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COST ANALYSIS OF FUZZY QUEUEING MODELS

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Abstract

In this paper, we study the fuzzy retrial queueing models with priority disciplines. It optimizes the retrial queueing model with no priority, preemptive priority, and non-preemptive priority. The rate of arrival, the rate of service, and the rate of retrial are fuzzy numbers. A new α cut approximation method defines the priority queuing system's performance measures' membership functions. A numerical example is also illustrated to check the validity of the model.

1. Introduction

Queueing theory is one of the branch studies of applied probability theory. A queue describes the waiting line of customers that claims service from a service station, and it constructs when service is not provided instantly. The queueing theory was first introduced by A. K. Erlang [4]. The principal purpose of the analysis of queueing systems is to understand their underlying processes' behavior so as informed and intelligent decisions can be made in their organization. Most of the queueing models were studied with the queueing discipline "First Come First Serve." However, situations that commonly occur that an arriving customer may be distinguished according to some critical measure.

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2. Model Description

We consider a priority queuing system with a single server, infinite calling population with arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$, and retrial rate $\tilde{\theta}$. Studying the queuing model aims to reduce customers' waiting time in the queue and the system's cost. Here, the system's cost represents long-run average cost per unit time, such as waiting for space, utilization cost of system's facility, cost of insurance, etc. [8], [9]. To incorporate the priority discipline fuzzy queuing model, we must compare the system's average total cost for the three cases. No priority discipline, preemptive priority, and non-preemptive priority discipline are denoted by C', C'' and C''' respectively [10].

3. Crisp Results

(i) No Priority Retrial Queueing Model. Average total cost of the system, when there is No Priority Discipline C',

$$C' = (C_1\tilde{\lambda}_1 + C_2\tilde{\lambda}_2)W,$$

where

$$W = \frac{\tilde{\lambda} + \tilde{\theta}}{(\tilde{\mu} - \tilde{\lambda})\tilde{\theta}}.$$

(ii) Preemptive Priority Retrial Queueing Model. Average total cost of the system when there is Preemption Priority, C'.

$$C'' = C_1 T_1 + C_2 T_2,$$

where

$$T_1 = \frac{\rho_1^2(1 + \rho_2 - \rho)}{(1 - \rho_1)^2}$$

$$T_2 = \rho_2 + \frac{\rho_1}{1 - \rho_1}\left(\rho_2 + \frac{\tilde{\lambda}_2}{\tilde{\theta}}\right) + \frac{\rho_2}{(1 - \rho)(1 - \rho_1)}\left(\rho_2 + \frac{\rho\rho_2}{1 - \rho_1}\right) + \frac{3\rho\rho_3}{2(1 - \rho_1)^2}.$$

(iii) Non-Preemptive Priority Retrial Queueing Model. Average total cost of the system, when there is Non-Preemptive Priority C''

$$C''' = C_1 L_1 + C_2 L_2,$$

where

$$L_1 = \frac{\rho\rho_1}{1-\rho_1} + \rho_1$$

$$L_2 = \frac{\rho\rho_2}{(1-\rho_1)(1-\rho)} + \frac{\tilde{\lambda}_2\rho}{\tilde{\theta}(1-\rho)} + \rho_2.$$

Comparing the three total costs shows which of priority disciplines minimize the system's average total cost function [10].

4. Proposed Algorithm

A constant range can express any continuous membership function α cut from $\alpha = 0$ to $\alpha = 1.$ This algorithm consists of the following steps:

(i) Select α cut value where $0 \leq \alpha \leq 1$.

(ii) Finding the intervals in the input membership functions that correspond to this α.

(iii) Using the ranking method $\left[R(A) = \frac{c-a}{2}\right]$, convert the fuzzy number into crisp values.

(iv) Evaluate the crisp values for the output membership function for the selected α-cut level.

(v) Repeat steps (i) to (iv) for different values of α to complete α-cut representation of the solution α.

5. Numerical Illustrations

Consider a telecommunication system in which calls arrive in two classes. With 20% and 80% utilization, calls come at this system following a poison process. The service times and the retrial times follow an exponential distribution. The arrival rate, service rate, and retrial rate are triangular fuzzy numbers given by $\tilde{\lambda} = [20, 26, 35]\tilde{\mu} = [25, 40, 50]$ and $\tilde{\theta} = [12, 22, 25]$ per minute respectively. The possibility distribution of the unit cost of inactivity of two classes is triangular fuzzy number $C_1 = [10, 25, 28]$ and $C_2 = [6, 16, 17]$, respectively. The system manager enquires to evaluate the total cost of the system when there is

(i) no priority discipline C',

(ii) preemptive priority discipline C'',

(iii) non-preemptive priority discipline in the retrial queue C''',

S. No.	α	Queue Discipline		
		C'	C''	C'''
1	0	20.03081	12.96227	16.57167
2	0.2	17.82229	11.31235	14.66927
3	0.4	15.74156	9.77622	12.88043
4	0.6	13.78881	8.40285	11.2052
5	0.8	11.96385	7.044423	9.64375
6	1	10.26656	5.847723	8.195691

6. Conclusion

Comparison results show the three total costs of which of the priority disciplines minimize the system's average total cost function. Even though they overlap fuzzy numbers, the minimum average total cost of the system is achieved with the non-preemptive priority discipline. The method proposed enables a reasonable solution for each case, with a different level of feasibility. This approach offers more details to help to design a fuzzy priority discipline queuing system.

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