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DISCRIMINATING INDEX ON FUZZY NEAR SETS

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Abstract

In this paper the concept of fuzzy near sets, lower and upper approximation and the boundary region of fuzzy near sets are introduced. Nearness measure on fuzzy near sets is defined. Using nearness measure and topology of nearness measure, discriminating index is calculated. Further discriminating index is plotted graphically.

1. Introduction

A perceptual object is either something presented to the senses or knowable by the mind. Objects that have the same appearance are considered qualitatively near each other, i.e., objects with matching descriptions. The solution to the problem of approximating sets of perceptual objects results from a generalization of the classification of objects introduced by Pawlak [2]. This generalization lead to the introduction of near set by James F. Peters [3, 4]. Peters considered the problem of approximation of sets of perceptual objects that have matching descriptions.

The theory of fuzzy set was introduced by Zadeh [6]. Patnaik et al. [1], introduced a near set approach to image analysis. They measured the degree of nearness of histograms of all the blocks of one image with the corresponding blocks of another image by using near system. In [5], discrimination index is used to find out significant differences shown according to cognitive levels for multiple-choice questions.

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Our aim in this paper is to introduce the notions of fuzzy near set, lower and upper approximation and the boundary region of fuzzy near sets. Nearness measure on fuzzy near sets is defined. Using nearness measure and topology of nearness measure on fuzzy near sets, discriminating index is calculated. Further discriminating index is plotted graphically.

2. Fuzzy Near Set

Notation. Throughout this paper U denotes a non-empty finite universe.

Definition 2.1. Let X be a non-empty set and $A \subseteq X$. Let μ be the membership function. A set denoted by $FN(A)$ is called a fuzzy near set if at least one pair of objects has the same membership value, i.e., there exists at least one pair $x, x', \in X \ni \mu_{FN(A)}(x) = \mu_{FN(A)}(x')$.

Definition 2.2. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. A class $[x]_{FN(A)} = \{x \in X : \mu_{FN(A)}(x) = \mu_{FN(A)}(x')\}$.

Definition 2.3. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The lower approximation of a fuzzy near set $FN(A)$ is defined as:

$$\underline{FN}(A) = \bigcup_{x:[x]_{FN(A)} \subseteq X} [x]_{FN(A)}.$$

Definition 2.4. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The upper approximation of a fuzzy near set $FN(A)$ is defined as:

$$\overline{FN}(A) = \bigcup_{x:[x]_{FN(A)} \cap X \neq \varnothing} [x]_{FN(A)}.$$

Definition 2.5. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The boundary of a fuzzy near set is defined as:

$$B_{FN(A)} = \overline{FN}(A) - \underline{FN}(A).$$

3. Nearness Measure on Fuzzy Near Sets

Definition 3.1. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The nearness measure on fuzzy near set is denoted by $NM_{FN(A)}$ and is defined as:

$$NM_{FN(A)} = \bigcup \{x, x' \in X : \mu_{FN(A)}(x) \neq \mu_{FN(A)}(x')\}.$$

Definition 3.2. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. Let τ be the collection of all subsets of $NM_{FN(A)}$ satisfying the following axioms:

(i) $\emptyset, X \in \tau$

(ii) The union of the elements of any sub collection of τ is in τ

(iii) The intersection of the elements of any finite sub collection of τ is in τ.

τ forms a topology called as the fuzzy near topology on X.

Definition 3.3. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The Discriminating Index (DI) of nearness measure on fuzzy near sets is denoted by $DI(NM)$ and is defined as:

$$DI(NM) = \frac{|U - B_{NM}|}{|U|}.$$

Definition 3.4. Let X be a non-empty set, $A \subseteq X$ and $FN(A)$ be a fuzzy near set. The Discriminating index on topology of nearness measure on fuzzy near set is denoted by $DI(\tau NM)$ and is defined as:

$$DI(\tau NM) = \frac{|U - B_{\tau NM}|}{|U|}.$$

Example 3.5. Let $FN(A1)$, $FN(A2)$ and $FN(A3)$ be three fuzzy near sets defined on a nonempty set $X = \{x_1, x_2, x_3, x_4\}$ as in Table 1.

Table 1. Three fuzzy near sets.

X	$FN(A1)$	$FN(A2)$	$FN(A3)$
x_1	0.2	0.8	0.1
x_2	0.2	0.4	0.9
x_3	0.3	0.4	0.1
x_4	0.7	0.6	1

$NM_{FN(A1)} = \{\{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$

$NM_{FN(A2)} = \{\{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}\}$

$NM_{FN(A3)} = \{\{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_2, x_3, x_4\}\}$

$\tau NM_{FN(A1)} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\},$

$\{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, X, \varnothing\}$

$\tau NM_{FN(A2)} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_4\},$

$\{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, X, \varnothing\}$

$\tau NM_{FN(A3)} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\},$

$\{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\},$

$\{x_2, x_3, x_4\}, X, \varnothing\}.$

Table 2. DI based on nearness measure of fuzzy near sets.

$P(X)$	$NM_{FN(A1)}$	$NM_{FN(A2)}$	$NM_{FN(A3)}$
$\{x_1\}$	0.25	0	0.25
$\{x_2\}$	0.25	0.25	0
$\{x_3\}$	0	0.25	0.25
$\{x_4\}$	0	0	0
$\{x_1, x_2\}$	0	0.5	0.5
$\{x_1, x_3\}$	0.5	0.5	0
$\{x_1, x_4\}$	0.5	0.5	0.5
$\{x_2, x_3\}$	0.5	0	0.5
$\{x_2, x_4\}$	0.5	0.5	0.5
$\{x_3, x_4\}$	0.5	0.5	0.5
$\{x_1, x_2, x_3\}$	0.75	0.75	0.75
$\{x_1, x_2, x_4\}$	0.75	0.75	0.75
$\{x_2, x_3, x_4\}$	0.75	0.75	0.75
$\{x_2, x_3, x_4\}$	0.75	0.75	0.75
$\{x_1, x_2, x_3, x_4\}$	1	1	1

DI-Nearness Measure

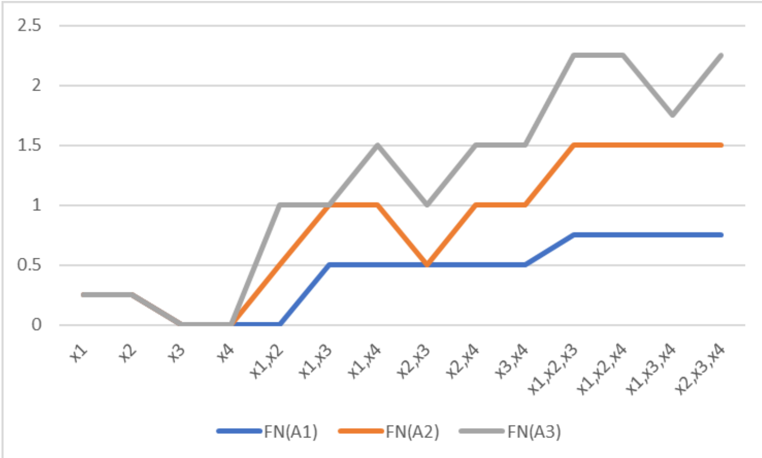
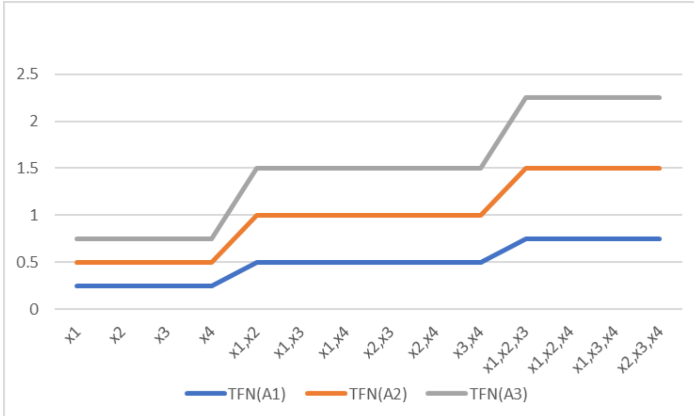


Table 3. DI based on topological nearness measure of fuzzy near set.

$P(X)$	$NM_{\tau FN(A1)}$	$NM_{\tau FN(A2)}$	$NM_{\tau FN(A3)}$
$\{x_1\}$	0.25	0.25	0.25
$\{x_2\}$	0.25	0.25	0.25
$\{x_3\}$	0.25	0.25	0.25
$\{x_4\}$	0.25	0.25	0.25
$\{x_1, x_2\}$	0.5	0.5	0.5
$\{x_1, x_3\}$	0.5	0.5	0.5
$\{x_1, x_4\}$	0.5	0.5	0.5
$\{x_2, x_3\}$	0.5	0.5	0.5
$\{x_2, x_4\}$	0.5	0.5	0.5
$\{x_3, x_4\}$	0.5	0.5	0.5
$\{x_1, x_2, x_3\}$	0.75	0.5	0.75
$\{x_1, x_2, x_4\}$	0.75	0.75	0.75
$\{x_1, x_3, x_4\}$	0.75	0.75	0.75
$\{x_2, x_3, x_4\}$	0.75	0.75	0.75
$\{x_1, x_2, x_3, x_4\}$	1	1	1

DI-Topological nearness measure



Comparing the discriminating index using nearness measure and topology of nearness measure from Table 2 and Table 3, we find that the values are more consistent in Table 3 than in Table 2. This shows that the discriminating index using topology of nearness measure is more appropriate.

Example 3.6. Consider three fuzzy near sets $FN(A1)$, $FN(A2)$, $FN(A3)$ as in Table 1 of Example 3.5. Let the fuzzy near sets $FN(A1)$, $FN(A2)$, $FN(A3)$ represent plant growth, flowering and yield, respectively, of coconut palm.

Let $X = \{x_1, x_2, x_3, x_4\}$ be the macro nutrients: Nitrogen, Phosphorous, Potassium and Magnesium required for plant growth, flowering and yield of coconut palm. The discriminating index in the first column of Table 3 indicates that when Nitrogen, Phosphorous, Potassium and Magnesium are supplied independently, then the growth rate is 0.25, when supplied in combinations of two (viz. Nitrogen, Phosphorous) and so on the growth rate is 0.5, when supplied in combinations of three the growth rate is 0.75 and when all the nutrients are supplied the growth rate is 1 indicating 100 percent growth. Similarly, for flowering and yield of coconut palm.

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