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EFFICIENT EXACT SOLUTION OF BLAST WAVES IN MAGNETO-GAS-DYNAMIC FLOW AT STELLAR SURFACES

AKMAL HUSAIN, SYED AFTAB HAIDER and V. K. SINGH

Department of Mathematics
SoE University of Petroleum and Energy Studies
Dehradun-248007, India
E-mail: ahusain@ddn.upes.ac.in

Department of Mathematics
Shia P. G. College
Lucknow-226020, India
E-mail: aftabhaidershiapgc2019@gmail.com

Department of Applied Sciences
Institute of Engineering and Technology
Lucknow-226021, India
E-mail: singhvijaikrishna9@gmail.com

Abstract

The main emphasis of the present analysis is to construct an analytical solution of the blast wave problem, which govern one-dimensional non-homogeneous unsteady adiabatic flow with generalized geometries at stellar surface under the significant and specific effects of magnetic field. Here we consider the plasma be an inviscid perfect gas permeated by transverse component of magnetic field, under the assumption that the gas has infinite electrical conductivity. A detailed investigation has been made for the cases of planar and non-planar flows under the specific influence of magnetic field. A unified analytical approach has been used to obtain the explicit solution of flow profiles velocity, density, pressure and magnetic pressure in the presence of the magnetic field, which represents time-space dependence. Furthermore, an analytic expression for the total energy of blast wave problem in magnetogasdynamics regime has been obtained at the stellar surface.

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Introduction

Due to generation of high temperature and pressure, the study of blast wave propagation has gained undivided attention in many research areas such as plasma physics, astrophysics, detonation and fusion initiation, nozzle flow, stochastic dynamical systems, nuclear blasts, lunar ash flow, and many other diverse problems of engineering. In the past few decades, various mathematical and theoretical studies provided useful insights related to the theory of blast wave propagation in different material media. It is well known that after an explosion, due to the effect of high temperature and pressure at the centre of convergence the gas particles are ionized. These ionized gas particles generate the magnetic field. Therefore, the influence of the magnetic field on non-linear systems involving blast wave phenomenon has remained an important topic for capturing the physics of the process well.

The pioneering work concerning physics of blast wave propagation was made by several authors ([11], [12], [13], [18], [19]). In his theoretical study of blast wave phenomenon Taylor [18, 19] discussed self-similar solutions for spherically symmetric blast waves. Sakurai [12, 13] used similarity analysis technique, and found a series solution for shock waves produced by intense explosion. Rogers [11] gave an analytical solution for spherically symmetric flows to point source explosion dynamics and examined the nature of discontinuity. Lerche [5, 6] reported the existence of self-similar solution for blast waves under isothermal condition in planar and cylindrical geometry and observed that the effect of magnetic field plays a significant role in the theory of blast waves. Besides, Kleine et al. [4] experimentally and numerically investigated blast wave reflections from straight surfaces, using small particles with mass in the milligram range.

Many attempts have been performed in different material media in the context of explicit determination of exact solution of nonlinear hyperbolic partial differential equations governing blast wave phenomenon. Sharma et al. [15] analytically described entire flow profile for the case of a decaying shock wave and gave a uniformly valid exact solution to the problem of shock-rarefaction interaction. Much later Oliveri et al. [8-9] presented a systematic exact solution for both gas dynamics and magnetogasdynamics equation by using group theoretic invariance transformation method. Murata [7] has obtained a new class of exact solution to the quasi-hyperbolic system of PDEs that governs the one dimensional, non-steady flow of a perfect gas involving strong discontinuities. Singh et al. [16] used group invariant solution to PDE's and explicitly presented an analytical solution for compressible gas dynamic equation driven by planar and radially symmetric strong shocks in a real medium. Singh et al. [17] described the analytical solution of the blast wave propagation in a real medium and observed that the blast waves propagating outwards from source of eruption. Ram et al. [10] analytically analyzed the influence of magnetic field on flow parameters driven by strong shock waves in non-ideal gas-dynamic regime with generalized geometries. Haider et al. [2] considered point explosion problem at stellar surface in a perfect gas and reported an explicit exact solution of thermodynamic variables (density, velocity and pressure) exhibiting time-space dependence.

In this paper, our goal is to determine the analytical solution of blast wave problem in an ideal gas for generalized geometries under the influence of magnetic field and examine how the magnetic field affect the flow parameters which exhibits space time dependence. Here, it is assumed that the mass density distribution obeying a power law of the radial distance from the centre of explosion. It may be pointed out here that the analytical solution obtained in terms of physical parameters such as density, velocity, pressure and magnetic pressure, confirms the pattern of the theoretical results. Furthermore, we have derived an analytical expression for the total energy of the blast wave, influenced by the magnetic field.

Problem Formulation and Discontinuous (Rankine-Hugoniot) Conditions

The fundamental equations describing the one-dimensional unsteady planar $(m = 1)$ and radially symmetric [cylindrically symmetric $(m = 2)$ and spherically symmetric $(m = 3)$] motion of a perfect gas at the stellar surfaces under the assumption that the electrical conductivity of the gas has taken to be infinite subject to the transverse component of the magnetic field, have the form ([1], [3], [14], [20], [21])

$$\rho_t + v\rho_r + \rho v_r - \frac{(m-1)\rho v}{1-r} = 0, \tag{1}$$

$$v_t + vv_r + \frac{1}{\rho}(p_r + h_r) = 0, \tag{2}$$

$$p_t + vp_r + \rho a^2 \left\{ v_r - \frac{(m-1)v}{1-r} \right\} = 0, \tag{3}$$

$$h_t + vh_r + 2h \left\{ v_r - \frac{(m-1)v}{1-r} \right\} = 0, \tag{4}$$

in which t and r represents the time and space coordinates, ρ, v, p and h are the density, velocity, pressure and magnetic pressure of the gas respectively. The magnetic pressure h is defined as $h = \mu H^2/2$, where μ and H denotes the magnetic permeability and magnetic field strength. The entity $a = (\gamma p/\rho)^{1/2}$ is the equilibrium speed of sound and γ is the adiabatic index with $1 \leq \gamma \leq 2$. Here, non-numeric subscripts with respect to the indicated flow parameters denotes partial derivatives unless stated otherwise.

The governing hyperbolic system of equations (1)-(4) is supplemented with an equation of state to be of the form $p = \rho \mathfrak{R} T$, where $\mathfrak{R} > 0$ and T denotes the gas constant and absolute temperature respectively.

Let us consider, $r = R(t)$ be the position of the shock front from source of explosion then the propagation velocity V at the shock front, is defined as

$$\frac{dR(t)}{dt} = V. \tag{5}$$

If ρ_0 is the density ahead of the shock front. Then, for an infinite Mach number, the boundary conditions across the shock front $r = R(t)$ is given by

$$\rho = \frac{\gamma + 1}{\gamma - 1}\rho_0, \tag{6}$$

$$v = \frac{2}{\gamma + 1}V, \tag{7}$$

$$p = \frac{[4(\gamma - 1)^2 - (\gamma + 1)^3 C_0]}{2(\gamma + 1)(\gamma - 1)^2}\rho_0 V^2, \tag{8}$$

$$h = \frac{C_0}{2}\left[\frac{\gamma + 1}{\gamma - 1}\right]^2 \rho_0 V^2, \tag{9}$$

where $C_0 = 2h_0/\rho_0 V^2$ is the shock Cowling number.

In the present study, we consider the density in undisturbed region ρ_0 is assumed to vary according to the power law of the radius of the shock front $R(t)$ and is given by

$$\rho_0 = \rho_\theta R^\kappa, \tag{10}$$

where ρ_θ and κ are constants. The constant κ is to be determined with the help of subsequent analysis.

The total energy (sum of the kinetic and internal energy of the gas) E exerted by the blast wave during its propagation in ideal magneto-gas dynamic flow at stellar surface is given by

$$E = 4\pi \int_1^R \left(\frac{1}{2}\rho v^2 + \frac{1}{\gamma - 1} p + h\right)(1 - r)^{(m-1)}\, dr. \tag{11}$$

Exact Solution to the Blast Wave Problem

Using Rankine-Hugoniot jump conditions (6)-(9), the flow variables pressure and magnetic pressure in terms of fluid velocity and density can be written as

$$p = \frac{[4(\gamma - 1)^2 - (\gamma + 1)^3 C_0]}{8(\gamma - 1)} \rho v^2, \tag{12}$$

$$h = \frac{C_0}{8} \frac{(\gamma + 1)^3}{(\gamma - 1)} \rho v^2. \tag{13}$$

With the help of relations (12) and (13), the nonlinear basic equations (2), (3) and (4) can be transformed to

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} - \frac{(\gamma - 1)v}{2\rho}\left(\frac{\partial \rho}{\partial t} - \rho\frac{\partial v}{\partial r} - \frac{(m - 1)\rho v}{1 - r}\right) = 0, \tag{14}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{(\gamma - 1)v}{2}\left(\frac{\partial v}{\partial r} - \frac{(m-1)v}{1-r}\right) = 0, \tag{15}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{v}{2}\left(v_r - \frac{(m-1)v}{1-r}\right) = 0. \tag{16}$$

Plugging equations (14) and (15), and after integration the resulting expression can be written as

$$\rho v(1-r)^{-(m-1)} = \mathcal{N}(t), \tag{17}$$

where $\mathcal{N}(t)$ is an arbitrary function of integration.

From equation (17), the governing equation (1) can be transformed in the form

$$\frac{1}{v}\frac{\partial v}{\partial t} + \frac{2(m-1)}{1-r}v - \frac{1}{\mathcal{N}(t)}\frac{d\mathcal{N}(t)}{dt} = 0. \tag{18}$$

On solving equations (15) and (18), we have

$$v = \frac{2}{[(\gamma+3)m-2]}\frac{(1-r)}{\mathcal{N}(t)}\frac{d\mathcal{N}(t)}{dt}. \tag{19}$$

Substituting equation (19) in equation (18) and then integrating, we obtained

$$\mathcal{N}(t) = \mathcal{N}_0 t^{-\lambda}, \tag{20}$$

where $\lambda = \left[1 + \frac{4(m-1)}{[(\gamma-1)(m-1)+(\gamma+1)]}\right]$ and $\mathcal{N}_0$ is an arbitrary constant.

With the help of Rankine-Hugoniot shock condition (6), we can obtain the explicit expression for the radius of the shock front as

$$R(t) = 1 - t^{\frac{\gamma+1}{(\gamma-1)(m-1)+(\gamma+1)}}. \tag{21}$$

The value of the constant κ with the help of Rankine-Hugoniot condition (7) is given by

$$\kappa = \left[\frac{(\gamma-1)(m-2)-2m}{(\gamma+1)}\right]. \tag{22}$$

Finally, the particular analytical solution of governing hyperbolic system of equations (1)-(4) is given by

$$\rho = -\frac{\mathcal{N}_0[(\gamma-1)m+2]}{2}(1-r)^{(m-2)} t^{\frac{-4(m-1)}{[(\gamma-1)(m-1)+(\gamma+1)]}},$$

$$v = -\frac{2(1-r)}{t[(\gamma-1)(m-1)+(\gamma+1)]},$$

$$p = -\frac{[4(\gamma-1)^2-(\gamma+1)^3 C_0]}{4(\gamma-1)[(\gamma-1)(m-1)+(\gamma+1)]}\mathcal{N}_0\cdot(1-r)^m\cdot t^{-\frac{2m(\gamma+1)}{[(\gamma-1)(m-1)+(\gamma+1)]}},$$

$$h = -\frac{C_0(\gamma+1)^3}{4(\gamma-1)[(\gamma-1)(m-1)+(\gamma+1)]}\mathcal{N}_0\cdot(1-r)^m\cdot t^{-\frac{2m(\gamma+1)}{[(\gamma-1)(m-1)+(\gamma+1)]}},$$

$$\rho_0(R) = -\frac{1}{2}\left(\frac{\gamma-1}{\gamma+1}\right)\mathcal{N}_0[(\gamma-1)(m-1)+(\gamma+1)](1-R)\left[\frac{(\gamma-1)(m-2)-2m}{(\gamma+1)}\right]. \tag{23}$$

With the help of physical parameters (density, velocity, pressure and magnetic pressure in terms of spatial coordinates) from equation (23), the analytic expression for total energy at stellar surface is given by

$$E = \frac{4\pi}{m[(\gamma-1)(m-1)+(\gamma+1)]}\left[1+\frac{c_0}{8}\frac{(\gamma^2-(\gamma+2))^2\left(1+\frac{3}{(\gamma-2)}\right)}{(\sqrt{\gamma}(\sqrt{\gamma}+2)+1)\cdot(\sqrt{\gamma}(\sqrt{\gamma}-2)+1)}\right]\mathcal{N}_0. \tag{24}$$

Result and Discussion

Equation (23), explicitly represent the exact solution of blast waves problem at the stellar surfaces under the significant influence of magnetic field in the form of shock cowling number, and exhibiting the dependence of time and spatial co-ordinate. It is worth mentioning that in the absence of magnetic field and stellar surface, the results reported shows a closer agreement with the earlier results carried out by various approaches in different material media [11, 14].

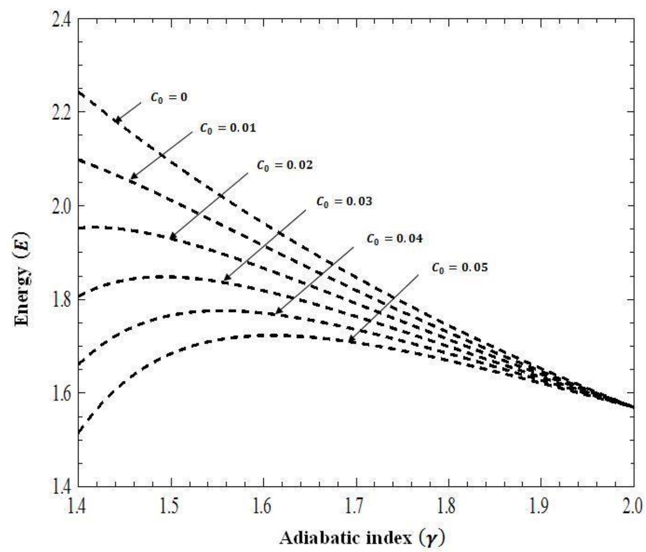

Figure 1. The effect of magnetic field (C_0) on energy (E) for various values of adiabatic index γ.

Figure 1 represents the behavior of energy carried by cylindrically symmetric blast wave in magneto-gas-dynamic regime with the variation in shock cowling number (C_0) at different values of adiabatic index γ. From Figure 1, it is evident that the energy of the cylindrical symmetric blast waves under the influence of the magnetic field decays much faster than that in the non-magnetic case subject to increasing value of adiabatic index. Simultaneously, for strong magnetic cases, the energy of the blast waves causes to decrease after increasing within a certain interval, subject to the increasing value of the adiabatic index. In particular, it may be pointed here that the effect of increasing value of magnetic field in terms of shock cowling number causes to decrease the energy of the blast waves, which confirms the corresponding theoretical results.

Conclusions

In the current study, we have considered the problem of propagation of blast waves, which govern one-dimensional inviscid compressible ideal magnetogasdynamics flow at stellar surface. In addition, we assume that the plasma be a perfect gas having infinite electrical conductivity subject to a transverse magnetic field. A simple and efficient analytical technique has been used to obtain the explicit particular solution to flow parameters under the significant influence of magnetic field exhibiting the dependence of space and time. The influence of magnetic field (in terms of Cowling number; $C_0 = 2h_0/\rho_0 V^2$) on the flow variables is noticeable from equation (23). It may be noted here that in the absence of magnetic field, the results reported shows a closer agreement with the earlier results. Moreover, in view of equation (24), an analytical result for the total energy of the blast wave problem under the influence of magnetic field has been obtained in the magnetogasdynamics flow on the stellar surface, clearly resembles the pattern of the theoretical results.

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