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ON $\Lambda_{\alpha g}$-SETS AND $\Lambda^{*}_{\alpha g}$-SETS IN TOPOLOGICAL SPACES

S. SUBHALAKSHMI and N. BALAMANI

Research Scholar, Department of Mathematics
Avinashilingam Institute for Home Science
and Higher Education for Women
Coimbatore -641043, India
E-mail: subhamanu2013@gmail.com

Department of Mathematics
Avinashilingam Institute for Home Science
and Higher Education for Women
Coimbatore-641043, India
E-mail: nbalamani77@gmail.com

Abstract

In this paper, a new class of sets called $\Lambda_{\alpha g}$-sets and $\Lambda^{*}_{\alpha g}$-sets are introduced and studied with corresponding examples by utilizing the notions of αg-open sets and αg-closed sets. Also, several fundamental properties and theorems of such sets are investigated. Further $(\Lambda, \alpha g)$-closed sets and $(\Lambda, \alpha g)$-open sets are established and their interesting properties and some characterizations are derived.

I. Introduction

Levine [2] introduced the notion of generalized closed sets in topological spaces. Following this, many researchers introduced several variations of generalized closed sets and investigated some stronger and weaker forms of them. The notion of α-sets in topological spaces was introduced by Njastad [7] and studied several fundamental properties. The complement of $\alpha-$*sets*

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called α-closed sets were defined and studied by Mashhour et al. [6]. The generalized α-closed sets and α-generalized closed sets were proposed and studied by Maki et al. [4, 5].

Maki [3] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel, i.e., to the intersection of all open supersets of A. Caldas et al. [1] introduced the concept of Λ_{α}-sets and Λ_{α}^{*}-sets, which is the set equal to intersection of all α-open subsets and the set equal to the union of all α-closed supersets respectively and studied their fundamental properties. In this paper, we introduce two new notions namely $\Lambda_{\alpha g}$-sets and $\Lambda_{\alpha g}^{*}$-sets using the concept of αg-open set and αg-closed set due to Maki et al. [4, 5]. Some fascinating properties and characterization theorems are established. Also, the concept of $(\Lambda, \alpha g)$-closed sets and $(\Lambda, \alpha g)$-open sets are introduced and their essential properties are discussed with corresponding examples.

II. Preliminaries

Definition 2.1 [6, 7]**.** Let (X, τ) be a topological space. A subset A of (X, τ) is called α-open if $\mathrm{A} \subseteq \operatorname{int}(cl(\operatorname{int}(A)))$. The complement of an α-open set is called an α-closed set if $cl(\operatorname{int}(cl(A))) \subseteq A$.

Definition 2.2 [6]**.** The α-closure of a subset A of a topological space (X, τ) is the intersection of all α-closed sets containing A and is denoted by $cl_{\alpha}(A)$. The α-interior of a subset A of a topological space (X, τ) is the union of all α-open sets contained in A and is denoted by $\operatorname{int}_{\alpha}(A)$.

Definition 2.3 [5]**.** A subset A of a topological space (X, τ) is called an $\alpha-$generalized closed set $(\alpha g-$closed$)$ if $cl_{\alpha}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The family of all $ag-$closed subsets of the topological space (X, τ) is denoted by $\alpha GC(X, \tau)$. The complement of an $ag-$closed set is an $ag-$open set. The family of all $ag-$open subsets of a topological space (X, τ) is denoted by $\alpha GO(X, \tau)$.

III. $\Lambda_{\alpha g}$-sets and $\Lambda^*_{\alpha g}$-sets

Definition 3.1. Let A be a subset of a topological space (X, τ). A subset $\Lambda_{\alpha g}(A)$ is defined as $\Lambda_{\alpha g}(A) = \bigcap\{M \mid A \subseteq M$ and $M \in \alpha GO(X, \tau)\}$.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$. Then $\alpha GO(X, \tau) = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\Lambda_{\alpha g}(A) = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$.

Lemma 3.3. *For subsets* A, B *and* $A_i (i \in I)$ *of a topological space* (X, τ) *the following properties hold*:

(1) $A \subseteq \Lambda_{\alpha g}(A)$.

(2) If $A \subseteq B$, *then* $\Lambda_{\alpha g}(A) \subseteq \Lambda_{\alpha g}(B)$.

(3) $\Lambda_{\alpha g}(\Lambda_{\alpha g}(A)) = \Lambda_{\alpha g}(A)$.

(4) $\Lambda_{\alpha g}(\bigcap \{A_i / i \in I\}) \subseteq \bigcap\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

(5) $\Lambda_{\alpha g}(\bigcup \{A_i / i \in I\}) \supseteq \bigcup\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

(6) If $A \in \alpha GO(X, \tau)$, then $A = \Lambda_{\alpha g}(A)$.

Proof. (1). Let $x \notin \Lambda_{\alpha g}(A)$. Then $\exists$ an αg-open set $M \ni A \subseteq M$ and $x \notin M$. Hence $x \notin A$ and so $A \subseteq \Lambda_{\alpha g}(A)$.

(2) Let $A \subseteq B$ and let $x \notin \Lambda_{\alpha g}(B)$. Then $\exists$ an αg–open set $M \ni B \subseteq M$ and $x \notin M$. Now $A \subseteq B \Rightarrow A \subseteq B \subseteq M$ and $x \notin M \Rightarrow x \notin \Lambda_{\alpha g}(A)$. Hence $\Lambda_{\alpha g}(A) \subseteq \Lambda_{\alpha g}(B)$.

(3) Since $A \subseteq \Lambda_{\alpha g}(A)$, $\Lambda_{\alpha g}(A) \subseteq (\Lambda_{\alpha g}(A))$. Suppose $x \notin \Lambda_{\alpha g}(A)$ then $\exists$ an αg–open set $M \ni A \subseteq M$ and $x \notin M$. By Definition 3.1, $\Lambda_{\alpha g}(A) \subseteq M$ and $x \notin M$. Hence $x \notin (\Lambda_{\alpha g}(\Lambda_{\alpha g}(A))$. Therefore, $(\Lambda_{\alpha g}(\Lambda_{\alpha g}(A)) \subseteq \Lambda_{\alpha g}(A)$. Hence $\Lambda_{\alpha g}(\Lambda_{\alpha g}(A)) = \Lambda_{\alpha g}(A)$.

(4) Suppose that $x \notin \bigcap\Lambda_{\alpha g}(A_i / i \in I)\}$ then $\exists\, i_0 \in I \ni x \notin \Lambda_{\alpha g}(A_{i_0})$. Therefore, $\exists$ an $\alpha g - open$ set $M \ni Ai_0 \subseteq M$ and $x \notin M$. Now

$\bigcap A_i \subseteq A_{i_0} \subseteq M$ and $x \notin M$. Thus, $x \notin \Lambda_{\alpha g}(\bigcap\{A_i / i \in I\})$. Hence $\Lambda_{\alpha g}(\bigcap\{A_i / i \in I\}) \subseteq \bigcap\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

(5) From (1), $A_i \subseteq \Lambda_{\alpha g}(A_i)$. From (2), $A_i \subseteq \bigcup_{i \in I} A_i \Rightarrow \Lambda_{\alpha g}(A_i) \subseteq \Lambda_{\alpha g}(\bigcup_{i \in I} A_i)$. Hence $\Lambda_{\alpha g}(\bigcup\{A_i / i \in I\}) \supseteq \bigcup\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

(6) By Definition 3.1 and by (1), $A = \Lambda_{\alpha g}(A)$.

Remark 3.4. In Lemma 3.2, the reverse inclusion of (4) is not true as seen from the following example.

Example 3.5. Consider X and τ as in Example 3.2. Let A_1, A_2 and A_3 respectively be $\{a, b\}, \{b, c\}$ and X. Then $\Lambda_{\alpha g}(\bigcap A_i) = \{b\}$ and $\bigcap \Lambda_{\alpha g}(A_i) = \{a, b\}$. Hence $\Lambda_{\alpha g}(\bigcap\{A_i / i \in I\}) \not\supseteq \bigcap\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

Remark 3.6. In Lemma 3.2, the reverse inclusion of (5) is not true as seen from the following example.

Example 3.7. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, X\}$. Then $\Lambda_{\alpha g}(A) = \{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. Let A_1 and A_2 respectively be $\{b\}$ and $\{c\}$. Then $\Lambda_{\alpha g}(\bigcup A_i) = X$ and $\bigcup \Lambda_{\alpha g}(A_i) = \{b, c\}$. Hence $\Lambda_{\alpha g}(\bigcup\{A_i / i \in I\}) \not\subseteq \bigcup\{\Lambda_{\alpha g}(A_i / i \in I)\}$.

Definition 3.8. Let A be a subset of a topological space (X, τ). A subset $\Lambda^*_{\alpha g}(A)$ is defined as $\Lambda^*_{\alpha g}(A) = \bigcup\{N / N \subseteq A \text{ and } N \in \alpha GC(X, \tau)\}$.

Example 3.9. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a, b\}, X\}$. Then $\alpha GC(X, \tau) = \{\varphi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $\Lambda^*_{\alpha g}(A) = \{\varphi, \{c\}, \{b, c\}, \{a, c\}, X\}$.

Lemma 3.10. *For subsets* A, B *and* $A_i (i \in I)$ *of a topological space* (X, τ) *the following properties hold*:

(1) $A \subseteq \Lambda^*_{\alpha g}(A)$.

(2) *If* $A \subseteq B$, *then* $\Lambda^*_{\alpha g}(A) \subseteq \Lambda^*_{\alpha g}(B)$.

(3) $\Lambda^*_{\alpha g}(\Lambda^*_{\alpha g}(A)) = \Lambda^*_{\alpha g}(A)$.

(4) $\Lambda_{\alpha g}^{*}(\{A_i / i \in I\}) \subseteq \bigcap \{\Lambda_{\alpha g}^{*}(A_i / i \in I)\}$.

(5) $\Lambda_{\alpha g}^{*}(\bigcup \{A_i / i \in I\}) \supseteq \bigcup \{\Lambda_{\alpha g}^{*}(A_i / i \in I)\}$.

(6) *If* $A \in \alpha GC(X, \tau)$, *then* $A = \Lambda_{\alpha g}^{*}(A)$.

(7) $\Lambda_{\alpha g}(A^c) = (\Lambda_{\alpha g}^{*}(A))^c$

Proof. (1) to (6) are obvious.

(7) $(\Lambda_{\alpha g}^{*}(A))^c = (\bigcup \{N / N \subseteq A$ and $N \in \alpha GC(X, \tau)\})^c = \bigcap \{N^c / A^c \subseteq N^c$ and $N^c \in \alpha GO(X, \tau)\}$. Let $N^c = M$. Then $M \in \alpha GO(X, \tau)$. Therefore, $(\Lambda_{\alpha g}^{*}(A))^c = \bigcap \{M / A^c \subseteq M$ and $M \in \alpha GO(X, \tau)\} = \Lambda_{\alpha g}^{*}(A^c)$.

Remark 3.11. In Lemma 3.10 the reverse inclusion of (4) and (5) are not true as seen from the following example.

Example 3.12. Let $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a, b\}, X\}$. Then $\alpha GC(X, \tau) = \{\varphi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \{b, c, d\}, X\} = \Lambda_{\alpha g}^{*}(A)$.

Let A_1, A_2 and A_3 respectively be $\{a, d\}, \{a, b, c\}$ and X. Then $\Lambda_{\alpha g}^{*}(\bigcap \{A_i / i \in I\}) = \varphi$ and $\bigcap \{\Lambda_{\alpha g}^{*}(A_i / i \in I)\} = \{a\}$. Hence $\Lambda_{\alpha g}^{*}(\bigcap \{A_i / i \in I\})$

Let A_1, A_2, A_3 and A_4 respectively be $\{a\}, \{b\}, \{d\}$ and $\{a, b\}$. Then $\Lambda_{\alpha g}^{*}(\bigcup \{A_i / i \in I\}) = \{a, b, d\}$ and $\bigcup \{\Lambda_{\alpha g}^{*}(A_i / i \in I)\}) = \{d\}$. Hence $\Lambda_{\alpha g}^{*}(\bigcup \{A_i / i \in I\}) \not\subseteq \bigcup \{\Lambda_{\alpha g}^{*}(A_i / i \in I)\}$.

Definition 3.13. A subset A of a topological space (X, τ) is called a $\Lambda_{\alpha g}$-set if $A = \Lambda_{\alpha g}(A)$. The set of all $\Lambda_{\alpha g}$-sets is denoted by $\Lambda_{\alpha g}(\mathrm{X}, \tau)$.

Example 3.14. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then $\alpha GO(X, \tau) = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Therefore, $\Lambda_{\alpha g}$-sets are $\varphi, \{a\}, \{a, b\}, \{a, c\}, X$.

Lemma 3.15. *For a topological space* (X, τ) *the following properties hold*:

(1) *X and* φ *are* $\Lambda_{\alpha g}$*-sets.*

(2) *If A is an* αg*–open set, then A is a* $\Lambda_{\alpha g}$*-set.*

(3) $\Lambda_{\alpha g}(\mathrm{A})$ *is a* $\Lambda_{\alpha g}$*-set.*

(4) *Every intersection of* $\Lambda_{\alpha g}$*-sets is a* $\Lambda_{\alpha g}$*-set.*

Proof. (1) is obvious.

(2) Let $A \in \alpha GO(X, \tau)$. Then by Lemma 3.3 (6), $A = \Lambda_{\alpha g}(A)$. Therefore, A is a $\Lambda_{\alpha g}$-set.

(3) $\Lambda_{\alpha g}(\mathrm{A})$ is the smallest αg-open set containing A. Since $\Lambda_{\alpha g}(\mathrm{A})$ is αg-open and from (2), $\Lambda_{\alpha g}(\mathrm{A})$ is a $\Lambda_{\alpha g}$-set.

(4) Let $A_i (i \in I)$ be a family of $\Lambda_{\alpha g}$-sets in (X, τ). Then $A_i = \Lambda_{\alpha g}(A_i)$, for all $i \in I$. Let $A = \bigcap A_i$. Then by Lemma 3.3 (4), $\Lambda_{\alpha g}(\mathrm{A}) = \Lambda_{\alpha g}(\bigcap A_i) \subseteq \bigcap(\Lambda_{\alpha g}(A_i)) = \bigcap A_i = A$. Therefore, $\Lambda_{\alpha g}(A) \subseteq A$. Also from Lemma 3.3 (1), $A \subseteq \Lambda_{\alpha g}(A)$. Therefore, $A = \Lambda_{\alpha g}(A)$. Hence every intersection of $\Lambda_{\alpha g}$-sets is a $\Lambda_{\alpha g}$-set.

Remark 3.16. In general finite union of $\Lambda_{\alpha g}$-sets need not be a $\Lambda_{\alpha g}$-set as seen from the following example.

Example 3.17. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, X\}$. Then $\Lambda_{\alpha g}$ - sets are $\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X$. Here $\{b\}$ and $\{c\}$ are $\Lambda_{\alpha g}$ - sets but their union $\{b\ c\}$ is not a $\Lambda_{\alpha g}$ - set.

Definition 3.18. A subset A of a topological space (X, τ) is called a $\Lambda^*_{\alpha g}$-set if $A = \Lambda^*_{\alpha g}(A)$. The set of all $\Lambda^*_{\alpha g}$-sets is denoted by $\Lambda^*_{\alpha g}(X, \tau)$.

Example 3.19. Consider (X, τ) as in Example 3.14. Then $\Lambda^*_{\alpha g}$-sets are $\varphi, \{b\}, \{c\}, \{b, c\}, X$.

Theorem 3.20. *For a topological space* (X, τ) *the following properties hold*:

(1) *X and* φ *are* $\Lambda^*_{\alpha g}$*-sets.*

(2) *If A is an* αg*–closed set, then A is a* $\Lambda^*_{\alpha g}$-set.

(3) $\Lambda^*_{\alpha g}(\mathrm{A})$ *is a* $\Lambda^*_{\alpha g}$*-set.*

(4) *Every union of* $\Lambda^*_{\alpha g}$*-sets is a* $\Lambda^*_{\alpha g}$-set.

(5) *A subset A is a* $\Lambda^*_{\alpha g}$*-set if* A^c *is a* $\Lambda_{\alpha g}$*-set.*

Proof. Proofs of (1) to (4) are similar to Lemma 3.15.

(5) Let A^c be a $\Lambda^*_{\alpha g}$-set. Then $\Lambda^*_{\alpha g}(A^c) = A^c$. Therefore, $(\Lambda^*_{\alpha g}(A^c))^c = (A^c)^c \Rightarrow (\Lambda^*_{\alpha g}(A^c))^c = A$. By Lemma 3.10 (7), $(\Lambda_{\alpha g}(A^c)^c) = A \Rightarrow \Lambda_{\alpha g}(A) = A$. Hence A is a $\Lambda^*_{\alpha g}$-set.

Remark 3.21. In general finite intersection of $\Lambda^*_{\alpha g}$-sets need not be a $\Lambda^*_{\alpha g}$-set as seen from the following example.

Example 3.22. Consider (X, τ) as in Example 3.17. The $\Lambda^*_{\alpha g}$ - sets are $\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X$. Now $\{a, b\}$ and $\{a, c\}$ are $\Lambda^*_{\alpha g}$ - sets but their intersection $\{a\}$ is not a $\Lambda^*_{\alpha g}$ - set.

Proposition 3.23. *For a topological space* (X, τ)

(1) *Every* Λ_{α}*-set is a* $\Lambda_{\alpha g}$*-set.*

(2) *Every* Λ^*_{α}*-set is a* $\Lambda^*_{\alpha g}$*-set.*

Proof. (1) $\Lambda_{\alpha}(A) = \bigcap\{M/M \in \tau^{\alpha}$ and $A \subseteq M\} = \bigcap C$ and $\Lambda_{\alpha g}(A) = \bigcap\{M/M \in \alpha GO(X, \tau)$ and $A \subseteq M\} = \bigcap D$. Further, $C \subseteq D \Rightarrow \bigcap D \subseteq \bigcap C \Rightarrow \Lambda_{\alpha g}(A) \subseteq \Lambda_{\alpha}(A) = A$. By Lemma 3.3 (1), $A \subseteq \Lambda_{\alpha g}(A)$. Hence A is a $\Lambda_{\alpha g}$-set.

(2) is similar to (1).

Definition 3.24. Let (X, τ) be a topological space. Then the αg-closure of A is denoted by $\alpha GCl(A)$ is defined by $\alpha GCl(A) = \bigcap\{E/E \in \alpha GC(X, \tau)$ and $A \subseteq E\}$.

Lemma 3.25. *Let* (X, τ) *be a topological space and let* $x, y \in X$. *Then* $y \in \Lambda_{\alpha g}(\{x\})$ *iff* $x \in \alpha GCl(\{y\})$.

Proof (Necessity). Let $y \in \Lambda_{\alpha g}(\{x\})$. Then $y \in M$, whenever $x \in M$, where M is αg-open. Suppose $x \notin \alpha GCl(\{y\})$ then $\exists$ an αg-closed set $N \ni \{y\} \subseteq M$ and $x \notin N$. This implies $x \in X - N$ and $y \notin X - N$, where $X - N \in \alpha GO(X, \tau)$. Take $X - N = M$ then $M \in \alpha GO(X, \tau) \ni x \in M$ and $y \notin M$. This is a contradiction and hence $x \in \alpha GCl(\{y\})$.

(Sufficiency) Let $x \in \alpha GCl(\{y\})$. Then $x \in N$, whenever $y \in N$, where N is αg-closed. Suppose $y \notin \Lambda_{\alpha g}(\{x\})$ then $\exists$ an αg-open set $M \ni \{x\} \subseteq M$ and $y \notin M$ so that $y \in X - M$ and $x \notin X - M$. Take $X - M = N$ then $N \in \alpha GC(X, \tau) \ni y \in N$ and $x \notin N$. This is a contradiction and hence $y \in \Lambda_{\alpha g}(\{x\})$.

Theorem 3.26. *For any two points x and y in a topological space* (X, τ) *the following statements are equivalent*:

(1) $\Lambda_{\alpha g}(\{x\}) \neq \Lambda_{\alpha g}(\{y\})$.

(2) $\alpha GCl(\{x\}) \neq \alpha GCl(\{y\})$.

Proof. $(1) \Rightarrow (2)$ Let $\Lambda_{\alpha g}(\{x\}) \neq \Lambda_{\alpha g}(\{y\})$. Then $\exists z \in X \ni z \in \Lambda_{\alpha g}(\{x\})$ but $z \notin \Lambda_{\alpha g}(\{y\})$. By Lemma 3.25, $x \in \alpha GCl(\{z\})$ and $y \notin \alpha GCl(\{z\})$ $\Rightarrow \alpha GCl(\{x\}) \subseteq \alpha GCl(\{z\})$ and $\{y\} \bigcap \alpha GCl(\{z\}) = \varphi$. Thus $y \notin \alpha GCl(\{x\})$. Since $\{y\} \subseteq \alpha GCl(\{y\})$, $\alpha GCl(\{x\}) \neq \alpha GCl(\{y\})$.

$(2) \Rightarrow (1)$ Let $\alpha GCl(\{x\}) \neq \alpha GCl(\{y\})$. Then $\exists z \in X \ni z \in \alpha GCl(\{x\})$ and $z \notin \alpha GCl(\{y\})$. By Lemma 3.25, $x \in \Lambda_{\alpha g}(\{z\})$ but $y \notin \Lambda_{\alpha g}(\{z\})$. Since $x \in \Lambda_{\alpha g}(\{z\})$ and $\Lambda_{\alpha g}(\{z\}) = \bigcap\{M/M \in \alpha GO(X, \tau)$ and $z \in M\}$ we get $x \in M$ whenever $z \in M$, where M is $\alpha g - open \rightarrow$ (i). Since, $y \notin \Lambda_{\alpha g}(\{z\})$, $\exists$

an $\alpha g-open$ set $M \ni z \in M$ but $y \notin M$. Let it be $M_x \rightarrow$(ii). By (i) and (ii), $x \in M_x$ and $y \notin M_x$ and therefore $y \notin \Lambda_{\alpha g}(\{x\})$. Hence $\Lambda_{\alpha g}(\{x\}) \neq \Lambda_{\alpha g}(\{y\})$.

Proposition 3.27. *Let* (X, τ) *be a topological space and* $A \in \alpha GO(X, \tau)$. *Then* $\Lambda_{\alpha g}(A) = \{x \in X \mid \alpha GCl(\{x\}) \cap A \neq \varphi\}$.

Proof. Let $A \in \alpha GO(X, \tau)$ and let $x \in \Lambda_{\alpha g}(A)$. Since $A \in \alpha GO(X, \tau)$ by Lemma 3.15 (2), $A = \Lambda_{\alpha g}(A)$. Also $x \in \alpha GCl(\{x\}) \Rightarrow \alpha GCl(\{x\}) \cap A \neq \varphi$. Conversely, let $x \in X \ni \alpha GCl(\{x\}) \cap A \neq \varphi$. Suppose $x \notin \Lambda_{\alpha g}(A)$ then $\exists M \in \alpha GO(X, \tau) \ni A \subseteq M$ and $x \notin M$. Let $y \in \alpha GCl(\{x\}) \cap A$. Since $y \in \alpha GCl(\{x\})$, by Lemma 3.25, $x \in \Lambda_{\alpha g}(\{y\})$. Therefore, for every αg–open set $M \ni \{y\} \subseteq M$, $x \in M$. Since $y \in A$ and $A \subseteq M$, $y \in M$, where M is an $\alpha g - open$ set in (X,τ). Hence $x \in M$. This is a contradiction and hence $x \in \Lambda_{\alpha g}(A)$.

IV. $(\Lambda, \alpha g)$-Closed **Sets**

Definition 4.1. A subset A of a topological space (X, τ) is called a $(\Lambda, \alpha g)$-closed set if $A = T \cap C$, where T is a $\Lambda_{\alpha g}$-set and C is an α-closed set. The family of all $(\Lambda, \alpha g)$-closed sets is denoted by $\Lambda\alpha GC(X, \tau)$.

Example 4.2. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a, b\}, \{X\}\}$. Then $\Lambda\alpha GC(X, \tau) = \{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, X\}$.

Proposition 4.3. *Every* α*-closed set is a* $(\Lambda, \alpha g)$*-closed set but not conversely*.

Proof. Obvious.

Example 4.4. Consider (X, τ) as in example 4.2. Here $\{a\}$ is a $(\Lambda, \alpha g)$-closed set but not α-closed.

Proposition 4.5. *Every* Λ_α *(resp* $\Lambda_{\alpha g}$*)-set is a* $(\Lambda, \alpha g)$*-closed set but not conversely*.

Proof. Let A be a $\Lambda_\alpha(\text{resp.}\,\Lambda_{\alpha g})$-*set*. Then $A = A \cap X$ where A is a $\Lambda_\alpha(\textit{resp}.\,\Lambda_{\alpha g})$-set and X is an α-closed set. Therefore A is a $(\Lambda, \alpha g)$-closed set.

Example 4.6. Consider (X, τ) as in example 4.2. Here $\{c\}$ is a $(\Lambda, \alpha g)$-closed set but not a $\Lambda_\alpha(\textit{resp}\ \Lambda_{\alpha g})$-*set*.

Proposition 4.7. *Every* αg-*open set is a* $(\Lambda, \alpha g)$-*closed set but not conversely*.

Proof. Let A be an αg-open set. Then by Lemma 3.15 (2), A is a $\Lambda_{\alpha g}$-set. Then $A = A \cap X$ where X is a $\Lambda_{\alpha g}$-set and A is an α-closed set. Therefore A is a $(\Lambda, \alpha g)$-closed set.

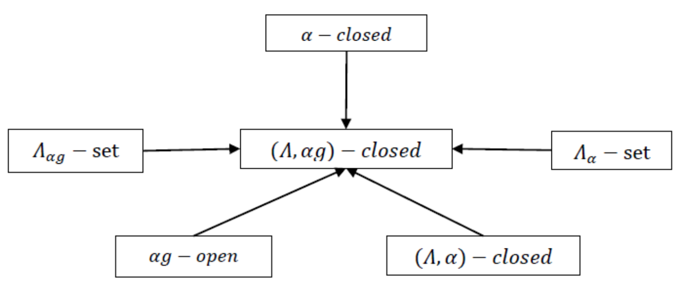
Example 4.8. Consider (X, τ) as in example 4.2. Here $\{c\}$ is a $(\Lambda, \alpha g)$-closed set but not an αg-open set.

Proposition 4.9. *Every* (Λ, α)-*closed set is a* $(\Lambda, \alpha g)$-*closed set but not conversely*.

Proof. Obvious.

Example 4.10. Let $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a, b\}, X\}$. Then (Λ, α)-closed sets are φ, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, X and $(\Lambda, \alpha g)$-closed sets are φ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{a, b, d\}$, $\{b, c, d\}$, X. Here $\{b, c\}$ is a $(\Lambda, \alpha g)$–closed set but not (Λ, α)–closed.

Remark 4.11. The above discussion is exhibited in the following diagram.



Theorem 4.12. *For a subset A of a topological space* (X, τ), *the following statements are equivalent*.

(1) *A is a* $(\Lambda, \alpha g)$*–closed set.*

(2) $A = T \cap Cl_{\alpha}(A)$, *where T is a* $\Lambda_{\alpha g}$*-set.*

(3) $A = \Lambda_{\alpha g}(A) \cap Cl_{\alpha}(A)$.

Proof. $(1) \Rightarrow (2)$ Let $A = T \cap C$, where T is a $\Lambda_{\alpha g}$-set and C is an α-closed set. Now, $A \subseteq C$ and C is α-closed $\Rightarrow Cl_{\alpha}(A) \subseteq Cl_{\alpha}(C) = C$. There for $Cl_{\alpha}(A) \subseteq C$ and $T \cap C \supseteq T \cap Cl_{\alpha}(A) \supseteq A$. Hence $A = T \cap Cl_{\alpha}(A)$, where T is a $\Lambda_{\alpha g}$-set.

$(2) \Rightarrow (3)$ Let $A = T \cap Cl_{\alpha}(A)$, where T is a $\Lambda_{\alpha g}$-set. Now, $A \subseteq T$ and T is a $\Lambda_{\alpha g}$*-set* $\Rightarrow \Lambda_{\alpha g}(A) \subseteq \Lambda_{\alpha g}(T) = T$. Thus, $\Lambda_{\alpha g}(A) \subseteq T$ and $A \subseteq \Lambda_{\alpha g}(A) \cap Cl_{\alpha}(A) \subseteq T \cap Cl_{\alpha}(A) = A$. Hence $A = \Lambda_{\alpha g}(A) \cap Cl_{\alpha}(A)$.

$(3) \Rightarrow (1)$ Since $\Lambda_{\alpha g}(A)$ is a $\Lambda_{\alpha g}$-set, $Cl_{\alpha}(A)$ is α-closed and $A = \Lambda_{\alpha g}(A) \cap Cl_{\alpha}(A)$, by Definition 4.1 we have A is a $(\Lambda, \alpha g)$-closed set.

Definition 4.13. A subset A of a topological space (X, τ) is said to be $(\Lambda, \alpha g)$–open if the complement of A is $(\Lambda, \alpha g)$–closed. In other words, a subset A of a topological space (X, τ) is called $(\Lambda, \alpha g)$–open if $A = T \cup C$, where T is a $\Lambda^*_{\alpha g}$*-set* and C is an α–open set.

Theorem 4.14. *For a subset A of a topological space* (X, τ) *the following statements are equivalent*:

(1) *A is* $(\Lambda, \alpha g)$*–open.*

(2) $A = T \cup \text{int}_{\alpha}(A)$, *where T is a* $\Lambda^*_{\alpha g}$-set.

(3) $A = \Lambda^*_{\alpha g}(\text{A}) \cup \text{int}_{\alpha}(A)$.

Proposition 4.15. *For a subset* $A_i (i \in I)$ *of a topological space* (X, τ) *the following properties hold good*:

(1) *If* A_i *is* $(\Lambda, \alpha g)$*–closed for each* $i \in I$, *then* $\cap \{A_i / i \in I\}$ *is* $(\Lambda, \alpha g)$*–closed.*

(2) *If* A_i *is* $(\Lambda, \alpha g)$–*open for each* $i \in I$, *then* $\bigcup\{A_i / i \in I\}$ *is* $(\Lambda, \alpha g)$–*closed*.

Proof. (1) Let A_i be a $(\Lambda, \alpha g)$ – closed set for each $i \in I$. Therefore, for each $i \in I \, \exists$ a $\Lambda_{\alpha g}$-set T_i and an α-closed set $C_i \ni A_i = T_i \cap C_i$. $\bigcap_{i \in I} A_i = \bigcap_{i \in I} (T_i \cap C_i) = (\bigcap_{i \in I} T_i) \cap (\bigcap_{i \in I} C_i)$. By Lemma 3.15 (4), $\bigcap_{i \in I} T_i$ is a $\Lambda_{\alpha g}$-set and $\bigcap_{i \in I} C_i$ is an α–closed set. Therefore, $\bigcap_{i \in I} A_i$ is the intersection of a $\Lambda_{\alpha g}$-set and an α–closed set. Hence $\bigcap_{i \in I} A_i$ is $(\Lambda, \alpha g)$–closed.

(2) Let A_i be a $(\Lambda, \alpha g)$–open set for each $i \in I$. Then $X - A_i$ is $(\Lambda, \alpha g)$–closed and $X - \bigcup_{i \in I} A_i = \bigcap_{i \in I} (X - A_i)$. Therefore by (1) $\bigcup_{i \in I} A_i$ is $(\Lambda, \alpha g)$–open.

Definition 4.16. If A is a subset of the topological space (X, τ), then a point $x \in X$ is called $(\Lambda, \alpha g)$ – *cluster* point of A if for every $(\Lambda, \alpha g)$ – *open* set U containing x, $A \cap U \neq \varphi$. The set of all $(\Lambda, \alpha g)$–cluster points is called the $(\Lambda, \alpha g)$-closure of A and is denoted by $(\Lambda, \alpha g) Cl(A)$.

Lemma 4.17. *For subsets* A, B *and* $A_i (i \in I)$ *of a topological space* (X, τ) *the following properties hold*:

(1) $(\Lambda, \alpha g) Cl(A) = \bigcap\{K / A \subseteq K \text{ and } K \text{ is } (\Lambda, \alpha g)\text{–closed}\}$.

(2) $(\Lambda, \alpha g) Cl(\varphi) = \varphi$ *and* $(\Lambda, \alpha g) Cl(X) = X$.

(3) *If* $A \subseteq B$, *then* $(\Lambda, \alpha g) Cl(A) \subseteq (\Lambda, \alpha g) Cl(B)$.

(4) $A \subseteq (\Lambda, \alpha g) Cl(A)$.

(5) $(\Lambda, \alpha g) Cl(\bigcup_{i \in I} (A_i)) \supseteq \bigcup_{i \in I} ((\Lambda, \alpha g) Cl(A_i))$.

(6) $(\Lambda, \alpha g) Cl(\bigcap_{i \in I} (A_i)) \subseteq \bigcap_{i \in I} ((\Lambda, \alpha g) Cl(A_i))$.

(7) $(\Lambda, \alpha g) Cl((\Lambda, \alpha g) Cl(A)) = (\Lambda, \alpha g) Cl(A_i)$.

Proof. (1) Let $x \notin (\Lambda, \alpha g)Cl(A)$. Then $\exists$ a $(\Lambda, \alpha g)$-open set U containing $x \ni A \cap U = \varphi$. Take $K = U^c$, then K is $(\Lambda, \alpha g)$-closed. Also $A \subseteq K$ and $x \notin K$. Therefore, $x \notin \cap\{K/A \subseteq K$ and K is $(\Lambda, \alpha g)$-closed$\}$ and hence $\exists$ a $(\Lambda, \alpha g)$-closed set $K \ni x \notin K$ and $A \subseteq K$. Take $K^c = U$, then U is a $(\Lambda, \alpha g)$-open set containing $x \ni A \cap U = \varphi \Rightarrow x$ is not a cluster point of A Thus, $x \notin (\Lambda, \alpha g)Cl(A)$. Hence $(\Lambda, \alpha g)Cl(A) = \cap\{K/A \subseteq K$ and K is $(\Lambda, \alpha g)$–closed$\}$.

(2) Obvious.

(3) Let $x \notin (\Lambda, \alpha g)Cl(B)$. Then $\exists$ a $(\Lambda, \alpha g)$-open set U containing $x \ni B \cap U = \varphi$. Since $A \subseteq B$, $A \cap U = \varphi$ and thus x is not a $(\Lambda, \alpha g)$-cluster point of A. Therefore, $x \notin (\Lambda, \alpha g)Cl(A)$. Hence $(\Lambda, \alpha g)Cl(A) \subseteq (\Lambda, \alpha g)Cl(B)$.

(4) Let $x \notin (\Lambda, \alpha g)Cl(A)$. Then x is not a $(\Lambda, \alpha g)$-cluster point of A which $\Rightarrow \exists$ a $(\Lambda, \alpha g)$–open set U containing $x \ni A \cap U = \varphi \Rightarrow x \notin A$. Hence $A \subseteq (\Lambda, \alpha g)Cl(A)$.

(5) Since $A_i \subseteq \bigcup_{i \in I}(A_i), (\Lambda, \alpha g)Cl(A_i) \subseteq (\Lambda, \alpha g)Cl(\bigcup_{i \in I} A_i) \Rightarrow \bigcup_{i \in I}(\Lambda, \alpha g)Cl(A_i)$ $\subseteq (\Lambda, \alpha g)Cl(\bigcup_{i \in I} A_i))$.

(6) Since $\subseteq \bigcap_{i \in I}(A_i) \subseteq A_i$, $(\Lambda, \alpha g)Cl(\bigcap_{i \in I} A_i) \subseteq (\Lambda, \alpha g)Cl(A_i)$

$$\Rightarrow (\Lambda, \alpha g)Cl(\bigcap_{i \in I}(A_i)) \subseteq \bigcap_{i \in I}((\Lambda, \alpha g)Cl(A_i)).$$

(7) By (3) and (4), $(\Lambda, \alpha g)Cl(A) \subseteq (\Lambda, \alpha g)Cl(A, \alpha g)Cl(A))$. Conversely, $x \in (\Lambda, \alpha g)Cl((\Lambda, \alpha g)Cl(A))$ then x is a cluster point of $(\Lambda, \alpha g)Cl(A) \Rightarrow$ for every $(\Lambda, \alpha g)$–open set U containing x, $(\Lambda, \alpha g)Cl(A) \cap U \neq \varphi$. Let $y \in (\Lambda, \alpha g)Cl(A) \cap U$. Then Y is a cluster point of $A \Rightarrow$ for every $(\Lambda, \alpha g)$-open set M containing y, $A \cap M \neq \varphi$. Since U is a $(\Lambda, \alpha g)$-open set and $y \in U$, $A \cap U \neq \varphi$. Therefore, $x \in (\Lambda, \alpha g)Cl(A)$. Hence $(\Lambda, \alpha g)Cl((\Lambda, \alpha g)Cl(A)) = (\Lambda, \alpha g)Cl(A)$.

Proposition 4.18. *A is* $(\Lambda, \alpha g)$*-closed iff* $A = (\Lambda, \alpha g)Cl(A)$.

Remark 4.19. From Lemma 4.17, we get $(\Lambda, \alpha g)$–closure is a closure operator.

Remark 4.20. The following example shows that generally the reverse inclusion of Lemma 4.17 (5) and (6) are not true.

Example 4.21. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a, b\}, X\}$.

Let $A_1 = \varphi$, $A_2 = \{a\}$ and $A_3 = \{c\}$. Then $\bigcup_{i \in I}(A_i) = \{a, c\}$ and $(\Lambda, \alpha g)Cl(\bigcup_{i \in I}(A_i)) = \{a, b, c\}$. But, $\bigcup_{i \in I}((\Lambda, \alpha g)Cl(A_i)) = \{a, c\}$. Thus, $(\Lambda, \alpha g)Cl(\bigcup_{i \in I}(A_i)) \not\subseteq \bigcup_{i \in I}((\Lambda, \alpha g))Cl(A_i))$.

Let $A_1 = \{a, b\}$ and $A_2 = \{b, c\}$. Then $\bigcap_{i \in I}(A_i) = \{b\}$ and $(\Lambda, \alpha g)Cl(\bigcap_{i \in I}(A_i)) = \{b\}$; But, $\bigcap_{i \in I}(\Lambda, \alpha g)Cl(A_i) = \{a, b\}$. Thus, $(\Lambda, \alpha g)Cl(\bigcap_{i \in I}(A_i)) \not\supseteq \bigcap_{i \in I}((\Lambda, \alpha g)Cl(A_i))$.

References

[1] M. Caldas, D. N. Georgiou and S. Jafari, Study of (Λ, α)- closed sets and the related notions in topological spaces, Bull. Malays. Math. Sci. Soc. (2) 30(1) (2007), 23-36.

[2] N. Levine, Generalized closed sets in topology, Rend. Cir. Math. Palermo 19 (1970), 89-96.

[3] H. Maki, Generalized Λ-sets and the associated closure operator, Special Issue in Commemoration of Prof. Kazuasada Ikeda's Retirement (1986), 139-146.

[4] H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed. Part III 42 (1993), 13-21.

[5] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math. 15 (1994), 51-63.

[6] A. S. Mashhour, I. A. Hasanein and S. N. EI-Deeb, On α-continuous and α-open mappings, Acta. Math. Hungarica 41 (1983), 213-218.

[7] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15 (1965), 961-970.