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EFFECTS OF PENTATOPE NUMBER ON OTHER POLYGONAL AND FIGURATE NUMBERS

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Abstract

We initiate disparate associations in the company of Pentatope Number and Other Figurate Numbers.

1. Introduction

Pentatope Numbers belongs in Pascal's triangle. It can be represented as regular, discrete geometric patterns (1-3). The nth Pentatope number is given by $(Ptop)_\beta = \frac{\beta(\beta+1)(\beta+2)(\beta+3)}{24}$. There are different types of figurate numbers which can be plotted by points and shows as polygonals. Here we consider Pentatope numbers and some special figurate numbers. Here we procure scores of rivet associations between these numbers are disclose by means of theorems (4-5).

Notations

Pentatope Number $= (Ptope)_\beta$

Hex Number $= (Hex)_\beta$

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Centered Pyramidal number of Rank β with side

$\gamma = (CPY)_{\gamma^{\beta}}$

Prism Number $= (Psm)_k^{\beta}$

Pronic Number $= (\Pr o)_{\beta}$

Gnomic Number $= (Gno)_{\beta}$

Centered Cube Number $= (Ccu)_{\beta}$

Centered Polygonal Numbers $= C(k, \beta)$

Polygonal Numbers $= P(m, \beta)$

Stella Octangula Number $= (Stoct)_{\beta}$

Tetrahedral Number $= (Tet)_{\beta}$

Octahedral Number $= (Octhe)_{\beta}$

Nexus Number $= (Nex)_{\beta}$

Rhombic Dodecahedral Number $= (Rhodo)_{\beta}$

m-gram Number $= (mgram)_{\beta}$

Hauy Octahedral Number $= (Hoct)_{\beta}$

Method of Analysis

Theorem 1. $48(ptop)_{\beta} - C(4, n^2) - 9(CPY)_8^{\beta} - 6(Hex)_{\beta} - P(6, \beta) + (Gno)_{\beta} + 8 = 0$

Proof of Theorem 1

$$48(Ptop)_{\beta} = (2\beta^4 + 2\beta^2 + 1) + 12\beta^3 + 20\beta^2 + 12\beta - 1$$

$$= C(4, \beta^2) + 9(CPY)_8^{\beta} + (18\beta^2 + 18\beta + 6) + 2\beta^2 - 3\beta - 7$$

$$= C(4, \beta^2) + 9(CPY)_8^\beta + (Hex)_\beta + P(6, \beta) - (2\beta - 1) - 8$$

$$= C(4, \beta^2) + 9(CPY)_8^\beta + 6(Hex)_\beta + P(6, \beta) - (Gno)_\beta - 8$$

Hence $48(ptop)_\beta - C(4, n^2) - 9(CPY)_8^\beta - 6(Hex)_\beta - P(6, \beta) + (Gno)_\beta + 8 = 0$

Theorem 2. $24(ptop)_\beta = P(3, \beta) + 2(Ccu)_\beta + 2(Gno)_{\beta^2}$

Proof of Theorem 2

$$24(ptop)_\beta = \beta^4 + 6\beta^3 + 11\beta^2 + 6\beta$$

$$= (\beta^4 + 2\beta^3 + \beta^2) + 4\beta^3 + 10\beta^2 + 6\beta$$

$$P^2(3, \beta) + 2(Ccu)_\beta + 4\beta^2 - 2$$

Hence $24(ptop)_\beta = P(3, \beta) + 2(Ccu)_\beta + 2(Gno)_{\beta^2}$

Theorem 3. *For all values of* β, $24(ptop)_\beta = 18(CPY)_\beta^4 - (\Pr o)_{\beta^2} - (\Pr o)_\beta$ *is an even integer.*

Proof of Theorem 3

$$24(Ptop)_\beta = (6\beta^3 + \beta^2 + 3\beta) + \beta^4 + 2\beta^2 + 3\beta$$

$$= 18(CPY)_\beta^4 + (\beta^4 + \beta^2) + \beta^2 + 3\beta$$

$$= 18(CPY)_\beta^4 + (\Pr o)_{\beta^2} + (\Pr o)_\beta + 2\beta$$

Therefore, $24(ptop)_\beta = 18(CPY)_\beta^4 - (\Pr o)_{\beta^2} - (\Pr o)_\beta$ is an even integer.

Theorem 4. $6(ptop)_{2\beta} - P(6, \beta)C(4, \beta) - 12(CPY)_\beta^7 - C(10, \beta) - 3(Gno)_\beta + 3\beta \equiv 0 \pmod 4$

Proof of Theorem 4

$$24(Ptop)_{\beta} = 16\beta^4 + 48\beta^3 + 44\beta^2 + 12\beta$$

Therefore $6(Ptop)_{2\beta} = (4\beta^4 + 2\beta^3 - \beta) + (10\beta^3 + 11\beta^2 + 4\beta)$

$$= P(6, \beta)C(4, \beta) + (10\beta^3 + 6\beta^2 - 4\beta) + 5\beta^2 + 8\beta$$

$$= P(6, \beta)C(4, \beta) + 12(CPY)_{\beta}^{7} + C(10, \beta) + (6\beta - 3) - 3\beta + 4$$

Hence $6(ptop)_{2\beta} - P(6, \beta)C(4, \beta) - 12(CPY)_{\beta}^{7} - C(10, \beta) - 3(Gno)_{\beta} + 3\beta \equiv 0(\mathrm{mod}\,4)$

Theorem 5. *$24(ptop)_{\beta} - (Psm)_{\beta}^{\beta} - 6(CPY)_{7}^{\beta} - 2P(13, \beta) - 7(Gno)_{\beta} + 17$ is a nasty number.*

Proof of Theorem 5

$$24(Ptop)_{\beta} = (\beta^4 - \beta^3 + 2\beta) + 7\beta^3 + 11\beta^2 + 4\beta$$

$$= (Psm)_{\beta}^{\beta} + 6(CPY)_{7}^{\beta} + 11\beta^2 + 5\beta$$

$$= (Psm)_{\beta}^{\beta} + 6(CPY)_{7}^{\beta} + 2P(13, \beta) + 14\beta$$

$$= (Psm)_{\beta}^{\beta} + 6(CPY)_{7}^{\beta} + 2P(13, \beta) + 7(Gno)_{\beta} + 7$$

Hence $24(ptop)_{\beta} - (Psm)_{\beta}^{\beta} - 6(CPY)_{7}^{\beta} - 2P(13, \beta) - 7(Gno)_{\beta} + 17$ is a nasty number.

Theorem 6. *$24(ptop)_{\beta} - 3(Stoct)_{\beta} - 2P(13, \beta) - 9(Gno)_{\beta} - 9$ is a bi-quadratic number*

Proof of Theorem 6

$$24(Ptop)_{\beta} = (6\beta^3 - 3\beta) + \beta^4 + 11\beta^2 + 9\beta$$

$$= 3(Stoct)_{\beta} + 2P(13, \beta) + \beta^4 + 9 + (18\beta - 9)$$

$24(Ptop)_{\beta} - 3(Stoct)_{\beta} - 2P(13, \beta) - 9(Gno)_{\beta} - 9$ is a bi-quadratic number.

Theorem 7. $48(Ptop)_{\beta}(Tet)_{\beta}[6(Gno)_{\beta}+42]$

Proof of Theorem 7

$$48(Ptop)_{\beta} = (2\beta^4 + 5\beta^3 + \beta^2 - 2\beta) + 7\beta^3 + 21\beta^2 + 14\beta$$

$$6(Tet)_{\beta}(Gno)_{\beta} + 7(\beta^3 + 3\beta^2 + 2\beta)$$

$$= 6(Tet)_{\beta}(Gno)_{\beta} + 42(Tet)_{\beta}$$

Hence $48(Ptop)_{\beta} = (Tet)_{\beta}[6(Gno)_{\beta} + 42]$

Theorem 8. $24(ptop)_{\beta} - 6(Octhe)_{\beta} - 6\beta(CPY)_{\beta}^{3} - (Psm)_{3}^{\beta} - 4P(5, \beta) \equiv 0(\bmod 7)$

Proof of Theorem 8

From theorem 6, $24(Ptop)_{\beta} = (6\beta^3 + 3\beta) + \beta^4 + 11\beta^2 + 3\beta$

$$= 6(Octhe)_{\beta} + 6\beta(CPY)_{\beta}^{3} - (3\beta^3 - 3\beta^2 + 2n) + 6\beta^2 + 5\beta$$

$$= 6(Octhe)_{\beta} + 6\beta(CPY)_{\beta}^{3} + (Psm)_{3}^{\beta} + (6\beta^2 - 2\beta) + 7\beta$$

$$= 6(Octhe)_{\beta} + 6\beta(Octhe)_{\beta}^{3} + (Psm)_{3}^{\beta} + 4P(5, \beta) + 7\beta$$

Hence $24(ptop)_{\beta} - 6(Octhe)_{\beta} - 6\beta(CPY)_{\beta}^{3} - 5(Psm)_{\beta}^{3} - 4P(5, \beta) \equiv (\bmod 7)$

Theorem 9. $120(Ptop)_{\beta} - (Nex)_{\beta} - 5(Rhodo)_{\beta} - 2P(17, \beta) - 9(Gno)_{\beta} \equiv 0(\bmod 3)$.

Proof of Theorem 9

$$120(Ptop)_{\beta} - (Nex)_{\beta} = 20\beta^3 + 45\beta^2 + 25\beta - 1$$

$$= 5(Rhodo)_{\beta} + (15\beta^2 - 13\beta) + 18\beta - 6$$

$$= 5(Rhodo)_{\beta} + 2P(17, \beta) + 9(Gno)_{\beta} + 3$$

Hence $120(Ptop)_\beta - (Nex)_\beta - 5(Rhodo)_\beta - 2P(17, \beta) - 9(Gno)_\beta \equiv 0(\mathrm{mod}\, 3)$

Theorem 10. $[(\Pr o)_{\beta^2} - 24(Ptop)_\beta + 3(Octhe)_\beta - 3(Stoct) + 2C(10, \beta) + 6(Gno)_\beta + \beta]$ *is a cubic integer.*

Proof of Theorem 10

From theorem 8, $24(Ptop)_\beta = (\Pr o)_{\beta^2} + 3(Octhe)_\beta - 6\beta^3 + 10\beta^2$

$$= (\Pr o)_{\beta^2} + 3(Octhe)_\beta - 3(Stoct)_\beta + (10\beta^2 + 10\beta + 2) - 13\beta - 2$$

$$= (\Pr o)_{\beta^2} + 3(Octhe)_\beta - 3(Stoct)_\beta + 2C(10, \beta) - 13\beta - 2$$

$$= (\Pr o)_{\beta^2} + 3(Octhe)_\beta - 3(Stoct)_\beta + 2C(10, \beta) + 6(Gno)_\beta - (\beta + 8)$$

$[(\Pr o)_{\beta^2} - 24(Ptop)_\beta + 3(Octhe)_\beta - 3(Stoct) + 2C(10, \beta) + 6(Gno)_\beta + \beta]$ is a cubic integer.

Corollary 1. $2[(\Pr o)_{\beta^2} - 24(Ptop)_\beta + 3(Octhe)_\beta - 3(Stoct) + 2C(10, \beta) + 6(Gno)_\beta + \beta]$ *is a perfect square.*

Corollary 2. $3[(\Pr o)_{\beta^2} - 24(Ptop)_\beta + 3(Octhe)_\beta - 3(Stoct) + 2C(10, \beta) + 6(Gno)_\beta + \beta]$ *is a Nasty number.*

Corollary 3. $[(\Pr o)_{\beta^2} - 24(Ptop)_\beta + 3(Octhe)_\beta - 3(Stoct) + 2C(10, \beta) + 6(Gno)_\beta + \beta] \equiv 0(\mathrm{mod}\, 8)$

Corollary 4. $[24(Ptop)_\beta - (\Pr o)_{\beta^2} + 24(Ptop)_\beta - 3(Octhe)_\beta + 3(Stoct) - 2C(10, \beta) - 6(Gno)_\beta - \beta]$ *is a negative cubic integer.*

Theorem 11. $120(Ptop)_\beta - (Nex)_\beta = 15(Coc)_\beta + 5(Hex)_\beta - 15(Gno)_\beta - 36$

Proof of Theorem 11

From theorem 9, $120(Ptop)_\beta - (Nex)_\beta = (20\beta^3 + 30\beta^2 + 40\beta + 15) + 15\beta^2 - 15\beta - 16$

$$= 15(Coc)_\beta + 5(Hex)_\beta - 30\beta - 21$$

$= 15(Coc)_\beta + 5(Hex)_\beta - 15(Gno)_\beta - 36$

Hence $120(Ptop)_\beta - (Nex)_\beta = 15(Coc)_\beta + 5(Hex)_\beta - 15(Gno)_\beta - 36$

Corollary 1. $(Nex)_\beta - 120(Ptop)_\beta + 15(Coc)_\beta + 5(Hex)_\beta - 15(Gno)_\beta$ *is a square integer.*

Corollary 2. $6[(Nex)_\beta - 120(Ptop)_\beta + 15(Coc)_\beta + 5(hex)_\beta - 15(Gno)_\beta]$ *is a Nasty number.*

Theorem 12. $120(Ptop)_\beta - (Nex)_\beta - 15(Hoct)_\beta - (75gram)_\beta - 60\beta \equiv 0(\text{mod}13)$

Proof of Theorem 12

From theorem 9, $120(Ptop)_\beta - (Nex)_\beta = (20\beta^3 - 30\beta^2 + 40\beta - 15) + 75\beta^2 - 15\beta + 14$

$= 15(Hoct)_\beta + (75\beta^2 - 75\beta + 1) + 60\beta + 13$

Hence $120(Ptop)_\beta - (Nex)_\beta - 15(Hoct)_\beta - (75gram)_\beta - 60\beta \equiv 0(\text{mod}13)$

Theorem 13. $8\beta(Ptop)_\beta - \beta(Tet)_\beta(Gno) = \beta(CPY)_7^\beta + \beta C(7, \beta) - (\text{Pr}o)_\beta$

Proof of Theorem 13

From theorem 7, $48\beta(Ptop)_\beta - 6(Tet)_\beta(Gno)_\beta = 7\beta^3 + 21\beta^2 + 14\beta$

$= 6(CPY)_7^\beta + (21\beta^2 + 21\beta + 6) - (6\beta + 6)$

$= 6(CPY)_7^\beta + 6C(7, \beta) - 6(\beta + 1)$

$8\beta(Ptop)_\beta - (Tet)_\beta(Gno)_\beta = \beta(CPY)_7^\beta + C(7, \beta) - (\beta + 1)$

Hence $8\beta(Ptop)_\beta - \beta(Tet)_\beta(Gno)_\beta = \beta(CPY)_7^\beta + \beta C(7, \beta) - (\text{Pr}o)_\beta$

Theorem 14. $24(Ptop)_\beta - (\text{Pr}o)_{\beta^2} - 3(Ccu)_\beta + 3(\beta + 1)$ *is a perfect square.*

Proof of Theorem 14. From theorem 10, $24(Ptop)_\beta = (\Pr o)_{\beta^2} + (6\beta^3 + 10\beta^2 + 6\beta)$.

$$24(Ptop)_\beta = (\Pr o)_{\beta^2} + 3(Ccu)_\beta + \beta^2 - 3(\beta + 1)$$

Therefore $24(Ptop)_\beta = (\Pr o)_{\beta^2} - 3(Ccu)_\beta + 3(\beta + 1)$ is a perfect square.

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