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INVERSE MAJORITY DOMINATION NUMBER ON SUBDIVISION GRAPHS

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Abstract

In this article, Majority domination number $\gamma_M(G)$ and Inverse Majority domination number $\gamma_M^{-1}(G)$ are found for some special graphs and its subdivision graphs. Then $\gamma_M^{-1}(G)$ for some families of the subdivision graphs $S(G)$ is determined. Some results on $\gamma_M(S(G))$ and $\gamma_M^{-1}(S(G))$ are also studied.

1. Introduction

The Domination theory in graphs was defined by ore and Berdge, in 1977, Cockayne et al., developed the domination concept and it has been discussed extensively in their seminal paper. Then many eminent graph theorists defined various domination parameters and produced many interesting results in this area. Also the new parameter inverse domination in graphs was initiated by Kulli et al., in 1991.

Let G be a simple, un directed and finite with p vertices and q edges. $N(V) = \{u \in V(G)/uv \in E(G)\}$ and $N[v] = N(v) \cup \{v\}$ be the open

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neighbourhood and the closed neighbourhood of v respectively.

A set $S \subseteq V(G)$ of vertices in a graph $G = (V, E)$ is called a majority dominating (MD) set of G if at least half of the vertices of $V(G)$ are either in S or adjacent to the elements of S the Majority dominating set S is minimal if no proper subset of S is a majority dominating set of a graph G.

A subdivision of an edge $e = uv$ of a graph G is the replacement of an edge e by a path (u, v, w). The graph obtained from a graph G by subdividing every edge e of G exactly once and is called the subdivision graph of G denoted by $S(G)$.

Let G be simple and finite graph with p vertices and q edges and D be a minimum majority dominating set of G. If the set $(V - D)$ contains a majority dominating set say D' then D' is called Inverse majority dominating (IMD) set with respect to D.

1.2. Results on $\gamma_M(G)$ and $\gamma_M^{-1}(G)$ [3] and [5]

The following are the results on $\gamma_M(G)$ and $\gamma_M^{-1}(G)$

1. For $G = P_p,\ p \geq 2$ and cycle $C_p,\ p \geq 3,\ \gamma_M(G) = \left\lceil \frac{p}{6} \right\rceil$.

2. Let $G = K_{m, n},\ m, n \geq 2$. Then $\gamma_M(G) = 1$.

3. For a Path $P_p,\ p \geq 2$ and cycle $C_p,\ p \geq 3,\ \gamma_M^{-1}(G) = \left\lceil \frac{p}{6} \right\rceil$

4. Let $G = K_{m, n},\ m, n \geq 2$. Then $\gamma_M(G) = 1$.

5. Let $G = K_{1, p-1},\ \gamma_M^{-1}(G) = \left\lfloor \frac{p-1}{2} \right\rfloor,\ p \geq 2$.

6. For a $G = K_p,\ \gamma_M^{-1}(K_p) = 1$.

7. Let $G = mk_2$. Then $\gamma_M^{-1}(G) = \left\lceil \frac{p}{4} \right\rceil$, where $p = 2m$.

8. For any regular graph with p vertices then $\gamma_M(G) = \gamma_M^{-1}(G)$.

2. Inverse Majority Domination number for some Special Graphs

Proposition 1. *Let G be the Dodecahedron graph and $G' = S(G)$ be the subdivision graph. Then*

(i) $\gamma_M(G) = 3 = \gamma_M^{-1}(G)$ *and*

(ii) $\gamma_M(G') = 7 = \gamma_M^{-1}(G')$.

Proof. Let G be the platonic solid dodecahedron, Then G is a 3-regular graph $p = 20$ and $q = 30$. Let $D = \{v_1, v_4, v_6\}$ such that $d(u_i, v_j) = 3$. Then $|N[D]| = 11 > \left\lceil \frac{p}{2} \right\rceil$. Hence D is a MD set of G and $\gamma_M(G) = 3$. Let $D' = \{v_2, v_5, v_8\} \subseteq V - D$, Such that $d(u_i, v_j) = 3$. By similar argument, D' is a IMD-set of G and $\gamma_M^{-1}(G) = 3$.

Let G' be a subdivision graph of dodecahedron with $p' = 50$ and $V(G') = \{v_1, \ldots, v_{20}, u_1, u_2, \ldots, u_q\}$ with $d(v_i) = 3$, $i = 1, 2, \ldots, 20$ and $d(u_i) = 2$, $i = 1, 2, \ldots, 30$. Let $S = \{v_1, v_3, v_6, v_7, v_{11}, v_{13}, v_{17}\}$, such that $d(u_i, v_j) = 4$, for $\forall i, j, i \neq j$. Then $|N[S]| = 28 > \left\lceil \frac{p}{2} \right\rceil$. It implies that S is a majority dominating set of G' and $\gamma_M(G') = |S| = 7$. Next choose $S' = \{v_2, v_4, v_5, v_7, v_{18}, v_8, v_{15}\}$ such that $d(u_i, v_j) = 4$, and $S' \subseteq V - S$. By the similar argument, S' is a Inverse majority dominating set of G and $\gamma_M^{-1}(G') = 7$. Hence, $\gamma_M(G') = 7 = \gamma_M^{-1}(G')$.

Proposition 2. *Let G be the Tetrahedron graph and Octahedron graph. Then*

(i) $\gamma_M(G) = 1 = \gamma_M^{-1}(G)$ *and*

(ii) $\gamma_M(G') = 2 = \gamma_M^{-1}(G')$.

Proof. The proof is obvious.

Proposition 3. *Let G be an icosahedral graph and $S(G)$ be the subdivision graph of G. Then*

(i) $\gamma_M(G) = 1 = \gamma_M^{-1}(G)$

(ii) $\gamma_M(G') = 2 = \gamma_M^{-1}(G')$, *if* $G' = S(G)$.

Proof. (i) Let G be an icosahedral graph with $p = 12$ vertices and it is a 5-regular graph. Since each vertex dominates six vertices $\gamma_M(G) = 1 = \gamma_M^{-1}(G)$.

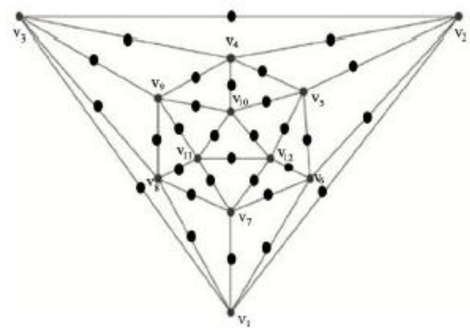

Figure 1. Subdivision of G - $S(G)$.

Let G' be the subdivision graph of icosahedral with $p' = 42$ and $V(G') = \{v_1, v_2, \ldots, v_{12}, u_1, u_2, \ldots, u_{30}\}$ Now G' is not a regular graph with $d(v_i) = 5$, for all $i = 1, 2, \ldots, 12$ and $d(v_j) = 2$, for all $i = 1, 2, \ldots, 30$. Let $S = \{v_1, v_4, v_{11}, v_{12}\}$ such that $d(u_i, v_j) = 4$. Then $|N[S]| = \sum d(v_i) + |S|$ $(3 \times 5) + 4 + 4 > \left\lceil \frac{p}{2} \right\rceil$ And $|N[S]| = 23 > \left\lceil \frac{p}{2} \right\rceil$. Then S is a majority dominating set of G' and $\gamma_M(G') = 4$.

Choose $S' = \{v_2, v_9, v_7, v_{10}\} \subseteq V - S$. By the similar argument, $\gamma_M^{-1}(G') = 4$.

Proposition 4. *Let G and G' be the Frucht graph and its subdivision graph respectively.*

(i) $\gamma_M(G) = 2 = \gamma_M^{-1}(G)$ and

(ii) $\gamma_M(G') = 4 = \gamma_M^{-1}(G')$

Proof. The proof is obvious.

Proposition 5. (i) *Let G be a Doyle graph. Then* $\gamma_M(G) = 3 = \gamma_M^{-1}(G)$ *and*

(ii) *If* $G' = S(G)$ *is the subdivision of G then* $\gamma_M(G') = 8 = \gamma_M^{-1}(G')$.

Proof. Let G be a Doyle graph with $p = 27$ vertices and G is a 4 regular graph. The vertex set $V(G)$ can be partitioned into three vertex sets V_1, V_2 and V_3, each comprising of 9 vertices with degree 4 and V_1, V_2 and V_3 are vertices of a outer cycle C_1, inner circle C_2, innermost cycle C_3 respectively.

Let $D = \{v_2, v_5, v_8\}$ and $D' = \{v_2, v_5, v_8\} \subseteq V - D$. Since G is a 4-regular graphs, $|N[D]| = |N[D']| = \sum d(v_i) + 3 = 15 > \left\lceil \frac{p}{2} \right\rceil$.

Hence, the sets D and D' are the MD-set and the IMD-set of G respectively and $\gamma_M(G) = \gamma_M^{-1}(G) = 3$.

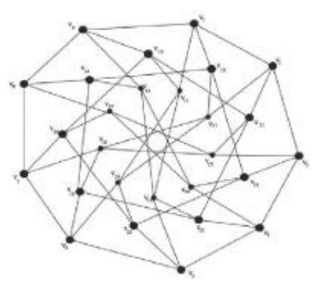

Figure 2. G: Doyle graph.

Let G' be the subdivision graph of a Doyle graph with $p' = 72$ vertices. Let $V(G') = \{v_1, v_2, \ldots, v_{27}, u_1, u_2, \ldots, u_{45}\}$ where $d(u_j) = 2$, $j = 1$ t for $i = 1$ to 27 and $d(u_j) = 2$, $j = 1$ to 45 and $V_1(G') = \{v_1, \ldots, v_9\}$, $V_2(G') = \{v_{10}, \ldots, v_{18}\}$, $V_3(G') = \{v_{19}, \ldots, v_{27}\}$

Let $S = \{v_1, v_4, v_7, v_{13}, v_{17}, v_{18}, u_{20}, u_{25}\}$ such that $d(u_i, v_j) = 4$. for

$i \neq j$ and $v_i, v_j \in S$.

Then $|N[S]| = \sum_{v_i \in S} d(v_i) + |S| = (8 \times 4) + 8 = 40 > \left\lceil \frac{p}{2} \right\rceil$. Hence S is a MD- set of G' and $\gamma_M(G') = |S| = 8$ In $V - S$, choose $S' = \{v_2, v_5, v_8, v_{10}, v_{13}, v_{16}, u_{18}, u_{22}\}$ and $|N[S']| = 40 > \left\lceil \frac{p}{2} \right\rceil$. Hence, S' is a IMD-set of G' and $\gamma_M(G') = |S| = 8$. Hence, $\gamma_M(G') = \gamma_M^{-1}(G') = 8$.

Proposition 6. (i) *Let G be a Folkman graph. Then* $\gamma_M(G) = 2 = \gamma_M^{-1}(G)$ *and* (ii) $\gamma_M(G') = 6$ *and* $\gamma_M^{-1}(G') = 8$.

Proof. Let G be a Folkman graph with $p = 20$ vertices. It is a bipartite, 4-regular, Hamiltonian graph and it is a four edge connected perfect graph. Let $D = \{(u_1, v_1) \geq$ and $D' = \{u_3, v_3\} \subseteq V - D$ such that $d(u_i, v_j) \geq 4$. Since each vertex dominates 5 vertices, $|N[D]| = |N[D']| = 10 = \left\lceil \frac{p}{2} \right\rceil$. Hence $\gamma_M(G') = \gamma_M^{-1}(G') = 2$.

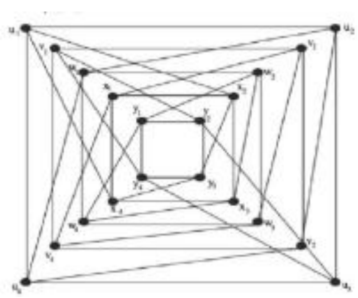

Figure 3. G: Folk man graph.

Let G' be a subdivision graph of a Folkman graph with the vertex set $V(G') = \{u_1, \ldots, u_4, v_1, \ldots, v_4, w_1, \ldots, w_4, x_1, \ldots x_4, y_1, \ldots y_4, z_1, z_2, \ldots, z_{40}\}$ and $|V(G')| = 60$, where the vertices u_i, v_i, w_i, x_i and y_i are in the outer square to inner square of totally 5 squares in G'_R and $z_i, i = 1, \ldots, 40$ denotes the newly added vertices in G. Let $S = \{u_1, v_1, w_1, x_1, y_1, y_3\}$ in which all are non-adjacent vertices in G' and $|N[S']| = \sum d(v_i) + 6 = 30$

$= \left\lceil \frac{p}{2} \right\rceil$. Hence S is a majority dominating set of G' and $\gamma_M(G') = |S| = 6$. In $V - S$, Choose $S' = \{u_3, v_3, w_3, x_3, u_4, v_4, w_4, x_4\} \subseteq V - S$ and $= 32 > \left\lceil \frac{p}{2} \right\rceil$. It implies that set S' is an inverse majority dominating set of G' and $\gamma_M^{-1}(G') = |S| = 8$.

Proposition 7. (i) *Let G be a Levi graph, Then* $\gamma_M(G) = 4 = \gamma_M^{-1}(G)$ *and* (ii) *Let* $G' = S(G)$ *be the subdivision of G. Then* $\gamma_M(G') = 9 = \gamma_M^{-1}(G')$.

Proof. Let G be a Levi graph with $p = 30$ and $q = 45$, and it is not a regular graph.

Let $V(G) = \{w_1, w_2, \ldots, w_{10}, v_1, \ldots, v_{10}, u_1, \ldots, u_{10}\}$ in which the vertices w_i, v_i and u_i, for $i = 1, \ldots, 10$ from a outer circle to inner circle and $d(w_i) = 3$, $d(v_i) = 2$, $d(u_i) = 4$, $i = 1, \ldots, 10$.

Let $D = \{u_1, u_3, w_2, w_5\}$ and $|N[D]| = \sum d(u_i) + \sum d(w_i) + 4$ $= 18 > \left\lceil \frac{p}{2} \right\rceil$. Hence D is a MD-set of G and $\gamma_M(G) = |D| = 4$.

Next, choose $D' = \{u_5, u_6, w_1, w_8\} \subseteq V - D$. Then $|N[D']| = 18 > \left\lceil \frac{p}{2} \right\rceil$. It implies that D' is aIMD-set of G and $\gamma_M^{-1}(G) = 4$.

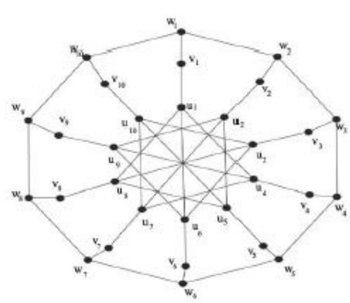

Figure 4. G: Levi graph.

Let G' be the subdivision graph of a Levi graph G with $p' = 75$.

$V(G') = \{w_1, \ldots, w_{10}, v_1, \ldots, v_{10}, u_1, \ldots, u_{10}, x_1, x_2, \ldots, x_{45}\}$ where $d(w_i) = 3$, $d(v_i) = 2$, $d(u_i) = 4$. $i = 1$ to 10 and $d(x_j) = 2$, $j = 1$ to 45.

Let $S = \{u_1, u_2, u_3, w_1, w_3, w_5, w_9, u_9\} \subseteq V(G')$ and $|N[D']| = \sum_1^3 d(u_i)$ $+ \sum_1^5 d(w_i) + d(u_9) + |S| = 39 > \left\lceil \frac{p}{2} \right\rceil$. It implies that $\gamma_M(G') = |S| = 9$.

Next, choose, $S' = \{u_4, u_5, u_6, w_2, w_4, w_6, w_8, w_{10}, u_{10}\} \subseteq V - S$.

By the above calculations, $|N[S']| = 39 > \left\lceil \frac{p}{2} \right\rceil$.

Hence S' is a IMD-set of G' and $\gamma_M^{-1}(G') = |S| = 9$.

Thus, $\gamma_M(G') = 9 = \gamma_M^{-1}(G')$.

Proposition 8. *Let G be Platonic Solid Cube and $S(G)$ be the subdivision of G. Then*

(i) $\gamma_M(G) = 1 = \gamma_M^{-1}(G)$ *and*

(ii) $\gamma_M(G) = 3 = \gamma_M^{-1}(G)$, *if* $G' = S(G)$.

Proof. The proof is obvious.

3. $\gamma_M^{-1}(G)$ for Some Classes of Subdivision Graphs

Proposition 1. *Let $G = S(C_p)$ be a subdivision graph of a cycle with p vertices then* $\gamma_M(G) = \gamma_M^{-1}(G) = \left\lceil \frac{p}{3} \right\rceil$.

Proof. Let $G = S(C_p)$ and $V(G) = \{u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p\}$ and $|V(G)| = 2p$. Then $S(C_p)$ is also a cycle with $2p$ vertices. By the result (1.3) 1 and (3) $\gamma_M(C_p)$ and $\gamma_M^{-1}(C_p) = \left\lceil \frac{p}{6} \right\rceil$, where $|V(C_p)| = p$. Hence $\gamma_M^{-1}(G) = \left\lceil \frac{2p}{6} \right\rceil = \left\lceil \frac{p}{3} \right\rceil = \gamma_M(C_p)$. Thus $\gamma_M(G) = \gamma_M^{-1}(G) = \left\lceil \frac{p}{3} \right\rceil$.

Proposition 2. *If G is a subdivision of a complete graph then* $\gamma_M(G) = 2$

and $\gamma_M^{-1}(G) = 2$.

Proof. Since K_p is complete graph, there are p vertices and $\frac{p(p-1)}{2} = q$ edges. Let $G = S(K_p)$ with $V(G) = \{u_1, \ldots, u_p, v_1, v_2, \ldots, v_q\}$ and $|V(G)| = p' = (p+q)$, where $d(u_i) = p - 1$, for $\forall u_i, \ldots, u_p$ and $d(v_i) = 2$, $\forall f$ or $v_i, \ldots, v_q$, $p' = \frac{p(p+1)}{2}$ and $\left\lceil \frac{p}{2} \right\rceil = \left\lceil \frac{p(p+1)}{4} \right\rceil$. Let $D = \{u_1, u_3\}$ such that $d(u_i, v_j) = 4$. Then $|N[D]| = (p-1) + (p-2) + 2 = 2p - 1$ $|N[D]| = 2p - 1 > \left\lceil \frac{p}{2} \right\rceil$. It implies that D is a MD-set of G and $\gamma_M(G) = |D| = 2$.

Let $D' = \{u_2, u_4\} \subseteq V - D$ such that $d(v_2, v_4) = 4$. Then $|N[D]| = (p-1) + (p-2) + 2 = 2p - 1 > \left\lceil \frac{p}{2} \right\rceil \Rightarrow D'$ is a IMD-set of G and $\gamma_M^{-1}(G) = |D'| = 2$. Hence $\gamma_M(G) = 2 = \gamma_M^{-1}(G)$.

Proposition 3. *Let* $G = S(K_{1,\, p-1})$. *Then* $\gamma_M(G) = 1$ *and* $\gamma_M^{-1}(G) = \left\lceil \frac{p-1}{2} \right\rceil$.

Proof. Let $V(G) = \{u, u_1, \ldots, u_{p-1}, v_1, \ldots, v_{p-1}\}$ where $d(u_i) = 2$, for $\forall u_i$ and v_i's are all pendants, $d(u) = p - 1$ and $|V(G)| = 2p - 1 = p'$. Since u is a MD vertex of G, $\gamma_M(G) = 1$.

Let $D' = \{v_1, \ldots, v_{\left\lceil \frac{p-1}{2} \right\rceil}\} \subseteq V - D$. Then $|N[D']| = 2\left\lceil \frac{p-1}{2} \right\rceil + 1 = 2\left(\frac{p-1}{2}\right) + 1p$ or $p + 1|\, N[D']| = p$ or $p + 1$, if p is odd or even. $|N[D']| \geq \left\lceil \frac{p'}{2} \right\rceil$. It implies that D' is a IMD-set of G. Hence $\gamma_M^{-1}(G) = \left\lceil \frac{p-1}{2} \right\rceil$.

Proposition 4. *Let* $G = S(P_p)$ *be a subdivision graph of a path with*

$p \geq 2$ vertices. Then $\gamma_M^{-1}(G) = \left\lceil \frac{p}{3} \right\rceil$.

Proof. Since the subdivision graph of path P_p is also a path, the result is obvious.

Proposition 5. *For a graph $G = S(W_p)$ be a subdivision graph of a wheel with $p \geq 5$ vertices. Then* $\gamma_M^{-1}(G) = \gamma_M(G) = \left\lceil \frac{p'}{8} \right\rceil$, *where* $p' = (p + q)$.

Proof. Let $G = S(W_p)$ and $V(G) = \{u, u_1, u_2, \ldots, u_{p-1}, v_1, v_2, \ldots, v_q\}$, and $|V(G)| = p + q = p'$, where u is a central vertex, $d(v_i) = 2$, for $\forall v_i, i = 1, \ldots, q$ and $d(u_i) = 3$, for $\forall u_i, i = 1, 2, \ldots, p - 1$. Let $D = \{u_1, u_3, \ldots, u_t\}$ such that $t = \left\lceil \frac{p'}{8} \right\rceil$ and $d(u_i, u_j) \geq 4$, for $i \neq j$. Then $|N[D]| = \sum_{i=1}^{t} d(v_i) + t = 4t = 4\left\lceil \frac{p'}{8} \right\rceil \geq \left\lceil \frac{p'}{2} \right\rceil$. Then D is a MD-set of G and $\gamma_M(G) = |D| = \left\lceil \frac{p'}{8} \right\rceil$. Now, choose the set $D' = \{u_2, u_5, \ldots, u_t\} \subseteq V - D$ Such that $t = \left\lceil \frac{p'}{8} \right\rceil$ and $d(u_i, u_j) \geq 4$, for $i \neq j$. By the above argument, the set D' is a IMD- set of G.

$$\gamma_M^{-1}(G) \leq |D'| = \left\lceil \frac{p'}{8} \right\rceil \tag{1}$$

Suppose $D_1 \subseteq V - D$ be a set with $|D_1| < |D'| = \left\lceil \frac{p'}{8} \right\rceil$. Then $|N[D_1]| < \left\lceil \frac{p'}{2} \right\rceil$. It implies that D_1 is not a IMD-set of G and

$$\gamma_M^{-1}(G) > |D_1| \text{ and } \gamma_M^{-1}(G) \geq \left\lceil \frac{p'}{8} \right\rceil \tag{2}$$

Hence It implies that $\gamma_M^{-1}(G) \geq \left\lceil \frac{p'}{8} \right\rceil$, and $\gamma_M(G) \geq \left\lceil \frac{p^1}{8} \right\rceil$, where $p' = p + q$.

4. Results on $\gamma_M(G')$ and $\gamma_M^{-1}(G')$

Observation 1.

(1) A full degree vertex of a graph becomes a majority dominating vertex in the subdivision graph $S(G)$ of G.

(2) The degree of a vertex of G will never change in the subdivision graph $S(G)$ and the degree of a newly added vertex is always two in $S(G)$.

(3) The regular graph G is not a regular in $S(G)$ except $G = C_p$.

(4) Any pendant edge becomes a path K_2.

Proposition 2. *Let G and G' be the disconnected graph and its Subdivision graph with p and p' vertices respectively. Then* $\gamma_M(G') = \gamma_M^{-1}(G')$.

For Example

(i) Let $G = 5K_2$ By the result, $\gamma_M(G) = \left\lceil \frac{p}{4} \right\rceil$, $\gamma_M(G) = 3 = \gamma_M^{-1}(G) = 3$. Let the subdivision graph $S(G) = G' = 5P_3$ with $p' = 15$. Now, the set $D = \{u_1, u_2, u_3\}$, where $d_G(u_i) = 2$, for $\forall, u_i$ and D is a MD-set of G' and $\gamma_M(G') = 3$. And Now the set $D' = \{u_4, u_5, u_1\} \subseteq V - D$ is a IMD- set of G' and $\gamma_M(G') = \gamma_M^{-1}(G')$.

(ii) Let $G = 5K_3$ with $p = 15$ vertices. Then $D = \{v_1, v_1', v_1''\}$ is a MD-set of G and $\gamma_M^{-1}(G) = \gamma_M(G) = 3$. Let $G' = 5K_6$ be the subdivision graph of G with $p = 30$. Now, the set $D = \{u_1, u_1', u_1'', u_3', v_3\}$ is a MD-set of G' and $\gamma_M(G') = |D| = 5$. Now the set $D' = \{u_2, u_2', u_2'', v_1', v_1\} \subseteq V - D$ is a IMD-set of G' and $\gamma_M(G') = \gamma_M^{-1}(G') = |D'| = 5$. Hence $\gamma_M(G') = \gamma_M^{-1}(G')$.

Proposition 3. *If the graph G is regular then* $\gamma_M(G') = \gamma_M^{-1}(G')$.

Proof. Since G is a regular graph, and G by the result [1.3] (10), $\gamma_M(G) = \gamma_M^{-1}(G)$. Let $G' = S(G)$ be the subdivision graph of G. Then G' is not a regular graph except $G = C_p$, a cycle Since the degree of the vertices of G is equal to the degree of the vertices of G' except the newly added vertex u_i with $d(u_i) = 2$, for $\forall$, $u_i I = 1, 2, \ldots, q$. To get the minimality select the vertices v_i of G with the distance $d(v_i, v_j) \geq 4$ in G' then it will form a minimum MD-set and minimum IMD-set in G' with the same cardinality. Hence, $\gamma_M(G') = \gamma_M^{-1}(G')$.

Proposition 4. *If the graph G contains a full degree vertex and others are pendants then* $\gamma_M(G') < \gamma_M^{-1}(G')$.

Proof. By the observation (4.1) (1), any full degree vertex v of G becomes a majority dominating vertex v of G'. Then $D = \{v\}$ is a majority dominating set of G' and $\gamma_M(G') = 1$. Since G has pendant vertices, $d(u_i) = 2$, for every u_i, $1 = 1, 2, \ldots, q$, and the vertices u_i of $G' \subseteq V - D$ will form a inverse majority dominating set of G' with respect to D. It implies that $\gamma_M^{-1}(G') > 1$. Hence $\gamma_M(G') < \gamma_M^{-1}(G')$.

Proposition 4.5. *If the graph G has exactly one majority dominating vertex and other are pendants then* $\gamma_M(G') > 1$ *and* $\gamma_M(G') < \gamma_M^{-1}(G')$.

Proof. Let $|V(G)| = p$ and $|V((G'))| = p + q = p'$. Since the majority dominating vertex v of degree $d_G(v) \geq \left\lceil \frac{p}{2} \right\rceil - 1$, $d_G(v) < \left\lceil \frac{p}{2} \right\rceil - 1$. Then the majority dominating set D will contain at least two vertices of G'. Hence $\gamma_M(G') = |D| > 1$. Since other vertices u_i are pendants, the newly added vertices $d(u_i) = 2$. Such that for $i = 1, 2, \ldots, q$. Now the IMD-set (D') contains the vertices of $u_i \subseteq V - D$ of G'. It implies that $\gamma_M^{-1}(G') > 2$. Hence $\gamma_M(G') < \gamma_M^{-1}(G')$.

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