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VERTEX PERFECT ANTI-MAGIC BIPOLAR FUZZY LABELING GRAPHS

K. KALAIARASI and L. MAHALAKSHMI

[1]PG and Research Department of Mathematics
Cauvery College for women (Autonomous)
Affiliated to Bharathidasan University
Trichy-18 Tamil Nadu, India
E-mail: Kalaishruthi12@gmail.com
and
Post-Doctoral Research Fellow
Department of Mathematics
Srinivas University, Surathkal
Mangaluru, Karnataka 574146

[2]PG and Research Department of Mathematics
Cauvery College for women (Autonomous)
Affiliated to Bharathidasan University
Trichy-18 Tamil Nadu, India
E-mail: mahaparthi19@gmail.com

Abstract

A Fuzzy labeling graph have been contained specific conditions to give the values of vertices or edges or both. Vertex perfect fuzzy labeling graph, vertex perfect bipolar fuzzy labeling graph, vertex, edge anti-magic fuzzy labeling graph, vertex and edge anti-magic bipolar fuzzy labeling graphs these are the new ideas of this paper. Additionally, we explain a few models and hypotheses.

1. Introduction

Fuzzy graph ideas were explores by Zadeh [1] in 1965. The connectivity

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concepts between fuzzy cut nodes and fuzzy bridges were established by Bhattacharya [2]. Hypothetical ideas of a few fuzzy graphs, for example, paths, cycles, connectedness were exposed by this. By the fuzzy graphs which can explained numerous issues.

The fuzzy graph has been developed quickly and numerous applications in different fields. An exponential development for research on fuzzy graphs both inside mathematics and its applications in science and technology. In many places, a fuzzy graph deviates from generalized crisp graphs.

The problem of labeling graphs is one which has attracted a number of researchers. A bibliography of almost all works on labeling is enlisted by Gallian [3] in 2014. A graph labeling has been containing certain conditions for vertices and edges. In 1963, the notation of magic graph was introduced by sedlacek [4]. The properties of magic graphs were defined by A. Kotzig and A. Rosa [4] in 1970. The applications of fuzzy labeling graphs are coding hypotheses, X-ray, radar, astronomy, circuit plan, communication networks etc. The idea of fuzzy labeling, magic fuzzy labeling graph, some of their properties was introduced by Nagoorgani [5-8]. K. Ameenal Bibi and M. Devi [9-[5] discussed fuzzy vertex graceful labeling in 2017. In 2016, R. Jebesty Shajila and S. Vimala [11-12] talked about some fuzzy labeling graphs N. Sujatha talked about triangular fuzzy graceful labeling in 2017. M. Fathalian [13] deliberated simple graphs for fuzzy magic labeling. M. Akram [14-16] deliberated many types of fuzzy graph in 2011 and 2013. In 2016, S. N. Mishra and Antipal [17] talked about the magic fuzzy labeling graph. Seema Mehra and Manjeet Singh [6] introduced an intuitionistic fuzzy magic labeling graph in 2017. In the same year P. K. Kishore Kumar [19] talked about on interval valued intuitionistic fuzzy labeling graphs. K. Kalaiarasi and P. Geethanjali [21] discussed some new concepts of Arc sequences in fuzzy graphs.

Here we talked about, the first section contains preliminaries, the second section contains vertex perfect edge anti-magic fuzzy labeling graphs, and vertex perfect vertex anti-magic fuzzy labeling graphs. Likewise, the third section vertex perfect edge anti-magic bipolar fuzzy labeling graphs and vertex perfect vertex anti-magic bipolar fuzzy labeling graphs examine.

2. Preliminaries

Definition 2.1 [8]. A fuzzy labeling graph is said to be a edge anti-magic fuzzy labeling graph if $\sigma(u)+\mu(u, v)+\sigma(v)$ has some specific values for all $u, v \in V$ which is indicated by $M_A(G)$.

Definition 2.2 [8]. A fuzzy labeling graph is said to be vertex anti-magic fuzzy labeling graph if $\mu(u, v)+\sigma(v)+\mu(v, w)$ has some specific values for all $u, v, w \in V$ which is indicated by $M_A(G)$.

Definition 2.3 [9]. A fuzzy labeling graph is said to be a magic bipolar fuzzy labeling graph if

(i) $\sigma^P(u)+\mu^P(uv)+\sigma^P(v)$ has a same value for all $u, v \in V$ which is indicated by $m_0^P(G)$.

(ii) $\sigma^N(u)+\mu^N(uv)+\sigma^N(v)$ has a same value for all $u, v \in V$ which is indicated by $m_0^N(G)$.

Definition 2.4 [8]. A fuzzy labeling graph is said to be a anti-magic bipolar fuzzy labeling graph if

(i) $\sigma^P(u)+\mu^P(uv)+\sigma^P(v)$ has some specific values for all $u, v \in V$ which is indicated by $m_A^P(G)$.

(ii) $\sigma^N(u)+\mu^N(uv)+\sigma^N(v)$ has some specific values for all $u, v \in V$ which is indicated by $M_A^N(G)$.

3. Vertex Perfect Vertex Anti-Magic and Vertex Perfect Edge Anti-Magic Fuzzy Labeling Graphs

In this section we consider,

$G-$ The Fuzzy graph

G_L- The Fuzzy labeling graph

$G_{AL}-$ The Fuzzy anti-labeling graph

G_L^B – Bipolar Fuzzy labeling graph

G_{AL}^B – Bipolar Fuzzy anti-labeling graph

V – Vertices

E – Edges

C_n – The Fuzzy Cycle graph

Definition 3.1. A fuzzy labeling graph is represented by vertex perfect vertex anti-magic fuzzy labeling graph if

(i) In C_n, $n > 4$ any one of the vertex value must be one.

(ii) $\Im(a, b) + \wp(b) + \Im(b, c)$ has some specific values for all $a, b, v \in V$ and it is indicated by $VPVM(G_{AL})$.

Example 3.1.

G_L

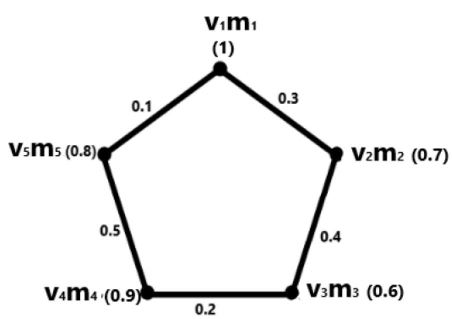

Figure 3.1. $VPVM(G_{AL})$.

In Figure 3.1,

$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4, v_5m_5\} = \{1, 0.7, 0.6, 0.9, 0.8\}$

$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4, e_5m_5\} = \{0.3, 0.4, 0.2, 0.5, 0.1\}$

$\Im(v_1m_1, v_2m_2) + \wp(v_1m_1) + \Im(v_1m_1, v_5m_5)$

$$= \Im(v_1m_1, v_2m_2) + \wp(v_2m_2) + \Im(v_2m_2, v_3m_3)$$

$$= \Im(v_1m_1, v_5m_5) + \wp(v_5m_5) + \Im(v_4m_4, v_5m_5) = 1.4$$

$$\Im(v_2m_2, v_3m_3) + \wp(v_3m_3) + \Im(v_3m_3, v_4m_4) = 1.2$$

$$\Im(v_5m_5, v_3m_3) + \wp(v_4m_4) + \Im(v_4m_4, v_5m_5) = 1.6$$

Hence the graph G_L is $VPVM_{AL}(G)$.

Remark 3.1. A fuzzy labeling graph is represented by vertex perfect vertex magic fuzzy labeling graph if

(i) In C_3 and C_4 any one of the vertex value must be one.

(ii) $\Im(a, b) + \wp(b) + \Im(b, c)$ has same values for all $a, b, c \in V$ and it is indicated by $VPVM(G_L)$.

Example 3.2.

G_L

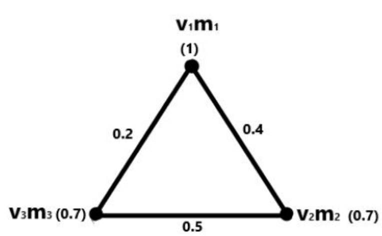

Figure 3.2. $VPVM(G_L)$.

In Figure 3.2,

$$V = \{v_1m_1, v_2m_2, v_3m_3\} = \{1, 0.7, 0.9\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3\} = \{0.4, 0.5, 0.2\}$$

$$\Im(v_1m_1, v_2m_2) + \wp(v_1m_1) + \Im(v_1m_1, v_3m_3)$$

$$= \Im(v_2m_2, v_3m_3) + \wp(v_2m_2) + \Im(v_2m_2, v_1m_1)$$

$$= \Im(v_3m_3, v_1m_1) + \wp(v_3m_3) + \Im(v_3m_3, v_2m_2) = 1.6$$

Hence the graph G_L is $VPVM(G_L)$.

Example 3.3.

G_L

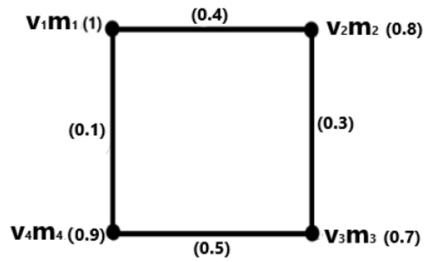

Figure 3.3. $VPVM(G_L)$.

In Figure 3.3,

$$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4\} = \{1, 0.8, 0.7, 0.9\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4\} = \{0.4, 0.3, 0.5, 0.1\}$$

$$\Im(v_1m_1, v_2m_2) + \wp(v_1m_1) + \Im(v_1m_1, v_4m_4)$$

$$= \Im(v_2m_2, v_3m_3) + \wp(v_2m_2) + \Im(v_2m_2, v_1m_1)$$

$$= \Im(v_3m_3, v_4m_4) + \wp(v_3m_3) + \Im(v_3m_3, v_2m_2)$$

$$= \Im(v_4m_4, v_3m_3) + \wp(v_4m_4) + \Im(v_4m_4, v_1m_1) = 1.5$$

Hence the graph G_L is $VPVM(G_L)$.

Definition 3.2. A fuzzy labeling graph is represented by vertex perfect edge anti-magic fuzzy labeling graph if

(i) In C_n, $n > 4$ any one of the vertex value must be one

(ii) $\wp(a) + \Im(a, b) + \wp(b)$ has some specific values for all $a, b \in V$ and it is indicated by $VPVM(G_{AL})$.

Example 3.4.

G_L

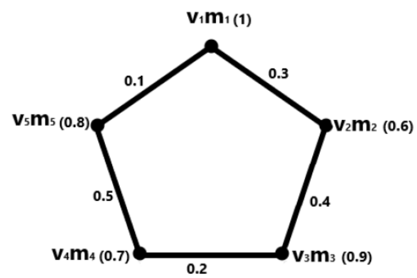

Figure 3.4. $VPVM(G_{AL})$.

In Figure 3.4,

$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4, v_5m_5\} = \{1, 0.6, 0.9, 0.7, 0.8\}$

$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4, e_5m_5\} = \{0.3, 0.4, 0.2, 0.5, 0.1\}$

$$\wp(v_1m_1) + \Im(v_1m_1, v_2m_2) + \wp(v_2m_2)$$
$$= \wp(v_2m_2) + \Im(v_2m_2, v_3m_3) + \wp(v_3m_3)$$
$$= \wp(v_1m_1) + \Im(v_1m_1, v_5m_5) + \wp(v_5m_5)$$
$$= 1.9$$

$$\wp(v_3m_3) + \Im(v_3m_3, v_4m_4) + \wp(v_4m_4) = 1.8$$

$$\wp(v_4m_4) + \Im(v_4m_4, v_5m_5) + \wp(v_5m_5) = 2.0$$

Hence the graph G_L is $VPVM(G_{AL})$.

Remark 3.2. A fuzzy labeling graph is represented vertex perfect edge magic fuzzy labeling graph if

(i) In C_3 and C_4 any one of the vertex value must be one

(ii) $\wp(a) + \Im(a, b) + \wp(b)$ has same values for all $a, b \in V$ and it is indicated by $VPVM(G_L)$.

Example 3.5.

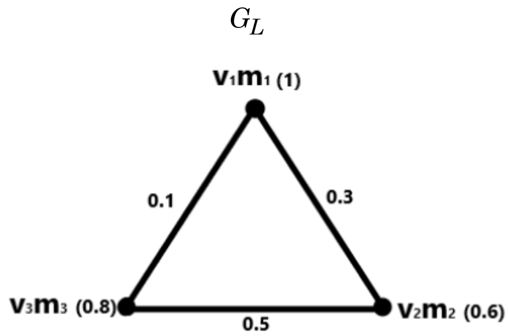

Figure 3.5. $VPVM(G_L)$.

In Figure 3.5,

$V = \{v_1m_1, v_2m_2, v_3m_3\} = \{1, 0.6, 0.8\}$

$E = \{e_1m_1, e_2m_2, e_3m_3\} = \{0.3, 0.5, 0.1\}$

$$\wp(v_1m_1) + \Im(v_1m_1, v_2m_2) + \wp(v_2m_2)$$

$$= \wp(v_2m_2) + \Im(v_2m_2, v_3m_3) + \wp(v_3m_3)$$

$$= \wp(v_3m_3) + \Im(v_3m_3, v_1m_1) + \wp(v_1m_1) = 1.9$$

Hence the graph G_L is $VPVM(G_L)$.

Example 3.6.

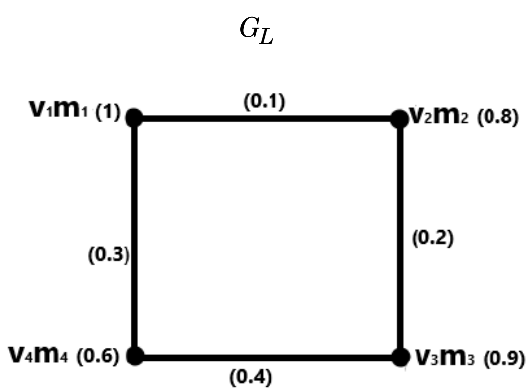

Figure 3.6. $VPVM(G_L)$.

In Figure 3.6,

$$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4\} = \{1, 0.8, 0.9, 0.6\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4\} = \{0.1, 0.2, 0.4, 0.3\}$$

$$\wp(v_1m_1) + \Im(v_1m_1, v_2m_2) + \wp(v_2m_2)$$

$$= \wp(v_2m_2) + \Im(v_2m_2, v_3m_3) + \wp(v_3m_3)$$

$$= \wp(v_3m_3) + \Im(v_3m_3, v_4m_4) + \wp(v_4m_4)$$

$$= \wp(v_4m_4) + \Im(v_4m_4, v_1m_1) + \wp(v_1m_1) = 1.9$$

Hence the graph G_L is $VPVM(G_L)$.

Theorem 3.1. *Let C_n, $n > 5$ be a vertex perfect fuzzy labeling graph cycle. Then it has vertex magic* (*or*) *edge magic fuzzy labeling graph.*

Proof. Suppose that C_n, $n > 5$ that means $n = 3$ and $n = 4$ then the fuzzy graph G_F is vertex perfect fuzzy labeling graph. We have to prove G_F is vertex magic (or) edge magic fuzzy labeling graph. If possible suppose the contrary.

Suppose that if $n < 3$ in that case by the definition of cycle if $v_1 = v_n$ and $n \geq 3$. which is impossible to our assumption. And $n > 4$ then the fuzzy graph has any of the vertex value (or) edge value is same. It is impossible by definition of fuzzy labeling graph the membership values of the vertex and edges has specific values which is also logical inconsistency to our assumption. Therefore, Our supposition that isn't right.

If $n = 3$ and $n = 4$ then the condition is hold.

Hence if C_n, $n < 5$ be a vertex perfect fuzzy labeling graph cycle. Then it has vertex magic (or) edge magic fuzzy labeling graph.

Preposition 3.1. *Every regular fuzzy graph is not vertex perfect edge anti-magic fuzzy labeling graph.*

Proof. Since $d_G(v) = M$ for all $v \in V$.

Also for $'S'$ regular fuzzy graph $\wp(a) = \text{constant}$ and $\Im(a, b) = \text{constant}$ for all $a, b \in V$.

Which is impossible in fuzzy labeling, since for fuzzy labeling graphs $\wp(a)$ and $\Im(a, b)$ are specific values for every vertex and edge.

But for vertex perfect edge anti- magic labeling fuzzy graph $\wp(a) + \Im(a, b) + \wp(b)$ has distinct and any one vertex value is perfect (i.e.) one.

Hence every regular fuzzy graph is not vertex perfect edge anti-magic fuzzy labeling graph.

Preposition 3.2. *Every regular fuzzy graph is not a vertex perfect vertex anti-magic fuzzy labeling graph.*

Note. (1) Every complete fuzzy graph is not vertex perfect edge magic fuzzy labeling graph.

(2) Every complete fuzzy graph is not a vertex perfect vertex magic fuzzy labeling graph.

(3) Every complete fuzzy graph is not vertex perfect edge anti- magic fuzzy labeling graph.

(4) Every complete fuzzy graph is not vertex perfect vertex anti-magic fuzzy labeling graph.

4. Vertex Perfect Vertex Anti-Magic and Vertex Perfect Edge Anti-Magic Bipolar Fuzzy Labeling Graphs

Definition 4.1. A fuzzy labeling graph is represented by a vertex perfect vertex anti-magic bipolar fuzzy labeling graph if

(i) In $C_n, n > 4$ any one of the vertex $\wp^P(a) = 1, \wp^N(a) = -1$ and $\Im^P(a, b) + \wp^P(a) + \Im^P(b, c)$ has some specific values for all $a, b, c \in V$ which is indicated as $VPVM^P(G_{AL})$.

(ii) In $C_n, n > 4$ any one of the vertex $\wp^P(a) = 1, \wp^N(a) = -1$ and $\Im^N(a, b) + \wp^N(a) + \Im^N(b, c)$ has some specific values for all $a, b, c \in V$ which is indicated as $VPVM^N(G_{AL})$.

The vertex perfect vertex anti-magic bipolar fuzzy labeling graph is indicated by $VPVM(G_{AL}^{B})$.

Example 4.1.

G_L

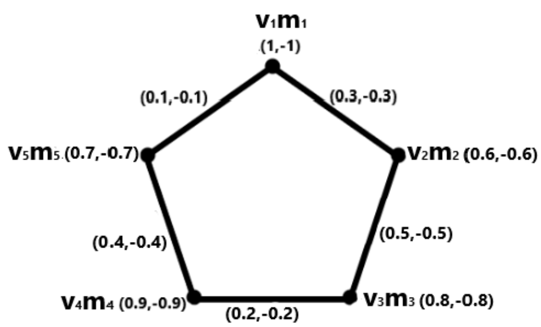

Figure 4.1. $VPVM(G_{AL}^{B})$.

In Figure 4.1,

$$V = \{v_1m_1,\, v_2m_2,\, v_3m_3,\, v_4m_4,\, v_5m_5\} = \{(1, -1), (0.6, -0.6), (0.8, -0.8), (0.9, -0.9), (0.7, -0.7)\}$$

$$E = \{e_1m_1,\, e_2m_2,\, e_3m_3,\, e_4m_4,\, e_5m_5\} = \{(0.3, -0.3), (0.5, -0.5), (0.2, -0.2), (0.4, -0.4), (0.1, -0.1)\}$$

$$\Im^{p}(v_1m_1, v_2m_2) + \wp^{p}(v_1m_1) + \Im^{p}(v_1m_1, v_5m_5)$$

$$= \Im^{p}(v_2m_2, v_1m_1) + \wp^{p}(v_2m_2) + \Im^{p}(v_2m_2, v_3m_3) = 1.4$$

$$\Im^{p}(v_3m_3, v_2m_2) + \wp^{p}(v_3m_3) + \Im^{p}(v_3m_3, v_4m_4) = 1.5$$

$$\Im^{p}(v_4m_4, v_3m_3) + \wp^{p}(v_4m_4) + \Im^{p}(v_4m_4, v_5m_5) = 1.5$$

$$\Im^{p}(v_5m_5, v_4m_4) + \wp^{p}(v_5m_5) + \Im^{p}(v_5m_5, v_1m_1) = 1.2$$

$$\Im^{N}(v_1m_1,\, v_2m_2) + \wp^{N}(v_1m_1) + \Im^{N}(v_1m_1,\, v_5m_5)$$

$$= \Im^N(v_2m_2, v_1m_1) + \wp^N(v_2m_2) + \Im^N(v_2m_2, v_3m_3) = 1.4$$

$$\Im^N(v_3m_3, v_2m_2) + \wp^N(v_3m_3) + \Im^N(v_3m_3, v_4m_4) = -1.5$$

$$\Im^N(v_4m_4, v_3m_3) + \wp^N(v_4m_4) + \Im^N(v_4m_4, v_5m_5) = -1.5$$

$$\Im^N(v_5m_5, v_4m_4) + \wp^N(v_5m_5) + \Im^N(v_5m_5, v_1m_1) = -1.2$$

Hence the graph G_L is $VPVM(G_{AL}^B)$.

Remark 4.1. A fuzzy labeling graph is represented by a vertex perfect vertex magic bipolar fuzzy labeling graph if

(i) In C_3 and C_4 any one of the vertex $\wp^p(a)=1, \wp^N(a)=-1$ and $\Im^p(a,b)+\wp^p(a)+\Im^p(b,c)$ has a same value for all $a, b, c \in V$ which is indicated as $VPVM^p(G_L)$.

(ii) In C_3 and C_4 any one of the vertex $\wp^p(a)=1, \wp^N(a)=-1$ and $\Im^N(a,b)+\wp^N(a)+\Im^N(b,c)$ has a same value for all $a, b, c \in V$ which is indicated as $VPVM^N(G_L)$.

The vertex perfect vertex magic bipolar fuzzy labeling graph is indicated by $VPVM(G_L^B)$.

Example 4.2.

G_L

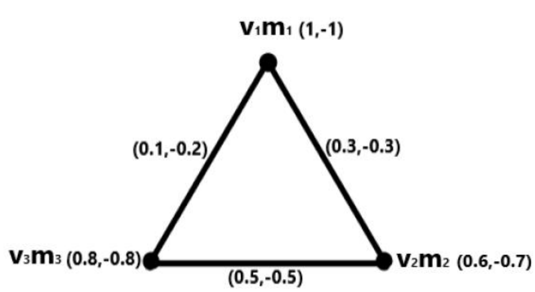

Figure 4.2. $VPVM(G_L^B)$.

In Figure 4.2,

$$V = \{v_1m_1,\ v_2m_2,\ v_3m_3\} = \{(1, -1), (0.6, -0.7), (0.8, -0.8)\}$$

$$E = \{e_1m_1,\ e_2m_2,\ e_3m_3\} = \{(0.3, -0.3), (0.5, -0.5), (0.1, -0.2)\}$$

$$\Im^p(v_1m_1, v_2m_2) + \wp^p(v_1m_1) + \Im^p(v_1m_1, v_3m_3)$$

$$= \Im^p(v_2m_2, v_1m_1) + \wp^p(v_2m_2) + \Im^p(v_2m_2, v_3m_3)$$

$$= \Im^p(v_3m_3, v_1m_1) + \wp^p(v_3m_3) + \Im^p(v_3m_3, v_2m_2) = 1.4$$

$$\Im^N(v_1m_1,\ v_2m_2) + \wp^N(v_1m_1) + \Im^N(v_1m_1,\ v_3m_3)$$

$$= \Im^N(v_2m_2,\ v_1m_1) + \wp^N(v_2m_2) + \Im^N(v_2m_2,\ v_3m_3)$$

$$= \Im^N(v_3m_3,\ v_1m_1) + \wp^N(v_3m_3) + \Im^N(v_3m_3,\ v_2m_2) = -1.5$$

Hence the graph G_L is $VPVM(G_L^B)$.

Example 4.3.

G_L

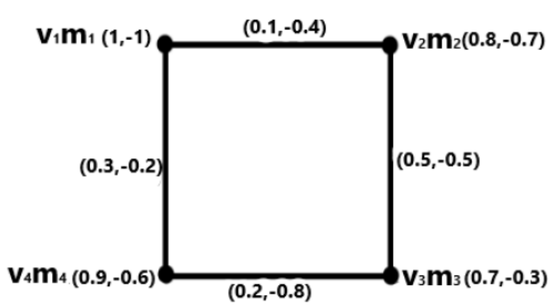

Figure 4.3. $VPVM(G_L^B)$.

In Figure 4.3,

$$V = \{v_1m_1,\ v_2m_2,\ v_3m_3,\ v_4m_4\} = \{(1, -1), (0.8, -0.7), (0.7, -0.3), (0.9, -0.6)\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4\} = \{(0.1, -0.4), (0.5, -0.5), (0.2, -0.8), (0.3, -0.2)\}$$

$$\Im^p(v_1m_1, v_2m_2) + \wp^p(v_1m_1) + \Im^p(v_1m_1, v_3m_3)$$

$$= \Im^p(v_2m_2, v_1m_1) + \wp^p(v_2m_2) + \Im^p(v_2m_2, v_3m_3)$$

$$= \Im^p(v_3m_3, v_2m_2) + \wp^p(v_3m_3) + \Im^p(v_3m_3, v_4m_4)$$

$$= \Im^p(v_4m_4, v_3m_3) + \wp^p(v_4m_4) + \Im^p(v_4m_4, v_1m_1) = 1.4$$

$$\Im^N(v_1m_1, v_2m_2) + \wp^N(v_1m_1) + \Im^N(v_1m_1, v_3m_3)$$

$$= \Im^N(v_2m_2, v_1m_1) + \wp^N(v_2m_2) + \Im^N(v_2m_2, v_3m_3)$$

$$= \Im^N(v_3m_3, v_2m_2) + \wp^N(v_3m_3) + \Im^N(v_3m_3, v_4m_4)$$

$$= \Im^N(v_4m_4, v_3m_3) + \wp^N(v_4m_4) + \Im^N(v_4m_4, v_1m_1) = -1.6$$

Hence the graph G_L is $VPVM(G_L^B)$.

Definition 4.2. A fuzzy labeling graph is represented by a vertex perfect edge anti-magic bipolar fuzzy labeling graph if

(i) In $C_n, n > 4$ any one of the vertex $\wp^p(a) = 1, \wp^N(a) = -1$ and $\wp^p(a) + \Im^p(a, b) + \wp^p(b)$ has some specific values for all $a, b, c \in V$ which is indicated as $VPVM^p(G_{AL})$.

(ii) In $C_n, n > 4$ any one of the vertex $\wp^p(a) = 1, \wp^N(a) = -1$ and $\wp^N(a) + \Im^N(a, b) + \wp^N(b)$ has some specific values for all $a, b, c \in V$ which is indicated as $VPVM^N(G_{AL})$.

The vertex perfect edge anti-magic bipolar fuzzy labeling graph is indicated by $VPVM(G_{AL}^B)$.

Example 4.4.

G_L

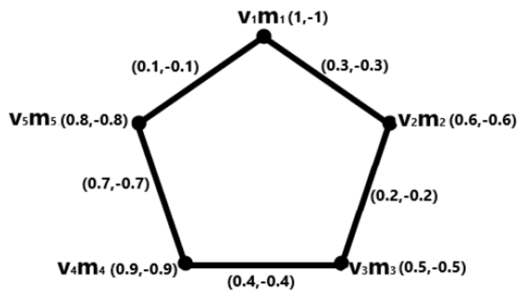

Figure 4.4. $VPVM(G_{AL}^{B})$.

In Figure 4.4,

$$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4, v_5m_5\} = \{(1, -1), (0.6, -0.6), (0.5, -0.5), (0.9, -0.9), (0.8, -0.8)\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4, e_5m_5\} = \{(0.3, -0.3), (0.2, -0.2), (0.4, -0.4), (0.7, -0.7), (0.1, -0.1)\}$$

$$\wp^p(v_1m_1) + \Im^p(v_1m_1, v_2m_2) + \wp^p(v_2m_2) = \wp^p(v_1m_1) + \Im^p(v_1m_1, v_5m_5) + \wp^p(v_5m_5) = 1.9$$

$$\wp^p(v_2m_2) + \Im^p(v_2m_2, v_3m_3) + \wp^p(v_3m_3) = 1.3$$

$$\wp^p(v_3m_3) + \Im^p(v_3m_3, v_4m_4) + \wp^p(v_4m_4) = 1.8$$

$$\wp^p(v_4m_4) + \Im^p(v_4m_4, v_5m_5) + \wp^p(v_5m_5) = 2.4$$

$$\wp^N(v_1m_1) + \Im^N(v_1m_1, v_2m_2) + \wp^N(v_2m_2) = \wp^N(v_1m_1) + \Im^N(v_1m_1, v_5m_5) + \wp^N(v_5m_5) = -1.9$$

$$\wp^N(v_2m_2) + \Im^N(v_2m_2, v_3m_3) + \wp^N(v_3m_3) = -1.3$$

$$\wp^N(v_3m_3) + \Im^N(v_3m_3, v_4m_4) + \wp^N(v_4m_4) = -1.8$$

$$\wp^N(v_4m_4) + \Im^N(v_4m_4, v_5m_5) + \wp^N(v_5m_5) = -2.4$$

Hence the graph G_L is $VPVM(G_{AL}^B)$.

Remark 4.2. A fuzzy labeling graph is represented by a vertex perfect edge magic bipolar fuzzy labeling graph if

(i) In C_3 and C_4 any one of the vertex $\wp^p(a) = 1, \wp^N(a) = -1$ (If $n = 3$ and $n = 4$) and $\wp^p(a) + \Im^p(a, b) + \wp^p(b)$ has same values for all $a, b, c \in V$ which is indicated as $VPVM_L^p(G)$.

(ii) In C_3 and C_4 any one of the vertex $\wp^P(a) = 1, \wp^N(a) = -1$ (If $n = 3$ and $n = 4$) and $\wp^N(a) + \Im^N(a, b) + \wp^N(b)$ has same values for all $a, b, c \in V$ which is indicated as $VPVM_L^N(G)$.

The vertex perfect edge magic bipolar fuzzy labeling graph is indicated by $VPVM(G_L^B)$.

Example 4.5.

G_L

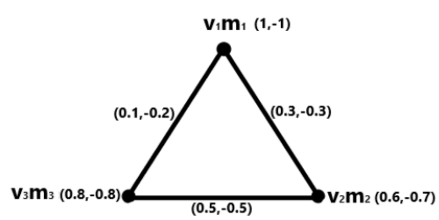

Figure 4.5. $VPVM(G_L^B)$.

In Figure 4.5,

$$V = \{v_1m_1, v_2m_2, v_3m_3\} = \{(1, -1), (0.6, -0.7), (0.8, -0.8)\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3\} = \{(0.3, -0.3), (0.5, -0.5), (0.1, -0.2)\}$$

$$\wp^p(v_1m_1) + \Im^p(v_1m_1, v_2m_2) + \wp^p(v_2m_2)$$

$$= \wp^p(v_2m_2) + \Im^p(v_2m_2, v_3m_3) + \wp^p(v_3m_3)$$

$$= \wp^{p}(v_3m_3) + \Im^{p}(v_3m_3, v_1m_1) + \wp^{p}(v_1m_1) = 1.9$$

$$\wp^{N}(v_1m_1) + \Im^{N}(v_1m_1, v_2m_2) + \wp^{N}(v_2m_2)$$

$$= \wp^{N}(v_2m_2) + \Im^{N}(v_2m_2, v_3m_3) + \wp^{N}(v_3m_3)$$

$$= \wp^{N}(v_3m_3) + \Im^{N}(v_3m_3, v_1m_1) + \wp^{N}(v_1m_1) = -2.0$$

Hence the graph G_L is $VPVM(G_L^B)$.

Example 4.6.

G_L

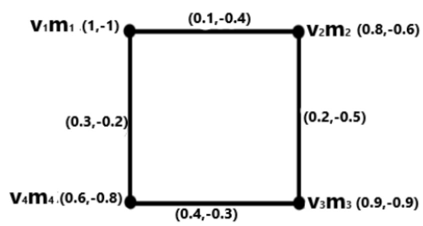

Figure 4.6. $VPVM(G_L^B)$.

In Figure 4.6,

$$V = \{v_1m_1, v_2m_2, v_3m_3, v_4m_4\} = \{(1, -1), (0.8, -0.6), (0.9, -0.9), (0.6, -0.8)\}$$

$$E = \{e_1m_1, e_2m_2, e_3m_3, e_4m_4\} = \{(0.1, -0.4), (0.2, -0.5), (0.4, -0.3), (0.3, -0.2)\}$$

$$\wp^{p}(v_1m_1) + \Im^{p}(v_1m_1, v_2m_2) + \wp^{p}(v_2m_2)$$

$$= \wp^{p}(v_2m_2) + \Im^{p}(v_2m_2, v_3m_3) + \wp^{p}(v_3m_3)$$

$$= \wp^{p}(v_3m_3) + \Im^{p}(v_3m_3, v_4m_4) + \wp^{p}(v_4m_4)$$

$$= \wp^{p}(v_4m_4) + \Im^{p}(v_4m_4, v_1m_1) + \wp^{p}(v_1m_1) = 1.9$$

$$\wp^{N}(v_1m_1) + \Im^{N}(v_1m_1, v_2m_2) + \wp^{N}(v_2m_2)$$

$$= \wp^{N}(v_2m_2) + \Im^{N}(v_2m_2, v_3m_3) + \wp^{N}(v_3m_3)$$

$$= \wp^N(v_3m_3) + \Im^N(v_3m_3, v_4m_4) + \wp^N(v_4m_4)$$

$$= \wp^N(v_4m_4) + \Im^N(v_4m_4, v_1m_1) + \wp^N(v_1m_1) = -2.0$$

Hence the graph G_L is $VPVM(G_L^B)$.

Theorem 4.1. *If vertex perfect bipolar fuzzy labeling graph* VPG_L^B *has an vertex anti-magic bipolar fuzzy labeling graph or edge anti-magic bipolar fuzzy labeling graph, then* VPG_L^B *is not edge perfect bipolar fuzzy labeling graph.*

Proof. Case (i). First suppose that the vertex perfect bipolar fuzzy labeling graph VPG_L^B has a vertex anti-magic bipolar fuzzy labeling graph at that point

$\Im^p(a, b) + \wp^p(a) + \Im^p(b, c) = VM_L^p(G)$ (specific value $\forall u, v \in V$)

$\Im^N(a, b) + \wp^N(a) + \Im^N(b, c) = VM_L^N(G)$ has specific value and any one of the vertex is $(1, -1)(u)$ (i.e.) $\wp^p(a) = 1, \wp^N(a) = -1$.

Conversely suppose that the edge perfect bipolar fuzzy labeling graph EPG_L^B has an vertex anti-magic bipolar fuzzy labeling graph then $\Im^p(a, b) + \wp^p(a) + \Im^p(b, c), \Im^N(a, b) + \wp^N(a) + \Im^N(b, c)$ has some values are specific and any one of the edge value must be $(1, -1)$. (i.e.) $\Im^p(a, b) = 1, \Im^N(a, b) = -1$. That is $\Im(a, b) = (1, -1)$ which is impossible since G_{FL} is a fuzzy labeling graph. The definition of fuzzy labeling graph is $\Im(a, b) < \wp(a) \wedge \wp(b)$ which is contradiction to our assumption.

Hence edge perfect bipolar fuzzy labeling graph has not an vertex anti-magic bipolar fuzzy labeling graph.

Case (ii). Suppose that the vertex perfect bipolar fuzzy labeling graph VPG_L^B has an edge antimagic bipolar fuzzy labeling graph then $\wp^p(a) + \Im^p(a, b) + \wp^p(b)$ and $\wp^N(a) + \Im^N(a, b) + \wp^N(b)$ has distinct value and any one of the vertex must be $(1, -1)$. $\wp^p(a) = 1, \wp^N(a) = -1$.

Conversely suppose that the edge perfect bipolar fuzzy labeling graph EPG_L^B has an edge anti-magic bipolar fuzzy labeling graph then $\wp^p(a)+\Im^p(a,b)+\wp^p(b)$ and $\wp^N(a)+\Im^N(a,b)+\wp^N(b)$ has specific value and any one of the edge value must be $(1,-1), \Im^p(a,b)=1, \Im^N(a,b)=-1.$ That is $\Im(a, b) = (1, -1)$ which is impossible since G_L is a fuzzy labeling graph. The definition of fuzzy labeling graph is $\Im(a, b) < \wp(a) \wedge \wp(b)$ which is contradiction to our assumption.

Hence edge perfect bipolar fuzzy labeling graph has not an edge anti-magic bipolar fuzzy labeling graph.

From case (i) and case (ii) we have every vertex perfect bipolar fuzzy labeling graph has an vertex anti-magic bipolar fuzzy labeling graph (or) edge magic bipolar fuzzy labeling graph.

Hence the theorem.

Theorem 4.2. *Let C_n, $n < 5$ be a vertex perfect bipolar fuzzy labeling graph cycle. Then it has vertex magic* (*or*) *edge magic bipolar fuzzy labeling graph*.

Proof. Suppose that C_n, $n < 5$. That means $n = 3$ and $n = 4$ then the fuzzy graph G_F is vertex perfect bipolar fuzzy labeling graph. We have to prove G_F is vertex magic (or) edge magic bipolar fuzzy labeling graph. If possible suppose the contrary.

Suppose that if $n < 3$ in that case by the definition of cycle if $v_1 = v_n$ and $n \geq 3.$ which is impossible to our assumption. And $n > 4$ then the fuzzy graph has any of the vertex value (or) edge value is same. It is impossible by definition of fuzzy labeling graph the membership values of the vertex and edges are specific values which is also logical inconsistency to our assumption. Therefore, Our supposition that isn't right.

If $n = 3$ and $n = 4$ then the condition is hold.

Hence if C_n, $n < 5$ be a vertex perfect bipolar fuzzy labeling graph cycle. Then it has vertex magic (or) edge magic bipolar fuzzy labeling graph.

Note. (1) Every regular fuzzy labeling graph is not a vertex perfect edge magic bipolar fuzzy labeling graph.

(2) Every regular fuzzy labeling graph is not vertex perfect vertex magic bipolar fuzzy labeling graph.

(3) Every complete fuzzy graph is not vertex perfect edge magic bipolar fuzzy labeling graph.

(4) Every complete fuzzy graph is not vertex perfect vertex magic bipolar fuzzy labeling graph.

(5) Every regular fuzzy labeling graph is not vertex perfect edge anti-magic bipolar fuzzy labeling graph.

(6) Every regular fuzzy labeling graph is not vertex perfect vertex anti-magic bipolar fuzzy labeling graph.

(7) Every complete fuzzy graph is not vertex perfect edge anti-magic bipolar fuzzy labeling graph.

(8) Every complete fuzzy graph is not vertex perfect vertex anti-magic bipolar fuzzy labeling graph.

Conclusion

The new idea has been explained of this paper, for perfect bipolar fuzzy labeling graphs, perfect vertex magic bipolar fuzzy labeling graph, perfect edge magic bipolar fuzzy labeling graph. Then we could be derived numerous fuzzy magic graphs and numerous hypotheses. In the future papers we will be talked about the remaining work.

References

[1] N. Honda and A. Ohsato, Fuzzy Set Theory and Its Applications 13(2) (1986), 64-89.

[2] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognit. Lett. 6(5) (1987), doi: 10.1016/0167-8655(87)90012-2.

[3] J. A. Gallian, A dynamic survey of graph labeling, Electron. J. Comb., 1 Dynamic Surveys, (2018).

[4] S. R. Kaufman, Notes on the Research, Ordinary Med. 7 (2021), 249-254, doi: 10.2307/j.ctv11cw8x0.14.

[5] A. Nagoorgani and D. Rajalakshmi, A Notes on Fuzzy Labeling, International journal of mathematical archive 4(2) (2014), 88-95.

[6] A. N. Gani, M. Akram and D. R. A. Subahashini, Novel Properties of Fuzzy Labeling Graphs, J. Math. (2014), doi: 10.1155/2014/375135.

[7] A. Nagoorgani and D. R. Subahashini, Fuzzy labeling tree, Int. J. Pure Appl. Math. 90(2) (2014), 131-141. doi: 10.12732/ijpam.v90i2.3.

[8] A. N. Gani, M. Akram and D. R. A. Subahashini, Novel Properties of Fuzzy Labeling Graphs, J. Math., 2014, January 2012, 2014, doi: 10.1155/2014/375135.

[9] K. Ameenal Bibi and M. Devi, Fuzzy anti-magic labeling on some graphs, Kongunadu Res. J., 5(1) (2018), doi: 10.26524/krj244.

[10] K. Ameenal Bibi and M. Devi, A note on fuzzy vertex graceful labeling on some special graphs, International Journal of Advanced Research in Computer Science 8(6) (2017), 175-180, doi: 10.26483/ijarcs.v8i6.4327.

[11] R. J. Shajila and S. Vimala, Fuzzy vertex graceful labeling on wheel and fan graphs, IOSR Journal Mathematics 12(2) (2016), 45-49, doi: 10.9790/5728-12214549.

[12] R. Shajila and S. Vimala, Graceful labelling for complete bipartite fuzzy graphs, Br. J. Math. Comput. Sci., 22(2) (2017), 1-9. doi: 10.9734/bjmcs/2017/32242.

[13] M. Fathalian, R. A. Borzooei and M. Hamidi, Fuzzy magic labeling of simple graphs, J. Appl. Math. Comput. 60(1-2) (2019), doi: 10.1007/s12190-018-01218-x.

[14] M. Akram, bipolar fuzzy graphs, Inf. Sci. (Ny). 181(24) (2011), 5548-5564.

[15] M. Akram and W. A. Dudek, Intuitionistic fuzzy hypergraphs with applications, Inf. Sci. (Ny)., 218 (2013), doi: 10.1016/j.ins.2012.06.024.

[16] M. Akram and W. A. Dudek, Interval-valued fuzzy graphs, Comput. Math. with Appl., 61(2) (2011), doi: 10.1016/j.camwa.2010.11.004.

[17] S. N. Mishrą and A. Pal, Magic labeling of interval-valued fuzzy graph, Ann. Fuzzy Math. Inform. 11(2) (2016), 273-282.

[18] S. Mehra and M. Singh, Some results on intuitionistic fuzzy magic labeling graphs, Aryabhatta Journal of Mathematics and Informatics 09(01) (2017), 931-938.

[19] P. K. Kishore Kumar, S. Lavanya, H. Rashmanlou and M. N. Jouybari, Magic labeling on interval-valued intuitionistic fuzzy graphs, J. Intell. Fuzzy Syst., 33(6) (2017), doi: 10.3233/JIFS-17847.

[20] M. Akram and A. Adeel, m-polar fuzzy graphs and m-polar fuzzy line graphs, J. Discret. Math. Sci. Cryptogr. 20(8) (2017), doi: 10.1080/09720529.2015.1117221.

[21] K. Kalaiarasi and P. Geethanjali, Arcs Sequence in complete and regular fuzzy graphs, International journal of Pure and Applied Mathematics 118(6) (2018), 95-104.