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APPLICATION OF NANO TOPOLOGICAL PROPERTIES IN FILTERING THE IMAGE

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Abstract

This main goal of this paper is to investigate how the conditions of nano topology is applied in image processing to remove noise, filtered, and edges are adjusted by smoothing to get the blurred image as a clear image.

1. Introduction

Lellis Thivagar and Richard established the notion of nano topology in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also make known about nano-closed sets, nano-interior, nanoclosure and weak form of nano open sets namely nano semi-open sets, nano preopen, nano α-open sets and nano β-open sets. Nasef et al. make known about some of nearly open sets in nano topological spaces. Revathy and Gnanambal Illango gave the idea about the nano β-open sets. Sathishmohan et al. bring up the idea about nano neighourhoods in nano topological spaces.

In this paper the conditions of nano topology is used in image processing for converting the blurred image into the clear image by removing noice, smoothing and edge adjustments.

The basic definitions for nano topology and filter (Laplace filtering) are recollected from the following papers [1, 2, 3, 4, 6, 7, 8, 9, 10, 11]. Throughout

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this paper neighbourhoods, closure, interior, boundary, limit point, isolated point are represents the nano neighbourhoods, nano closure, nano interior, nano boundary, nano limit point, nano isolated point respectively.

2. Preliminaries

Definition 2.1. Let $U 6 = \emptyset$ be a universe of discourse and X be a subset of U. An equivalence relation R, classifies U into a set of subsets $U/R = \{X_1, X_2, \ldots, X_n\}$ in which the following conditions are satisfied

(1) $X_i \subseteq U$; $X_i 6 = \emptyset$ for any i

(2) $X_i \cap X_i = \emptyset$ for any i, j

(3) $i = 1, 2, \ldots, n, X_i = U$.

Any subset X_i called a category, represents an equivalence class of R. A category in R containing an object $x \in U$ is denoted by $[x]_R$. An indiscernibility relation $I(R)$ is defined as follows

$$xI(R)_y = \{(x, y) \in U^2 \mid (x, y) \in P, P \in U/R\}.$$

For a family of equivalence relations $P \subset R$, $I(P)$ is defined as follows

$$I(P) = \bigcap_{R \in P} I(R).$$

Definition 2.2 [5]**.** Let U be the universe, R be an equivalence relation on U and $\{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms

(i) U and $\varphi \in \tau_R(X)$.

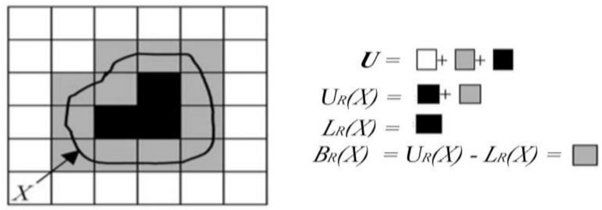
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as nano topological space. The elements of $\tau_R(X)$

are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Remark 2.3 [5]. If $\tau_R(X)$ is the nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.



3. Basic Relationship between Pixels

Neighbors of a Pixel 1. $N_4(P)$: 4-neighbors of P.

Any pixel $p(x, y)$ has two vertical and two horizontal neighbors, given by $(x+1, y), (x-1, y), (x, y-1)$

• This set of pixels are called the 4-neighbors of P, and is denoted by $N_4(P)$

• Each of them is at a unit distance from P.

• $ND(P)$.

• This set of pixels, called 4-neighbors and denoted by $ND(P)$.

• $ND(P)$: four diagonal neighbors of P have coordinates $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$.

• $N_8(P)$: 8-neighbors of P.

• $N_4(P)$ and $ND(P)$ together are called 8-neighbors of P, denoted by $N_8(P)$.

• $N_8 = N_4 \cup ND$ Some of the points in the N_4, ND and N_8 may fall outside image when P lies on the border of image. Neighbourhood operations simply operate on a larger neighbourhood of pixels than point operations

F (x-1, y-1)	F (x-1, y)	F (x-1, y+1)
F (x, y-1)	F (x, y)	F (x, y+1)
F (x+1, y-1)	F (x+1, y)	F (x+1, y+1)

N_8 (p)

Figure 1.

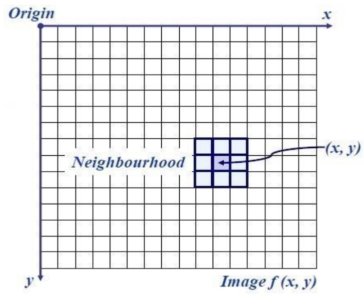


Figure 2

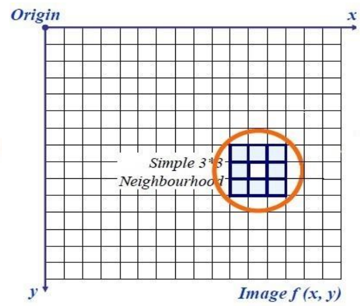


Figure 3

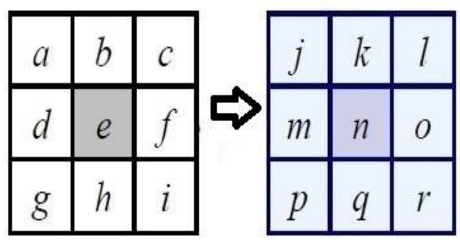


Figure 4.

$$e_{\text{processed}} = n \Rightarrow e + j \Rightarrow a + k \Rightarrow b + 1 \Rightarrow c + m$$

$$\Rightarrow d + 0 \Rightarrow f + p \Rightarrow g + q \Rightarrow h + r \Rightarrow i.$$

The above is repeated for every pixel in the original image to generate the filtered image. Since U is a universal set, it can represent various space such as a 2 or 3 dimensional rectangular lattice or triangular lattice space. neighborhood $N(x)$ can be considered as a set of points which has an adjacency relation with point x. Figure 5 shows various shapes of neighborhoods. The 5-neighborhood shown by Figure 5(1), and 9 neighborhood are shown in 5(6). As these examples, it is often assumed that the space is a set of $2D$ rectangles, and that the neighborhood is symmetric, $x \in N(x)$. Since the form of the space and of the neighborhoods in the nano topology has no restriction, it is possible to consider a neighborhood of a point n which does not include x itself as shown by Figure 5(4).

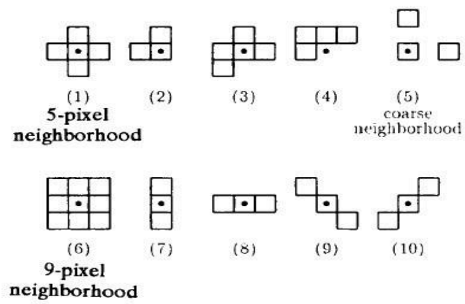


Figure 5.

Example 3.1. Let $U = \{a, b, c, d\}$, $U/R = \{\{a, c\}, \{b\}, \{d\}\}$, $X\{b, c\}$ and $\tau_R(X) = \{U, \emptyset, \{b\}, \{a, b, c\}, \{a, c\}\}$. Then $N(a) = \{U, \{a, c\}, \{a, b, c\}\}$, $N(b) = \{U, \{b\}, \{a, b, c\}\}$, $N(c) = \{U, \{a, c\}, \{a, b, c\}\}$, $N(d) = \{U\}$.

Definition 3.2. A nano topological space (U, K) is called a nano filled space $\forall x \in U : x \in N(x)$.

Note: the adjacency relation is not limited to adjacent pixels. In some cases, it is more appropriate to consider a coarse neighborhood, such as that shown in Figure 5(5) in which some pixels are outside the 9-neighborhood.

Definition 3.3. A nano topological space (U, K) is said to be nano symmetric if $\forall x, y \in U : y \in N(x) \Rightarrow x \in N(y)$. In Figure 5 (1, 6, 7, 8, 9, 10) are symmetric nano neighborhoods, and (2, 3, 4, 5) are asymmetric nano neighborhoods.

4. Relation between Nano Topology and Image Processing

The boundary of an image is a typical element in an image processing. It consists of a set of points which have an adjacency relation with both the inside and outside of the image.

Definition 4.1. The boundary set A^{∂} of subset A of U is defined by

$$A^{\partial} = \{x : N(x) \cap A6 = \emptyset \text{ and } N(x) \cap A^{c}6 = \emptyset\}.$$

A boundary set depends on a nano topology U. The boundary set with respect to (w.r.t) the 5-neighborhood of an image shown in Figure 6(2) is given as Figure 6(2). This figure looks like usual boundary of the image. When the vertical 3-pixel neighborhood (Figure 5(7)) is applied to an image (Figure 6(1)), a horizontal contour (Figure 7(1)) is obtained. Because a boundary set A^{∂} varies depending on the nano topology U, the notation $A^{\partial hN_i \setminus t_i}$ is sometimes used.

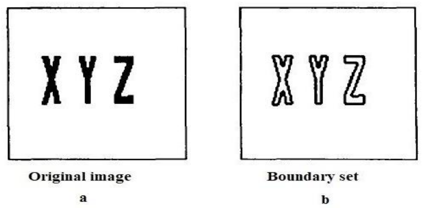


Figure 6.

Definition 4.2. The nano closure set of A is represented by A^{Ncl}, and given by

$$A^{Ncl} = \{x : N(x) \cap A6 = \emptyset\}.$$

Example 4.3. From Example 3.1, $Ncl(\{b\}) = \{b, d\}$, $Ncl(\{a, b, c\}) = \{U\}$, $Ncl(\{a, c\}) = \{a, cv, d\}$.

In the Figure 7, shows the closure for the 5-nano neighborhood shown in and this is an image expanded by the thickness of one pixel.

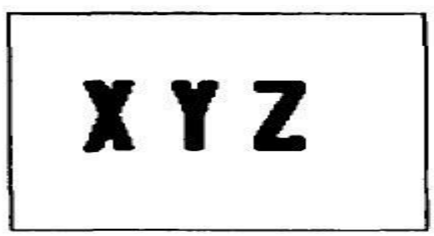


Figure 7.

The process of removing all boundary points from a picture is called shrinking. This can be represented by the set of all points whose nano neighborhoods are contained in the picture. Such a set is called an interior set in the nano topology.

Definition 4.4. The nano interior set of A is defined by

$$A^{N\,\mathrm{int}} = \{x : N(x) \in A\}.$$

Example 4.5. $N\mathrm{int}(\{b\}) = \{b\}, N\mathrm{int}(\{a, b, c\}) = \{a, b, c\}, N\mathrm{int}(\{a, c\}) = \{a, c\}$.

Figure 8 shows the nano interior set of the picture shown in Fig. 5 obtained by using the 5-nano neighborhood. This is thinner than the original image by the thickness of one pixel. The closure set and the interior set vary depending on the neighborhood as well as the boundary set. The former is represented by $A^{NclhN\mathrm{int}i}$, and the latter by $A^{N\,\mathrm{int}\,h\,N\,\mathrm{int}\,i}$, indicating they are based on a neighborhood N.

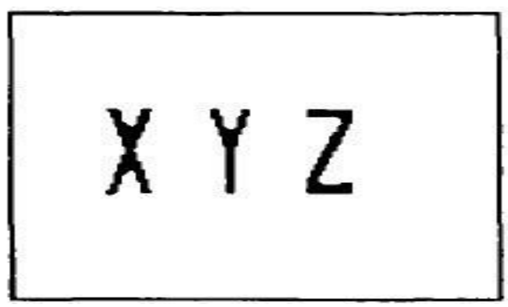


Figure 8.

Remark 4.6. The closure set and the interior set have the following relationship:

(1) $((Ae)\,\mathrm{Nint}\ t)c = ANcl$.

(2) $((Ac)\,Ncl)c = A\,\mathrm{Nint}$.

If the neighborhood of a pixel of a picture A contain no other pixel of A, this pixel is considered to be isolated in A, defined as follows.

Definition 4.7. The nano isolated point set of A, expressed by A^{NS}, is defined by $A^{NS} = \{x : x \in A \text{ and } (N(x) \setminus \{x\}) \cap A = \emptyset\}$.

Nano isolated points of a picture can be considered as noise, and this noise can be removed by eliminating the isolated points.

Definition 4.8. The nano limit point set A^l of A is defined by $A^l = A \setminus A^s$.

5. Smoothing of an Image

Filtering is a very important research field of digital image processing. An filtering algorithms involve so called neighborhood processing because they are based on the relationship between neighbouring pixels rather than a single pixel in point operations. Digital filtering is useful for enhancing lineaments that may represent significant geological structures such as faults, veins or dykes. It can also enhance image texture for discrimination of lithologies and drainage patterns. For land use studies, filtering may highlight the textures of urbanisation, road systems and agricultural areas and, for general visualisation, filtering is widely used to sharpen images. However, care should be taken because filtering is not so honest in retaining the information of the original image.

Most images are affected to some extent by noise, that is unexplained variation in data: disturbances in image intensity which are either uninterpretable or not of interest. Image analysis is often simplified if this noise can be filtered out. Filters change a pixel's value taking into account the values of neighbouring pixels too. They may either be applied directly to recorded images.

Each pixel has been replaced by the average of pixel values in a 5×5 square, or window centred on that pixel. The result is to reduce noise in the image, but also to blur the edges of the fibres. A similar effect can be produced by looking at Fig 9(a) through half-closed eyes.

If the output from the moving average filter is subtracted from the original image, on a pixel-by-pixel basis, then the result is as shown in Figure 9(c) (which has been displayed with the largest negative pixel values shown as black and the largest positive pixel values shown as white). This filter (the original image minus its smoothed version) is a Laplacian filter. It has had the effect of emphasising edges in the image.

Figure 9(d) shows the result produced when output from the Laplacian filter is added to the original image, again on a pixel-by-pixel basis. To the eye, this image looks clearer than Figure 9(a) because transitions at edges have been magnified an effect known as unsharp masking.

Image Smoothing Examples.

For point operations, we generally regard an image as a raster data stream and denote $x_{ij} \in U$ as a pixel at line i and column j in image U. For filtering and nano neighborhood processing, however, the pixel coordinates are very relevant and, in this sense, we regard an image as a two dimensional ($2D$) function. We therefore follow the convention of denoting an image with a pixel at image column x and line y as a $2D$ function $f(x, y)$ when introducing essential mathematical concepts of filtering. On the other hand, for the simplicity, the expression of $x_{ij} \in U$ is still used in describing some filters and algorithms. The image at the top left is an original image of size $500 * 500$ pixels. The subsequent images show the image after filtering with an averaging filter of increasing sizes.

Figure 9.

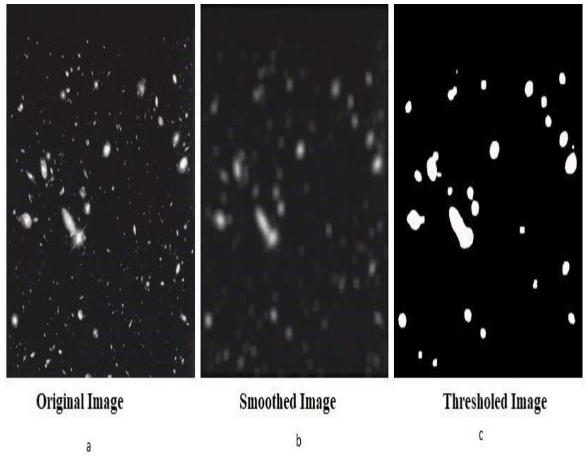


Figure 10

6. Conclusion

In this paper we have used the concept of nano topology and nano neighbourhoods and investigated how the nano neighbourhoods are used in images to change the blurred images into the clear images by filter, noice, edge adjustments.

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