row or column, find the smallest assignment cost and make the assignment, then cross off the corresponding row and column values, except assignment cost.

Step 7. Go to step 4 and repeat the procedure until an optimum is reached.

4. Numerical Illustration

Consider the Assignment problem

Table 4.1

PERSONS	JOBS			
	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

Solution:

Table 4.2. Locate smallest cost in each column and subtract the same cost from each cost in the corresponding column.

PERSONS	JOBS			
	A	B	C	D
I	0	0	0	0
II	8	3	4	6
III	3	1	5	4
IV	7	3	2	2

Since the first row has four zeros, therefore, the solution is not optimal solution.

Table 4.3. Find the penalty.

PERSONS	JOBS				PENALTY
	A	B	C	D	
I	0	0	0	0	[0]
II	8	3	4	6	[1]
III	3	1	5	4	[2]
IV	7	3	2	2	[0]
PENALTY	[3]	[1]	[2]	[2]	

The largest penalty is 3, choose the column and the smallest assignment costs in the column is 0. Make the assignment and cross off costs in the corresponding row and column.

Table 4.4.

PERSONS	JOBS			
	A	B	C	D
I	0	*	*	*
II	*	3	4	6
III	*	1	5	4
IV	*	3	2	2

Table 4.5. Again find the penalty.

PERSONS	JOBS				PENALTY
	A	B	C	D	
I	0	*	*	*	–
II	*	3	4	6	[1]
III	*	1	5	4	[3]
IV	*	3	2	2	[0]
PENALTY	–	[2]	[2]	[2]	

The largest penalty is 3 and smallest cost in the row is 1. Make the assignment and cross off the costs in the corresponding row and column.

Table 4.6.

PERSONS	JOBS			
	A	B	C	D
I	0	*	*	*
II	*	*	4	6
III	*	1	*	*
IV	*	*	2	2

Table 4.7. Again find the penalty.

PERSONS	JOBS				PENALTY
	A	B	C	D	
I	0	*	*	*	–
II	*	*	4	6	[2]
III	*	1	*	*	–
IV	*	*	*	2	[0]
PENALTY	–	–	[2]	[4]	

The largest penalty is 4 and smallest cost in the column is 2. Make the assignment and cross off the costs in the corresponding row and column.

Table 4.8.

PERSONS	JOBS			
	A	B	C	D
I	0	*	*	*
II	*	*	4	*
III	*	1	*	*
IV	*	*	*	2

Now each row and every column has only one assignment. Therefore, the optimum solution is attained.

The optimum assignment is

PERSONS	JOBS	ASSIGNMENT COST
I	A	1
II	C	10
III	B	5
IV	D	5
OPTIMUM ASSIGNMENT COST		21

Results:

Method	Optimum Assignment Cost
Hungarian method	21
Stephen's Algorithm	21

5. Conclusion

In this paper, an efficient algorithm named as Stephen's Algorithm is employed to solve assignment problem. The proposed algorithm consumes less time, less number of iterations and also provides an optimal solution. Since this algorithm is easy to understand, this technique may be extended to solve problems like traveling salesman problem, transportation problems, project scheduling problems and network problems.

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