STUDY OF MICROPOLAR THERMO-ELASTICITY

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Abstract

This paper illuminates the field of micropolar thermo-elasticity which was examined under different media relating to surface waves. This article particularly focuses on reviewing the extensive work done on Micropolar Thermo-elasticity. The micropolar thermo-elasticity was utilized and applied by numerous researchers to investigate the different impacts and especially for mechanical and temperature. This paper will be of extra ordinary interest for the rising researchers as this article deals with research work from the explorer work done by Eringen, Green and Lindsay, Lord and Shulman and many more are reviewed in detail.

Introduction

The elasticity theory is based on the phenomena of the response of elastic bodies to the action of forces. The elastic body is one which regains its original shape when the applied forces which cause the deformation are removed. When the force is applied on the body, the deformation is produced. Thermo-elasticity deals with the deformation in the materials which are considered as a thermodynamic system, with the alteration in temperature. The temperature in the body is affected by two factors: internal and external heat sources and deformation that take place inside the body. In thermo-elasticity, we study the thermo elastic displacement, thermal stresses, and heat conduction etc.

The thermo elastic problems are two types
(i) Direct thermo elastic problem

(ii) Inverse thermo elastic problem.

Direct thermo elastic problem is one in which temperature and heat transfer conditions are considered on the surface of the body and condition at any point of the body is to be determined, whereas the inverse thermo elastic problem is one which comprises of finding the temperature of the solid under thermal effect and the heat flux of the solid under known conditions for displacement and stresses at some points of the considered solid.

The Classical theory is inadequate to explain certain discrepancies which are mainly related to the problem that involves elastic vibrations which have large frequencies and small wavelengths. The main reason behind this is the microstructure of the materials which has a great effect at large frequencies and small wavelengths. Elastic vibrations which have high frequencies and small wavelengths are found mainly in composites, polymeric suspensions, liquid crystals and granular bodies etc., Some examples of the medium with microstructure are metals, polymers, soils, concrete etc.

Literature Survey

Duhamel [1] formulated a theory in which coupling of thermal and strain fields was taken into the account that results in coupled theory. An analysis of Duhamel's theory was conducted by Neuman [2], Voigt [3] and Jeffreys [4] and they solved many interesting problems in the concerned field. The theory of coupled thermo-elasticity was given by Biot [5] in which the basic equations were derived on the basis of Fourier’s law and using the thermo dynamics of irreversible processes formulated the various theorems of thermo elasticity. Lord and Shulman [6] formulated the first generalization which is also known as L-S theory. Muller [7], developed an entropy production inequality for thermodynamics of thermo elastic solid and used this inequality to apply the constraints to a class of constitutive relations. Green and Lindsay [8] introduced a theory called G-L theory in which, they generalized the constitutive relations for stresses and the entropy by considering two different relaxation times. Another generalization of this inequality was proposed by Green and Laws [9]. Suhubi [10] also obtained these constitutive relations in an explicit manner. In recent years the
Generalized thermo-elasticity theories was studied by various researchers Green and Nagdhi [11, 12], Sharma et al. [13], Othman [14], Ailawalia and Narah [15] and many others using some additional parameters in different mediums.

To eliminate the drawback of classical theory, Voigt [16] introduced a theory of micro-mechanics continuum in which he assumed that interactions within pair of material particles through an element of an area in the interior of the body is transmitted by both the force vector as well as the moment vector, which introduced the concept of couple stress in elasticity. This new theory was termed as “Couple Stress Theory”. After the introduction of couple theory, Cosserat and Cosserat [17] gave a unified theory which is based on the concept that during the deformation process, the material particles in addition to linear displacement, can rotate independently, thus introducing the concept of rotation. This theory proposed by Cosserat brothers was named as Cosserat theory of elasticity. This Cosserat theory did not get much attention for many years, may be due to the non-linear nature of the theory. After that many researchers developed Cosserat type theories independently. The detailed study of one dimensional, two dimensional and three-dimensional Cosserat model of the continuum was conducted by Gunther [18] and shown that Cosserat theory has a great significance in the dislocation problems. Various researchers like Grioli [19], Truesdell and Toupin [20], Mindlin and Tiersten [21] and Eringen [22] investigated the idea of Cosserat continuum in a special case named as the indeterminate couple-stress theory. The main limitation of the Cosserat theory of elasticity is that the microrotation is not considered as an independent vector.

By considering the law of conservation of microinertia, Eringen [23] developed the theory of simple micro fluids. Eringen and Suhubi [24] and Suhubi and Eringen [25] formulated a general theory of non-linear micro elastic continuum in which they supplemented the balance laws of continuum mechanics and the intrinsic motions of the microelement contained in a macro-volume were taken into consideration. In a similar way Mindlin [26] developed a theory of microstructure. Green and Rivlin [27] developed a multipolar continuum theory which in special cases appears to have similarities with the theory formulated by Eringen and Suhubi [24]. A micromorphic continuum consists of materials which possess classical motion.
as well as deformation, here the deformation is supposed to be affine. The theory developed by Eringen and Suhubi [24] was later renamed as the theory of micromorphic continuum by Eringen [28]. As a special case theory of micromorphic continuum contains both the indeterminate couple stress theory and Cosserat continuum theory. In his subsequent papers, Eringen [29, 30] simplified theory of micromorphic continuum and presented new term “micropolar elasticity”. In micropolar elasticity, the body was assumed to be consisting of interconnected material particles like small rigid bodies that can undergo both translational motion and rotational motion. In case of micro-isotropic solids, the general solutions for the micropolar elasticity obtained by Smith [31], Chiu and Lee [32] and Nowacki [33]. Later on, the micropolar elasticity was studied by Minagawa et. al. [34], Gauthier [35], Sládek and Sládek [36], Eringen [37], Scarpetta [38] and Singh and Kumar [39]. Kumar and Choudhary [40] investigated the time harmonic concentrated source in an orthotropic micropolar elastic solid. Kaur et. al. [41] discussed a problem on micropolar half-space with irregularity under dynamic moving load. Svanadze [42] obtained the solutions in case of the linear theory of micropolar viscoelasticity.

The thermal effects in the theory of micropolar materials were introduced by Nowacki [43] and Eringen [44] and named this theory as micropolar coupled thermo elasticity theory. This theory consists of conduction equation and stress-strain which is produced under thermal effects i.e. the effect of heat. Tauchert et al. [45] formulated the basic equations of the linear theory of micropolar thermo elasticity in which they formulated the constitutive equations, displacement components, microrotation and couple stress. Tauchert [46] derived a couple of general solutions of micropolar thermo elasticity theory, one using potentials and other using stress functions. Shanker and Dhaliwal [47] investigated the general solution of the dynamic micropolar coupled thermoelastic equations for an infinite body. Sladek and Sladek [48] investigated the micropolar thermo elasticity using boundary element method in the Laplace transform domain. Chandrasekharaih [49] developed a micropolar thermo elasticity in which constitutive variables are dependent on heat flux and deduced energy balance equation and uniqueness theorem for anisotropic materials. Dhaliwal and Singh [50] did a comprehensive study in the theory of micropolar thermo elasticity.

Kumar and Deswal [60] applied Laplace-Fourier transform techniques to study a problem on micropolar generalized thermo elastic solid under thermal and mechanical sources. Kumar et al. [61] investigated the effect of the thermal and mechanical source in a micropolar thermo elastic medium using eigen value approach. Tianmin [62] conducted a restudy of coupled field theories for micropolar thermo elasticity and derived the basic principle of micropolar thermo elasticity. Ciarletta and Scalia [63] investigated the behavior of thermo elastic microstretch continuum material. Svanadze [64] obtained the basic solutions of equations of equilibrium for micromorphic elastic solids with microtemperatures using elementary functions.

Sherief et al. [65] applied Laplace and Hankel transform techniques to study a problem of generalized micropolar thermo elasticity under an axisymmetric thermal shock. Kumar and Ailawalia [66] investigated the influence of time harmonic sources in case of micropolar thermo elastic solid having cubic symmetry under L-S theory. Kumar and Ailawalia [67] applied integral transforms techniques to investigate the effect of various sources in a micropolar thermo elastic solid possessing cubic symmetry under G-N theory. Kumar and Ailawalia [68] studied the influence of inclined load for the free surface of the micropolar thermo elastic medium which possesses cubic

Othman et al. [80] investigated a plane problem of rotating micropolar thermo elastic isotropic medium with two temperatures using DPL model. Bitsadze and Jaiani [81] discussed the two-dimensional problems of thermo elasticity with micro temperatures. Singh and Kumar [82] applied Laplace and Hankel transformation techniques to solve a problem on generalized thermo elastic medium with microstretch under the influence of mechanical source. Lotfy et. al. [83] investigated a problem in a micropolar thermo elastic solid which possess cubic symmetry for a mode-I crack under C-D, L-S, and G-L theories respectively.

Abbas et al. [84] investigate the effect of the normal, tangential and thermal source in a problem of micropolar thermo elastic solid with void by applying finite element method under C-D, L-S and G-L theories respectively. Othman et al. [85] investigated a problem on thermo elastic solid with voids
and micro-temperatures under the influence of initial stress by applying normal mode technique. Kumar et al. [86] studied the influence of Hall current in a rotating magneto-micropolar fractional order thermo elastic solid under ramp-type heating. Othman et al. [87] studied the influence of Hall current and gravity on the magneto-micropolar thermo elastic medium with micro-temperatures. Said et al. [88] investigated the effect of magnetic field in a problem of two-temperature micropolar thermo elastic medium with rotation under L-S and G-L theories. Partap and Chugh [89] investigated a problem of the deflection and thermo elastic damping analysis in microstretch thermo elastic rectangular plate. Othman and Abd-Elaziz [90] investigated the influence of gravitational field on a rotating micropolar magneto-thermo elastic solid in DPL model. Kumar et al. [91] discussed the propagation of waves in micropolar thermodiffusion elastic half-space.

Nomenclature

\( t_{kl} \) = stress tensor
\( e \) = mass density,
\( u_k \) = displacement vector,
\( \varepsilon_{klr} \) = alternating tensor,
\( l_k \) = body couple per unit mass,
\( \varepsilon \) = internal energy density,
\( \delta_{kl} \) = Kronecker delta,
\( f_i \) = body force per unit mass,
\( \varphi_k \) = microrotation vector,
\( j \) = microinertia,
\( v_k = \dot{u}_j \), \( v_k = \dot{\varphi}_k \),
\( \lambda, \mu, \chi, \alpha, \beta, \gamma \) = elastic constants.

Basic Equations

1. Balance of Momentum

\[
t_{kl,k} + e(f_i - \ddot{u}) = 0. \tag{1}
\]
2. Balance of moment of momentum
\[ m_{r,k} + \varepsilon_{k,lr} t_{ir} + \varepsilon(l_k - j\phi_k) = 0. \] (2)

3. Conservation of energy
\[ \varepsilon = t_{kl}(v_{l,k} - \varepsilon_{k,lr} v_r) + m_{kl}v_{l,k} \] (3)

4. Constitutive equations
\[ t_{kl} = \lambda u_{r,rl} \delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \chi(u_{l,k} - \varepsilon_{k,lr}\phi_r) \]  
\[ m_{kl} = \alpha\phi_{r,rl} \delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}. \] (4)

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