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UNISUM LABELING OF HYDRA HEXAGONS AND HYDRA OCTAGONS

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Abstract

In this paper, a variety of Hydra Hexagons are designed which assumes the shape of a Hydra. Hexagons are joined to another in even number to form the base stem of this Hydra Hexagons. The uppermost vertex of this base stem is extended to a number of tentacles which consists of the same number of hexagons as the base stem. This entire graph is labeled using unisum labeling. Also, this idea is extended to construction of Hydra Octagons which is found to admit unisum labeling.

1. Introduction

Hydra is a Genus of small, freshwater organism which have regenerative ability. A Hydra Hexagon (HH_n) is modelled by combining hexagons with one another by joining one vertex from each to form a chain of C_6 cell. Thus, the base post of Hydra Hexagon is constructed. Similar Hexagonal chains with equal number of hexagons are considered as tentacles of HH_n by extending the uppermost vertex of base post connecting it to multiple bridges. The edges which connect a single vertex of the topmost C_6 cell to various columns of identical Hexagonal chains are called bridges. Even and equal number of C_6 cells are present in each Hexagonal Chain. Thus, a number of Tentacles (T_n) is built (odd in number), to form the complete Hydra Hexagon.

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1. Theorem. *The Hydra Hexagon* $(3T_4HH_n)$ *is Unisum graph.*

Proof. The Hydra Hexagon $3T_4HH_n$ consists of four tentacles connected by 3 bridges. Consider m as the total count of vertices and n as the total count of edges of this graph and t as the total count of tentacles.

The vertices are labeled as follows:

$f(a_{li}) = i + s$, where $l = 1$, $i = 1$ and $s = \left\{0, 1, 2, 3, \ldots, \frac{m-t}{2t}\right\}$

$f(b_{lj}) = \{(n+1) - 25j\} - k$, where $l = \{1, 2, \ldots, t\}$ and $j = \{0, 1, 2, \ldots, t-1\}$

$f(a_{li}) = i + s + 1$, where $l = 2$, $i = 25$ and $s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}$

$f(a_{li}) = i + s + 1$, where $l = 3$, $i = 50$ and $s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}$

$f(a_{li}) = i + s + 1$, where $l = \{2, 3, \ldots, t\}$, $i = 25(t-1), \forall t \geq 2$ and $s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}$.

The edges have the labels from $n+1$ to 2, inclusive satisfying the Unisum inequality.

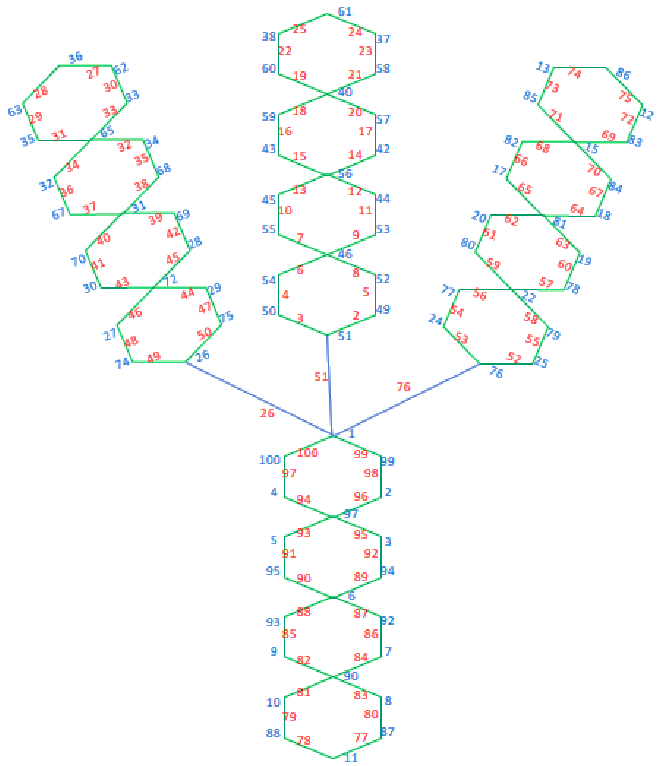

Figure 1. Unisum labeling of 3Tentacle 4Row Hydra Hexagon.

Illustration.

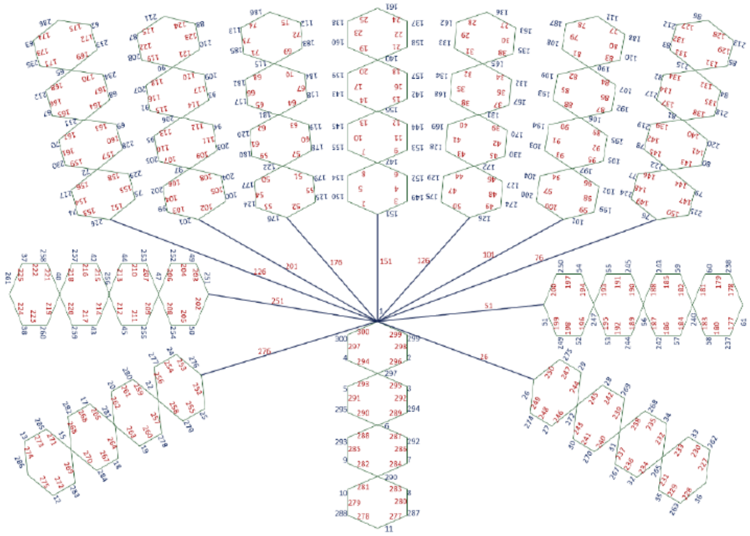

Figure 2. Unisum labeling of 11Tentacle 4Row Hydra Hexagon.

2. Theorem. *Hydra Hexagon* $(3T_4HH_n)$ *admits graceful labeling*.

Proof. The Hydra hexagon $3T_4HH_n$ consists of four tentacles connected by 3 bridges. Consider m as the total count of vertices and n as the total count of edges of this graph and t as the total count of tentacles.

The vertices are labeled as follows:

$f(a_{li}) = i + s$, where $l = 1$, $i = 1$ and $s = \left\{0, 1, 2, 3, \ldots, \frac{m-t}{2t}\right\}$

$f(b_{lj}) = \{(n) - 25j\} - k$, $l = \{1, 2, \ldots, t\}$ where $j = \{0, 1, 2, \ldots, t-1\}$ $k = \{0, 1, 3, 5, 6, 7, 8, 10, \ldots, 3t+1\}$

$f(a_{li}) = i + s$, where $l = 2$, $i = 25$ and $s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}$

$f(a_{li}) = i + s$, where $l = 3$, $i = 50$ and $s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}$

$f(a_{li}) = i + s$, where $l = \{2, 3, \ldots, t\}$, $i = 25(t-1)$, $\forall\, t \geq 2$,

$s = \left\{0, 1, 2, \ldots, \frac{m-t}{2t}\right\}.$

The edges take the labels from n to 1, inclusive adapting the inequality $| f^1(u) - f^1(v) |, \forall\, u, v \,\mathcal{E}\, V(G).$

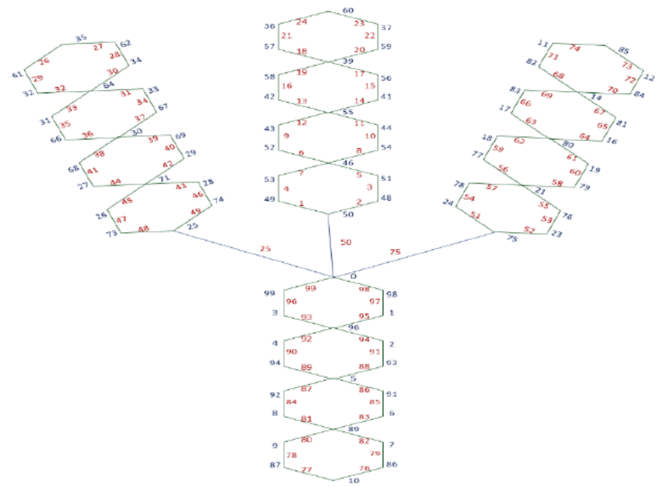

Figure 3. Graceful labeling of 3Tentacle 4Row Hydra Hexagon.

3. Theorem. *Hydra Hexagon* $(3T_4HH_n)$ *is Even Graceful graph*.

Proof. The Hydra Hexagon $3T_4HH_n$ consists of four tentacles connected by 3 bridges. Consider m as the total count of vertices and n as the total count of edges of this graph and t as the total count of tentacles.

The vertices are labeled as follows:

$f(a_{li}) = i + s,$ where $l = 1,\ i = 0$ and $s = \left\{0, 2, 4, 6, 8, \ldots, \frac{m-t}{t}\right\}$

$f(b_{lj}) = \{(2n) - 25j\} - k,\ l = \{1, 2, \ldots, t\}$ where $j = \{0, 1, 2, \ldots, t-1\}$ $k = \{0, 1, 3, 5, 6, 7, 8, 10, \ldots, 3t+1\}$

$f(a_{li}) = i + s,$ where $l = 2,\ i = 25$ and $s = \left\{0, 2, 4, 6, \ldots, \frac{m-t}{t}\right\}$

$f(a_{li}) = i + s,$ where $l = 3,\ i = 100,\ s = \left\{0, 1, 2, \ldots, \frac{m-t}{t}\right\}$

$$f(a_{li}) = i + s, \text{ where } l = \{2, 3, \ldots, t\}, i = 50(t-1), \forall t \geq 2, s = \left\{0, 1, 2, \ldots, \frac{m-t}{t}\right\}.$$

The edges take up the values between $2n$ and 2, inclusive acknowledging the condition of graceful labeling $| f^1(u) - f^1(v) |, \forall\, u, v \in V(G)$.

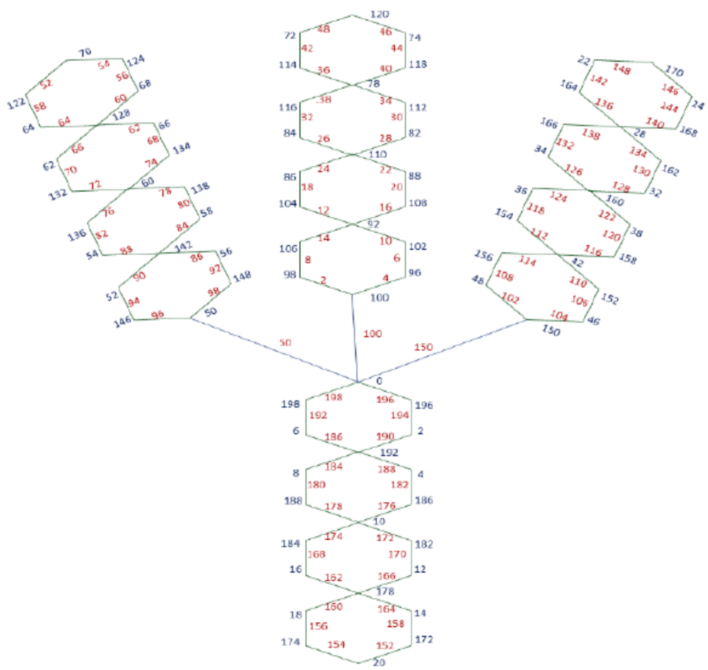

Figure 4. Even graceful labeling of 3Tentacle 4Row Hydra Hexagon.

4. Theorem. *Hydra Hexagon* $(3T_4HH_n)$ *admits Edge Odd graceful labeling.*

Proof. The Hydra Hexagon $3T_4HH_n$ consists of four tentacles connected by 3 bridges. Consider m as the total count of vertices and n as the total count of edges of this graph and t as the total count of tentacles.

The vertices are labeled as follows:

$$f(a_{li}) = i + s, \text{ where } l = 1, i = 0, s = \left\{0, 2, 4, 6, 8, \ldots, \frac{m-t}{t}\right\}$$

$$f(b_{lj}) = \{(2n-1) - 25j\} - k, \; l = \{1, 2, \ldots, t\}, \quad \text{where} \quad j = \{0, 1, 2, \ldots, t-1\}$$

$k = \{0, 1, 3, 5, 6, 7, 8, 10, \ldots, 3t + 1\}$

$$f(a_{li}) = i + s - 1, \text{ where } l = 2, i = 50, s = \left\{0, 2, 4, 6, \ldots, \frac{m-t}{t}\right\}$$

$$f(a_{li}) = i + s - 1, \text{ where } l = 3, i = 100, s = \left\{0, 1, 2, \ldots, \frac{m-t}{t}\right\}$$

$f(a_{li}) = i + s - 1,$ where $l = \{2, 3, \ldots, t\}, i = 50(t-1), \forall t \geq 2,$ $s = \left\{0, 1, 2, \ldots, \frac{m-t}{t}\right\}.$

The edges takes up the values between $2n - 1$ and 1, inclusive accepting the condition of graceful labeling.

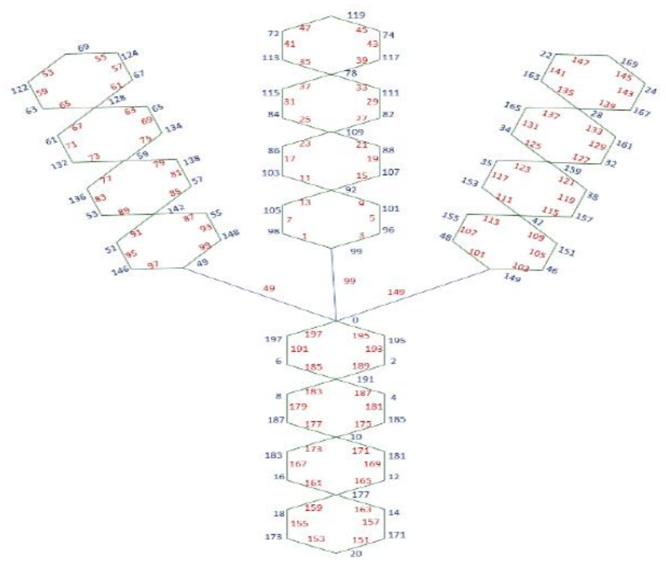

Figure 5. Edge odd graceful labeling of 3Tentacle 4Row Hydra Hexagon.

5. Corollary. *Hydra Hexagon* $(3T_4HH_n)$ *is cordial graph.*

Proof. A cordial graph is a weakened version of graceful graphs where, when vertices are labeled with ones and zeros and the edge labels are difference of their corresponding vertices, the number of edges labeled with ones and zeros differ at most by one.

Let $G = (V, E)$ be a graph where the set $\{0, 1\}$ is assigned to the vertices and edges. Thus, the labels of the vertices and edges takes only 0 or 1 as their labeling leading to cordial labeling of Hydra Hexagon.

In this graph there are 99 edges in total, out of which 50 edges are labeled as 1 and 49 edges are labeled as 0 satisfying the condition of cordial graph.

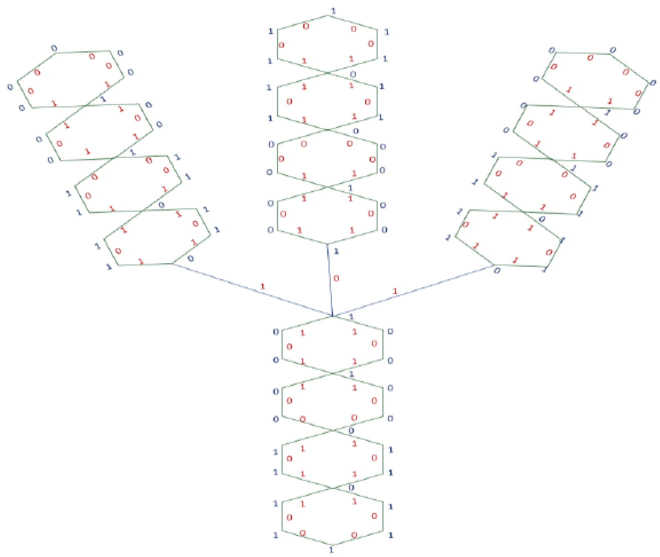

Figure 6. Cordial labeling of 3Tentacle 4Row Hydra Hexagon.

6. Theorem. *Hydra Octagon* $(3T_4HO_n)$ *admits Unisum labeling.*

Proof. The Hydra Octagon consists of four tentacles connected by 3 bridges. Consider m as the total count of vertices and n as the total count of edges of this graph and t as the total count of tentacles.

The vertices are labeled as follows:

$$f(a_{li}) = i + s, \text{ where } l = 1,\ i = 1,\ s = \left\{0, 1, 2, 3, \ldots, \frac{m+5t}{2t}\right\}$$

$f(b_{lj}) = \{(n+1) - 32j\} - k$, $l = \{1, 2, \ldots, t\}$, where $j = \{0, 1, 2, \ldots, t-1\}$ $k = \{0, 1, 3, 4, \ldots, 3t+3\}$.

$f(a_{li}) = i + s + 1$, where $l = \{2, 3, \ldots, t\}$, $i = 32(t-1)$, $\forall\, t \geq 2$, $s = \left\{0, 1, 2, \ldots, \frac{m+5t}{2t}\right\}$.

The edges have the labels from $n+1$ to 2, inclusive satisfying the Unisum inequality $|\, f^1(u) - f^1(v) \,|, \; \forall\, u,\, v \,\varepsilon\, V(G)$.

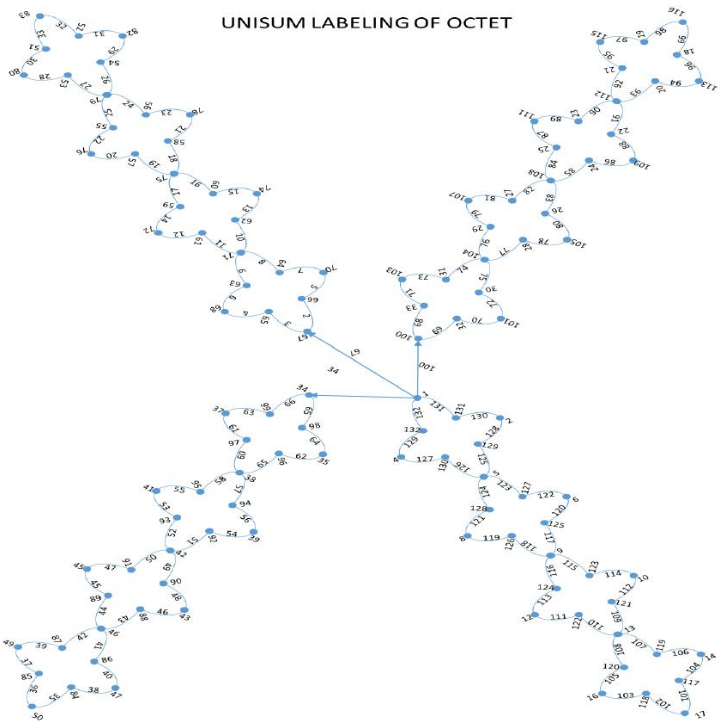

Figure 7. Unisum Labeling of Hydra Octagon $(3T_4HO_n)$.

2. Conclusion

This Unisum labeling can be defined as a graph that has vertices with distinct integers ranging from one to $m+1$ where m is the number of edges. The specialty of this labeling is that it is uniquely labeled by adding unity to the absolute difference between its corresponding vertices without any repetition. We then additionally extend the usage of Unisum labeling to label Hydra Hexagons and Hydra Octagons. These Hydra Hexagons and Hydra Octagons are peculiar in such a way that infinite number of hexagonal or the octagonal pattern can be diversified on any direction. Graph labeling that can deconstruct convoluted data and processes are required extensively in both qualitative and quantitative aspects. The Unisum labeling can be used to define structures of infinite values. The same is true for super Unisum labeling especially if repetitive nodes are present. Graph labeling can be used for any structure be it symmetrical or asymmetrical.

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