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CUBIC (1, 2)-IDEALS OF CUBIC NEAR-RINGS

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Abstract

Young Bae Jun made an effort in defining a remarkable structure namely cubic sets. The concept of cubic set is inter linked with interval – valued fuzzy set and fuzzy set. Inspired by the theory of cubic structure our aim of this paper is to introduce the notion of cubic (1, 2)-ideals of cubic near-rings and to investigate some of their properties. The characterizations of cubic (1, 2)-ideals of cubic near-rings are provided.

1. Introduction

The notion of fuzzy sets introduced by Zadeh [10] in 1965 laid the foundation for the development of fuzzy Mathematics. This theory has a wide range of application in several branches of Mathematics such as logic, set theory, group theory, semi group theory, real analysis, measure theory and topology. After a decade, the notaion of interval-valued fuzzy set was introduced by Zadeh [11] in 1975, as an extension of fuzzy sets. Jun et al., [5] have introduced a remarkable theory, namely the theory of cubic sets. This structure is comprised of an interval-valued fuzzy set and a fuzzy set. The

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idea of fuzzy ideals of near-rings was first proposed by Kim et al. [6]. N. Thillaigovindan et al., [9] introduced interval valued fuzzy ideals of near rings.

In this paper, we introduce the concept of cubic (1, 2)-ideals of cubic near-rings and investigate related properties.

2. Preliminaries

In this section, we list some concepts and results related to cubic (1, 2)-ideals of cubic near-rings needed in next section.

Definition 2.1 [8]**.** A near-ring N is a system with two binary operations '+' and ' • ' such that

1. $(N, +)$ is a group not necessarily abelian;

2. $(N, \cdot)$ is a semigroup;

3. $(x+y)z = xz + yz$ for all $x, y, z \in N$. We will use the word "near-ring" to mean "right distributive near-ring". We denote xy instead of $x.y$.

Definition 2.2 [11]**.** Let X be a non-empty set. A mapping $\bar{\mu} : X \to D[0, 1]$ is called interval-valued fuzzy set, where $D[0, 1]$ denote the family of all closed subintervals of $[0, 1]$.

Definition 2.3 [4]**.** Let X be a non-empty set. A cubic set A of X is a structure $\mathscr{A} = \{(x, \bar{\mu}(x), \gamma_A(x)) : x \in X\}$ which is briefly denoted by $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$, where $\bar{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy subset of X and γ is a fuzzy set in X.

Definition 2.4 [3]**.** Let $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$ be a cubic set of X. For any $r \in [0, 1]$ and $[s, t] \in D[0, 1]$, we define $U(\mathscr{A}; [s, t], r)$ as follows:

$$U(\mathscr{A}\,[s, t], r) = \{x / \mu_A(x) \geq [s, t], \gamma_A(x) \leq r\}$$

and we say it is a cubic level set of $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$

Definition 2.5 [1]**.** Let N be a near-ring, $(N, \bar{\mu})$ be an interval-valued fuzzy near-ring and (N, γ) be a fuzzy near-ring. A cubic set $\mathcal{A} = \langle \overline{\mu}, \gamma \rangle$ is called a cubic near-ring of N if it satisfies the following conditions,

1. $\overline{\mu}(x - y) \geq \min\{\overline{\mu}(x), \overline{\mu}(y)\}$,
2. $\overline{\mu}(xy) \geq \min\{\overline{\mu}(x), \overline{\mu}(y)\}$.
3. $\gamma(x - y) \leq \max\{\gamma(x), \gamma(y)\}$,
4. $\gamma(xy) \leq \max\{\gamma(x), \gamma(y)\}, \forall x, y \in N$.

Definition 2.6. A fuzzy subnear-ring A of N is called a fuzzy (1, 2)-ideal of N if it satisfies for all $w, x, y, z \in N$,

1. $\mathcal{A}(xw - (y - z)) \geq \min\{\mathcal{A}(x), \mathcal{A}(y), \mathcal{A}(z)\}$,
2. $\mathcal{A}(xw(yz)) \geq \min\{\mathcal{A}(x), \mathcal{A}(y), \mathcal{A}(z)\}$.

Definition 2.7. An interval-valued fuzzy subnear-ring $\overline{\mu}$ of N is called an interval-valued fuzzy (1, 2)-ideal of N if it satisfies for all $w, x, y, z \in N$,

1. $\overline{\mu}(xw - (y - z)) \geq \min\{\overline{\mu}(x), \overline{\mu}(y), \overline{\mu}(z)\}$,
2. $\overline{\mu}(xw(yz)) \geq \min\{\overline{\mu}(x), \overline{\mu}(y), \overline{\mu}(z)\}$.

Definition 2.8 [3]**.** A cubic set $\mathcal{A} = (\overline{\mu}_A, \lambda_A)$ of a near-ring N is called a cubic sub near-ring of N, if

1. $\overline{\mu}_A(x - y) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$,
2. $\overline{\mu}_A(xy) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$,
3. $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
4. $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$.

Definition 2.9 [2]**.** A cubic set $\mathcal{A} = (\overline{\mu}_A, \lambda_A)$ in X is a cubic bi-ideal of X if for all $x, y, z \in X$. If it satisfies the following conditions,

1. $\overline{\mu}_A(x - y) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$,
2. $\overline{\mu}_A(xyz) \geq \min\{\overline{\mu}_A(x), \overline{\mu}_A(z)\}$,

3. $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,

4. $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$.

In this section, we introduce the notion of cubic (1, 2)-ideal of cubic near-rings and discuss some of its properties:

Definition 3.1. A cubic subnear-ring $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ of a cubic near-ring N is called a cubic (1, 2)-ideal of N, if

1. $\bar{\mu}_A(xw - (y - z)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}$,

2. $\gamma_A(xw - (y - z)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}$,

3. $\bar{\mu}_A(xw(y - z)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}$,

4. $\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}\ \forall w, x, y, z \in N$.

Example 3.2. Let $N = \{0, 1, 2, 3\}$ be the Klein's four group. Define Multiplication in N as follows: (scheme 5: (0,1,1,0) see [8], p.407).

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	1	0
2	0	3	3	0
3	0	2	2	0

Then $(N, +, \cdot)$ is a cubic near-ring. Define $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ in N by $\bar{\mu}(0) = [0.4, 0.5]$, $\bar{\mu}(1) = [0.2, 0.3] = \bar{\mu}(3)$, $\bar{\mu}(2) = [0.3, 0.4]$ and $\gamma(0) = 0.25$, $\gamma(1) = 0.33 = \gamma(3)$, $\gamma(2) = 0.30$.

By routine calculation, we can check that $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of a cubic near-ring.

Theorem 3.3. *Every cubic bi-ideal is a cubic* (1, 2)-*ideal*.

Proof. Let $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ be a cubic bi-ideal of N and let $w, x, y, z \in N$. Then

$$\begin{aligned}\bar{\mu}_A(xw-(y-z)) &= \bar{\mu}_A((xw)-(y-z))\\ &\geq \min\{\bar{\mu}_A(xw), \bar{\mu}_A(y-z)\}\\ &\geq \min\{\bar{\mu}_A(x), \min\{\bar{\mu}_A(y), \bar{\mu}_A(z)\}\}\\ &= \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}\end{aligned}$$

$$\begin{aligned}\bar{\mu}_A(xw(yz)) &= \bar{\mu}_A((xwy)z)\\ &\geq \min\{\bar{\mu}_A(xwy), \bar{\mu}_A(z)\}\\ &\geq \min\{\min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}\\ &= \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}\end{aligned}$$

and

$$\begin{aligned}\gamma_A(xw-(y-z)) &= \gamma_A((xw)-(y-z))\\ &\leq \max\{\gamma_A(xw), \gamma_A(y-z)\}\\ &\leq \max\{\gamma_A(x), \max\{\gamma_A(y), \gamma_A(z)\}\}\\ &= \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}\end{aligned}$$

$$\begin{aligned}\gamma_A(xw(yz)) &= \gamma_A((xwy)z)\\ &\leq \max\{\gamma_A(xwy), \gamma_A(z)\}\\ &\leq \max\{\max\{\gamma_A(y), \gamma_A(y)\}, \gamma_A(z)\}\\ &= \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}\end{aligned}$$

Therefore $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of N.

Theorem 3.4. *If N is a regular cubic near-ring, then every cubic* (1, 2)-*ideal of N is a cubic bi-ideal of N.*

Proof. Assume that a cubic near-ring N is regular and let $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ be a cubic (1, 2)-ideal of N. Let $w, x, y \in N$. Since N is regular, we have $xw \in (xNx)N \subseteq xNx$, which implies that $xw = xnx$ for some $n \in N$. Thus

$$\bar{\mu}_A(xw-y) = \bar{\mu}_A((xnx)(-y))$$

$$= \bar{\mu}_A(xn(x-y))$$

$$\geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(x), \bar{\mu}_A(y)\}$$

$$= \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$$

$$\bar{\mu}_A(xwy) = \bar{\mu}_A((xnx)y)$$

$$= \bar{\mu}_A((xnx)y)$$

$$\geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(x), \bar{\mu}_A(y)\}$$

$$= \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}.$$

and

$$\gamma_A(xw-y) = \gamma_A((xnx)(-y))$$

$$= \gamma_A(xn(x-y))$$

$$\leq \max\{\gamma_A(x), \gamma_A(x), \gamma_A(y)\}$$

$$= \max\{\gamma_A(x), \gamma_A(y)\},$$

$$\gamma_A(xwy) = \gamma_A((xnx)y)$$

$$= \gamma_A(xn(xy))$$

$$\leq \max\{\gamma_A(x), \gamma_A(x), \gamma_A(y)\}$$

$$= \max\{\gamma_A(x), \gamma_A(y)\}.$$

Therefore $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic bi-ideal of N. □

Theorem 3.5. *A cubic set* $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ *is a cubic* (1, 2)*-ideal of N if and only if for any* $r \in [0, 1]$ *and* $[s, t] \in D[0, 1]$ *the cubic level set* $U(\mathcal{A}; [s, t], r) = \{x \in X / \bar{\mu}_A(x) \geq [s, t], \gamma_A(x) \in r\}$ *is* (1, 2)*-ideal of N.*

Proof. Let $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ be a cubic (1, 2)-ideal of N. For $[s, t] \in [0, 1]$ if $x, y \in U(\bar{\mu}_A; [s, t])$, then $\bar{\mu}_A(x) \geq [s, t], \bar{\mu}_A(y) \geq [s, t]$. So, $\bar{\mu}_A(x-y) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} \geq [s, t] = \min\{[s, t], [s, t]\} = [s, t]$. Thus $x - y \in U(\bar{\mu}_A; [s, t])$. $\bar{\mu}_A(xy) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}[s, t] = \min\{[s, t], [s, t]\} = [s, t]$. Thus $xy \in U(\bar{\mu}_A; [s, t])$.

Hence $U(\bar{\mu}_A; [s, t])$ is a sub near-ring of N.

For $[a_1, a_2] \in [0, 1]$, if $U(\bar{\mu}_A; [a_1, a_2])$ is non-empty and $x, y, z \in U(\bar{\mu}_A; [a_1, a_2]), w \in N$ then $\bar{\mu}_A(x) \geq [a_1, a_2], \bar{\mu}_A(y) \geq [a_1, a_2]$, $\bar{\mu}_A \geq [a_1, a_2]$, so $\bar{\mu}_A(xw - (y - z)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\}] \geq [a_1, a_2]$ and $\bar{\mu}_A(xw(yz)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\} \geq [a_1, a_2]$. This shows that, $xw - (y - z) \in U(\bar{\mu}_A; [a_1, a_2])$ and $xw - (yz) \in U(\bar{\mu}_A; [a_1, a_2])$.

Therefore, $U(\bar{\mu}_A; [s, t])$ is a (1, 2)-ideal of N.

For $r \in [0, 1]$, if $x, y \in U(\gamma_A; r)$ then $\gamma_A(x) \leq r, \gamma_A(y) \leq r$. So, $\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\} \leq r \max\{r, r\} = r$. Thus $x - y \in U(\gamma_A; r)$. and $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\} \leq r = \max\{r, r\} = r$. Thus $xy \in U(\gamma_A; r)$. Therefore $U(\gamma_A; r)$ is a sub near-ring of N.

For $\alpha \in [0, 1]$ if $U(\gamma_A; a)$ is non-empty and $x, y, z \in U(\gamma_A; a), w \in N, \gamma_A(y) \leq a, \gamma_A(z) \leq a$. So $\gamma_A(y) \leq a, \gamma_A(z) \leq a$. $\gamma_A(xw - (y - z)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\} \leq a$ and $\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\} \leq a$ This shows that $xw(y - z) \in U(\gamma_A; a)$ and $xw(yz) \in U(\gamma_A; a)$. Therefore $U(\gamma_A; r)$ is a (1, 2)-ideal of N.

Conversely, assume that for any $r \in [0, 1]$ and $[s, t] \in D[0, 1]$ the cubic level set $U(\mathscr{A}; [s, t], r)$ is (1, 2)-ideal of N, then $U(\bar{\mu}_A; [s, t])$ and $U(\gamma_A; r)$ are subnear-rings of N.

If there exist $a, b \in N$ such that $\bar{\mu}_A(a - b) < \max\{\bar{\mu}_A(a), \mu_A(b)\}$, $\bar{\mu}_A(ab) < \max\{\bar{\mu}_A(a), \bar{\mu}_A(b)\}$. Choose an interval $\bar{a}_0 = [a_1, a_2]$. Taking $\bar{a}_0 = \frac{1}{2}(\bar{\mu}_A(a - b) + \max\{\bar{\mu}_A(a), \bar{\mu}_A(b)\})$ and $\bar{a}_0 = \frac{1}{2}(\bar{\mu}_A(ab) + \max\{\bar{\mu}_A(a), \bar{\mu}_A(b)\})$ so $a, b \in U(\bar{\mu}_A; [a_1, a_2])$. Then $a - b \notin U(\bar{\mu}_A; [a_1, a_2])$ and $ab \notin U(\bar{\mu}_A; [a_1, a_2])$. But $U(\bar{\mu}_A; [a_1, a_2])$ is a subnear-ring of N. This is a contradiction. Hence for all $x, y \in N, \bar{\mu}_A(x - y) \geq \min\{\bar{\mu}_A(x), \mu_A(y)\}$ and $\bar{\mu}_A(xy) \geq \min\{\bar{\mu}_A(x), \mu_A(y)\}$.

If there exist $u, a, b, c \in N$ such that $\bar{\mu}_A(au - (b - c)) < \max\{\bar{\mu}_A(a), \bar{\mu}_A(b), \bar{\mu}_A(c)\}$ and $\bar{\mu}_A(au(bc)) < \max\{\bar{\mu}_A(a), \bar{\mu}_A(b), \bar{\mu}_A(c)\}$.

Then $\bar{\mu}_A(au-(b-c)) > \bar{a}_0 > \max\{\bar{\mu}_A(a), \bar{\mu}_A(b), \bar{\mu}_A(c)\}$ and $\bar{\mu}_A(au(bc)) > \bar{a}_0 > \max\{\bar{\mu}_A(a), \bar{\mu}_A(b), \bar{\mu}_A(c)\}$ and so $a, b, c \in U(\bar{\mu}_A; [a_1, a_2])$, $au-(b-c) \notin U(\bar{\mu}_A; [a_1, a_2])$, $au(bc) \notin U(\bar{\mu}_A; [a_1, a_2])$. But $U(\bar{\mu}_A; [a_1, a_2])$ is a (1, 2)-ideal of N. This is a contradiction. Hence for all $w, x, y, z \in N$, $\bar{\mu}_A(xw-(y-z)) \geq \min\{\bar{\mu}(x), \bar{\mu}(y), \bar{\mu}(z)\}$ and $\bar{\mu}_A(xw(yz)) \geq \min\{\bar{\mu}(x), \bar{\mu}(y), \bar{\mu}(z)\}$.

If there exist $a, b \in N$ such that $\gamma_A(a-b) > \min\{\gamma_A(a), \gamma_A(b)\}$, $\gamma_A(ab) > \min\{\gamma_A(a), \gamma_A(b)\}$. Taking $r_0 = \frac{1}{2}(\gamma_A(a-b) + \min\{\gamma_A(a), \gamma_A(b)\})$ and $r_0 = \frac{1}{2}(\gamma_A(ab) + \min\{\gamma_A(a), \gamma_A(b)\})$. We get $\gamma_A(a-b) > r_0 > \{\gamma_A(a), \gamma_A(b)\}$ and $\gamma_A(ab) > r_0 > \{\gamma_A(a), \gamma_A(b)\}$ and so $a, b \in U(\gamma_A; r_0)$ and $a-b \notin U(\gamma_A; r_0)$, $ab \notin U(\gamma_A; r_0)$. But $U(\gamma_A; r_0)$ is a subnear-ring of N. This is a contradiction. Hence for all $x, y \in N$, $\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ and $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

If there exist $u, a, b, c \in N$ such that $\gamma_A(au-(b-c)) > \min\{\gamma_A(a), \gamma_A(b), \gamma_A(c)\}$ and $\gamma_A(au(bc)) > \min\{\gamma_A(a), \gamma_A(b), \gamma_A(c)\}$. Then $\gamma_A(au-(b-c)) > r_0 > \min\{\gamma_A(a), \gamma_A(b), \gamma_A(c)\}$ and $\gamma_A(au(bc)) > r_0 > \min\{\gamma_A(a), \gamma_A(b), \gamma_A(c)\}$ and so $a, b, c \in U(\gamma_A; r_0)$, $au-(b-c) \notin U(\gamma_A; r_0)$. $au(bc) \notin U(\gamma_A; r_0)$. But $U(\gamma_A; r_0)$ is a (1, 2)-ideal of N. This is a contradiction. Hence for all $w, x, y, z \in N$, $\gamma_A(xw-(y-z)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}$ and $\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}$.

Therefore a cubic set $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of N.

Theorem 3.6. *Let I be a non empty subset of N. Let $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$ be a cubic set of N defined by*

$$\bar{\mu}_A(x) = \begin{cases} [s_1, s_2] & \text{if } x \in I \\ [r_1, r_2] & \text{otherwise} \end{cases},$$

$$\gamma_A(x) = \begin{cases} 1-s & \text{if } x \in I \\ 1-r & \text{otherwise} \end{cases}$$

for all $x \in N$, $[s_1, s_2], [r_1, r_2] \in [0, 1]$ and $s, r \in (0, 1]$ with $[s_1, s_2] > [r_1, r_2]$ and $s > r$. Then I is a (1, 2)-ideal of N if and only if $\mathscr{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of N.

Proof. Assume that I is a (1, 2)-ideal of N. We consider four cases:

(i) $x \in I$ and $y \in I$,

(ii) $x \in I$ and $y \notin I$,

(iii) $x \notin I$ and $y \in I$,

(iv) $x \notin I$ and $y \notin I$.

Case (i) If $x \in I$ and $y \in I$ then $\bar{\mu}_A(x) = [s_1, s_2] = \bar{\mu}_A(y)$ and $\gamma_A(x) = 1 - s = \gamma_A(y)$. So $x - y \in I$, $xy \in I$, since I is a (1, 2)-ideal of N. Thus $\bar{\mu}_A(x - y) = [s_1, s_2] = \min\{[s_1, s_2], [s_1, s_2]\} = \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$, $\bar{\mu}_A(xy) = \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ and $\gamma_A(x - y) = 1 - s = \max\{1 - s, 1 - s\} = \max\{\gamma_A(x), \gamma_A(y)\}$, $\gamma_A(xy) = \max\{\gamma_A(x), \gamma_A(y)\}$.

Case (ii) If $x \in I$ and $y \notin I$. Then $\bar{\mu}_A(x) = [s_1, s_2]$, $\bar{\mu}_A(y) = [r_1, r_2]$ and $\gamma_A(x) = 1 - s = \gamma_A(y)$. Clearly, $\min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} = [r_1, r_2]$ and $\max\{\gamma_A(x), \gamma_A(y)\} = 1 - r$. Then $\bar{\mu}_A(x - y), = [s_1, s_2]$ or $[r_1, r_2]$ and $\gamma_A(x - y) = 1 - s$ or $1 - r$ if $x - y \in I$ or $x - y \notin I$. Similarly, $\bar{\mu}(xy) = [s_1, s_2]$ or $[r_1, r_2]$ and $\gamma_A(xy) = 1 - s$ or $1 - r$ if $xy \in I$ or $xy \notin I$. By our assumption that $[s_1, s_2] > [r_1, r_2]$ and $s > r$, then $\bar{\mu}_A(x - y) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$, $\bar{\mu}_A(xy) > \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ and $\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$, $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

Similarly we can prove Case (iii).

Case (iii) If $x, y \notin I$ that is $\bar{\mu}_A(x) = [r_1, r_2] = \bar{\mu}_A(y)$ and $\gamma_A(x) = 1 - r = \gamma_A(y)$. Then, $\min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} = [r_1, r_2]$ and $\max\{\gamma_A(x), \gamma_A(y)\} = 1 - r$. This shows that $\bar{\mu}_A(x - y) = [s_1, s_2]$ or $[r_1, r_2]$, and $\gamma_A(x - y) = 1 - s$ or $1 - r$ if $x - y \in I$ or $x - y \notin I$, similarly $\bar{\mu}_A(xy) = [s_1, s_2]$ or $[r_1, r_2]$ and $\gamma_A(x - y) = 1 - s$ or $1 - r$ if $x - y \notin I$, or

$x - y \notin I$. Thus $\bar{\mu}_A(x-y) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}, \bar{\mu}_A(xy) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ and $\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\}, \gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$. This shows that $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic subnear-ring of N.

Let $w, x, y, z \in N$. We prove the result by following cases: If at least one of x, y, z does not belong to I, then $\bar{\mu}_A(xw - (y-z)) = [(xw(yz)) = [s_1, s_2]$ or $[r_1, r_2], \bar{\mu}_A(xw(yz)) = [s_1, s_2]$ or $[r_1, r_2]$ and $\gamma_A(xw - (y-z)) = 1 - s$ or $1 - r, (xw(yz)) = 1 - s$ or $1 - r$ if $x - (y-z) \in I, x(yz) \in I$ or $x - (y-z) \notin I, x(yz) \notin I$. By our assumption that $[s_1, s_2] > [r_1, r_2]$ and $s > r$ then,

$$\bar{\mu}_A(xw - (y-z)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\},$$

$$\bar{\mu}_A(xw(yz)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\} \text{ and}$$

$$\gamma_A(xw - (y-z)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\},$$

$$\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}.$$

Let $x, y, z \in I$. Since I is a (1, 2)-ideal of N, then $(xw - (y-z)) \in I$, $(xw(yz)) \in I$. So,

$$\bar{\mu}_A(xw - (y-z)) = \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\},$$

$$\bar{\mu}_A(xw(yz)) = \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\} \text{ and}$$

$$\gamma_A(xw - (y-z)) = \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\},$$

$$\gamma_A(xw(yz)) = \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\}.$$

In conclusion, we have a cubic set $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of N.

Conversely, assume that a cubic set $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic (1, 2)-ideal of N, then $\mathcal{A} = (\bar{\mu}_A, \gamma_A)$ is a cubic sub near-ring of N. Let $x, y \in I$ such that $\bar{\mu}_A(x) = [s_1, s_2] = \bar{\mu}_A(y)$ and $\gamma_A(x) = 1 - s = \gamma_A(y)$. By hypothesis, $\bar{\mu}_A(x-y)$ $\geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} = [s_1, s_2], \bar{\mu}_A(xy) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\} = [s_1, s_2]$ and

$\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\} = 1-s$, $\gamma_a(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\} = 1-s$. So $x-y \in I$ and $xy \in I$. and $xy \in I$ Therefore I is a sub near-ring of N. Let $x, y, z \in I, w \in N$ we have

$$\bar{\mu}_A(xw-(y-z)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\} = [s_1, s_2],$$

$$\bar{\mu}_A(xw(yz)) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y), \bar{\mu}_A(z)\} = [s_1, s_2] \text{ and}$$

$$\gamma_A(xw-(y-z)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\} = 1-s,$$

$$\gamma_A(xw(yz)) \leq \max\{\gamma_A(x), \gamma_A(y), \gamma_A(z)\} = 1-s.$$

It follows that $xw-(y-z) \in I$. and I is a (1, 2)-ideal of a cubic near-ring N.

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