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# INTUITIONISTIC FUZZY SEMI $\gamma^*$ GENERALIZED CONNECTEDNESS

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**Abstract**

In this paper, we have introduced the basic concepts of intuitionistic fuzzy semi $\gamma^*$ generalized connected space, intuitionistic fuzzy semi $\gamma^*$ generalized super connected space and intuitionistic fuzzy semi $\gamma^*$ generalized extremally disconnected space and discussed some of their properties.

## 1. Introduction

The concept of fuzzy sets was introduced by Zadeh [10] in 1965 and later Atanassov [3] generalized this idea to intuitionistic fuzzy sets. On the other hand Coker [4] has introduced intuitionistic fuzzy topological spaces. Connectedness in intuitionistic fuzzy topological spaces was introduced by Ozcag and Coker [6]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and coker [9]. Later Riya and Jayanthi [7] introduced intuitionistic fuzzy $\gamma^*$ generalized connected

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space in 2018. In this paper, we have introduced intuitionistic fuzzy semi $\gamma^*$ generalized connected space, intuitionistic fuzzy semi $\gamma^*$ generalized super connected space and intuitionistic fuzzy semi $\gamma^*$ generalized extremally disconnected space and discussed some of their properties.

## 2. Preliminaries

**Definition 2.1** [3]**.** An intuitionistic fuzzy set (IFS) $A$ is an object having the form

$$A = \{\langle x, \mu_A(x), \upsilon_A(x) \rangle x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ and $\upsilon_A : X \to [0, 1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in $X$. An intuitionistic fuzzy set $A$ in $X$ is simply denoted by $A = \langle x, \mu_A, \upsilon_A \rangle$ instead of denoting

$$A = \{\langle x, \mu_A(x), \upsilon_A(x) \rangle : x \in X\}.$$

**Definition 2.2** [3]**.** Let $A$ and $B$ be two IFSs of the form $A = \{\langle x, \mu_A(x), \upsilon_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \upsilon_B(x) \rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\upsilon_A(x) \geq \upsilon_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \upsilon_A(x), \mu_A(x) \rangle : x \in X\}$,

(d) $A \bigcup B = \{\langle x, \mu_A(x) \vee \mu_B(x) \wedge \upsilon_B(x) \rangle : x \in X\}$,

(e) $A \bigcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x) \vee \upsilon_B(x) \rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1 \rangle$ and $1_\sim = \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of $X$.

**Definition 2.3** [4]**.** An intuitionistic fuzzy topology (IFT) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$,

In this case the pair $(X, \tau)$ is called the intuitionistic fuzzy topological space (IFTS) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS) in $X$. The complement $A^c$ of an IFOS A in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS) in $X$.

**Definition 2.4** [5]**.** An IFS $A = \langle x, \mu, v_A \rangle$ in an IFTS $(X, \tau)$ is said to be an

(i) *intuitionistic fuzzy γ closed set* $(IF_\gamma CS)$ if $cl(\text{int}(A)) \cap \text{int}(cl(A)) \subseteq A$,

(ii) *intuitionistic fuzzy γ open set* $(IF_\gamma OS)$ if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$.

**Definition 2.5** [1]**.** An IFS $A$ of an IFTS $(X, \tau)$ is said to be an *intuitionistic fuzzy semi* $\gamma^*$ *generalized closed set* (IF semi $\gamma^* GCS$) if $\text{int}(cl(A)) \cap cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFSOS in $(X, \tau)$.

**Definition 2.6** [2]**.** A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy semi* $\gamma^*$ *generalized* (IF semi $\gamma^* G$) *continuous mapping* if $f^{-1}(V)$ is an IF semi $\gamma^* GCS$ in $(X, \tau)$ for every IFCS $V$ of $(X, \sigma)$.

**Definition 2.7** [1]**.** An IFTS $(X, \tau)$ is an intuitionistic fuzzy semi $\gamma_c^* T_{1/2}$ (IF semi $\gamma_c^* T_{1/2}$) space if every IF semi $\gamma^* GCS$ is an IFCS in $X$.

**Definition 2.8** [8]**.** Two IFSs $A$ and $B$ are said to *be q-coincident* $(A_q B)$ if and only if there exits an element $x \in X$ such that $\mu_A(x) > v_B(x)$ or $v_A(x) < \mu_B(x)$.

**Definition 2.9** [8]**.** Two IFSs $A$ and $B$ are said to be not *q-coincident* $(A_{q^c} B)$ if and only if $A \subseteq B^c$.

**Definition 2.10** [9]. An IFTS $(X, \tau)$ is said to be an *intuitionistic fuzzy* $C_5$-connected space if the only IFSs which are both an IFOS and an IFCS are $0_\sim$ and $1_\sim$.

**Definition 2.11** [9]. An IFTS $(X, \tau)$ is said to be an *intuitionistic fuzzy GO-connected* space if the only IFSs which are both an IFGOS and an IFGCS are $0_\sim$ and $1_\sim$.

**Definition 2.12** [6]. An IFTS $(X, \tau)$ is an intuitionistic fuzzy $C_5$-connected between two IFSs $A$ and $B$ if there is no IFOS $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E_{q^c}B$.

## 3. Intuitionistic Fuzzy Semi $\gamma^*$ Generalized Connected Spaces

In this section we introduce intuitionistic fuzzy semi $\gamma^*$ generalized connected space and investigate some of their properties.

**Definition 3.1.** An IFTS $(X, \tau)$ is said to be an *intuitionistic fuzzy semi* $\gamma^*$ *generalized* (IF semi $\gamma^*G$) connected space if the only IFSs which are both IF semi $\gamma^*$ GOS and IF semi $\gamma^*GCS$ are $0_\sim$ and $1_\sim$.

**Theorem 3.2.** *Every IF semi $\gamma^*G$ connected space is an $IFC_5$-connected space but not conversely in general.*

**Proof.** Let $(X, \tau)$ be an IF semi $\gamma^*G$ connected space. Suppose $(X, \tau)$ is not an $IFC_5$-connected space, then there exists a proper IFS $A$ which is both an IFOS and an IFCS in $(X, \tau)$. That is $A$ is both an IF semi $\gamma^*GOS$ and an IF semi $\gamma^*GCS$ in $(X, \tau)$. This implies that $(X, \tau)$ is not an IF semi $\gamma^*G$ connected space. This is a contradiction. Therefore $(X, \tau)$ must be an $IFC_5$-connected space.

**Example 3.3.** Let $X = \{a, b\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on $X$, where $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.5_b), (0.4_a, 0.4_b)\rangle$.

Here,

$IFSO(X) = \{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], v_b \in [0, 1] /$ whenever $0.3 \leq \mu_a \leq 0.4, 0.2 \leq \mu_b \leq 0.4$ and $0.6 \leq v_a \leq 0.7, 0.5 \leq v_b \leq 0.8$, whenever $0.6 \leq \mu_a \leq 0.7, 0.5 \leq \mu_b \leq 0.8$, and $0.3 \leq v_a \leq 0.4, 0.2 \leq v_b \leq 0.4$ and $0 \leq \mu_a + v_a \leq 1$ and $0 \leq \mu_b + v_b \leq 1\}$.

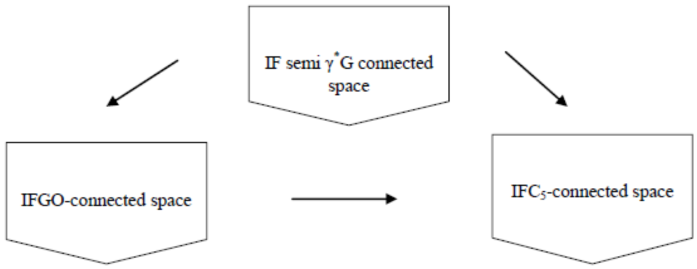
Then $(X, \tau)$ is an $IFC_5$-connected space but not an IF semi $\gamma^* G$ connected space, since the IFS $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.4_b)\rangle$ is both an IF semi $\gamma^* G$ open and an IF semi $\gamma^* G$ closed set in $(X, \tau)$.

**Theorem 3.4.** *Every IF semi $\gamma^* G$ connected space is an IFGO connected space but not conversely in general.*

**Proof.** Let $(X, \tau)$ be an IF semi $\gamma^* G$ connected space. Suppose $(X, \tau)$ is not an IFGO-connected space, then there exists a proper IFS A which is both an IFGOS and an IFGCS in $(X, \tau)$. That is $A$ is both an IF semi $\gamma^* GOS$ and an IF semi $\gamma^* GCS$ in $(X, \tau)$. This implies that $(X, \tau)$ is not an IF semi $\gamma^* G$ connected space, a contradiction. Therefore $(X, \tau)$ must be an IFGO-connected space.

**Example 3.5.** In Example 3.3,2 $(X, \tau)$ is an IFGO connected space but not an IF semi $\gamma^* G$ connected space, since the IFS $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is both IF semi $\gamma^* G$ open and IF semi $\gamma^* G$ closed in $(X, \tau)$.

The relation between various types of intuitionistic fuzzy connectedness is given in the following diagram.




In the above diagram the reverse implications are not true in general.

**Theorem 3.6.** *The IFTS* $(X, \tau)$ *is an IF semi* $\gamma^* G$ *connected space if and only if there exists no non-zero IF semi* $\gamma^* G$ *open sets A and B in* $(X, \tau)$ *such that* $A = B^c$.

**Proof. Necessity:** Let $A$ and $B$ be two IF semi $\gamma^* GOSs$ in $(X, \tau)$ such that $A \neq 0_\sim$, $B \neq 0_\sim$ and $A = B^c$. Therefore $A = B^c$ is an IF semi $\gamma^* GCS$. Since $B \neq 0_\sim$, $A = B^c \neq 1_\sim$. Hence $A$ is a proper IFS which is both IF semi $\gamma^* GOS$ and IF semi $\gamma^* GCS$ in $(X, \tau)$. Hence $(X, \tau)$ is not an IF semi $\gamma^* G$ connected space. But this is a contradiction to our hypothesis. Hence there exists no non-zero IF semi $\gamma^* GOSs$ $A$ and $B$ in $(X, \tau)$ such that $A = B^c$.

**Sufficiency.** Suppose $(X, \tau)$ is not an IF semi $\gamma^* G$ connected space. Then there exists an IFS A which is both an IF semi $\gamma^* GOS$ and an IF semi $\gamma^* GCS$ with $0_\sim \neq A \neq 1_\sim$. Now let $B = A^c$. Then $B$ is an IF semi $\gamma^* GOS$ and $B \neq A^c$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Hence $(X, \tau)$ is an IF semi $\gamma^* G$ connected space.

**Theorem 3.7.** *Let* $(X, \tau)$ *be an IF semi* $\gamma_c^* T_{1/2}$ *space, then the following are equivalent*:

(i) $(X, \tau)$ *is an IF semi* $\gamma^* G$ *connected space*,

(ii) $(X, \tau)$ *is an IFGO connected space*,

(iii) $(X, \tau)$ *is an* $IFC_5$- *connected space.*

**Proof.** (i) $\Rightarrow$ (ii) is obvious from the Theorem 3.4.

(ii) $\Rightarrow$ (iii) is obvious.

(iii) $\Rightarrow$ (i) Let $(X, \tau)$ be an $IFC_5$- connected space. Suppose $(X, \tau)$ is not an IF semi $\gamma^*G$ connected space, then there exists a proper IFS A in $(X, \tau))$ which is both an IF semi $\gamma^*GOS$ and an IF semi $\gamma^*GCS$ in $(X, \tau)$. But since $(X, \tau)$ is an IF semi $\gamma_c^*T_{1/2}$ space, $A$ is both an IFOS and an IFCS in $(X, \tau)$. This implies that $(X, \tau)$ is not an $IFC_5$- connected space, which is a contradiction to our hypothesis. Therefore $(X, \tau)$ must be an IF semi $\gamma^*G$ connected space.

**Theorem 3.8.** *If* $f : (X, \tau) \to (Y, \sigma)$ *is an IF semi* $\gamma^*G$ *continuous mapping and* $(X, \tau)$ *is an IF semi* $\gamma^*G$ *connected space, then* $(Y, \sigma)$ *is an* $IFC_5$- *connected space.*

**Proof.** Let $(X, \tau)$ be an IF semi $\gamma^*G$ connected space. Suppose $(Y, \sigma)$ is not an IFC5-connected space, then there exists a proper IFS A which is both an IFOS and an IFCS in $(Y, \sigma)$. Since f is an IF semi $\gamma^*G$ continuous IF semi $\gamma^*G$ connected space IFGO-connected space $IFC_5$- connected space mapping, $f^{-1}(A)$ is both a proper IF semi $\gamma^*GOS$ and an IF semi $\gamma^*GCS$ in $(X, \tau)$. But it is a contradiction to our hypothesis. Hence $(Y, \sigma)$ must be an $IFC_5$- connected space.

**Definition 3.9.** An IFTS $(X, \tau)$ is an IF semi $\gamma^*G$ connected between two IFSs $A$ and $B$ if there is no IF semi $\gamma^*GOS$ $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E_{q^c}B$.

**Example 3.10.** Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on $X$, where $G = \langle x, (0.6_a, 0.5_b), (0.4_a, 0.4_b)\rangle$. Then, the IFTS $(X, \tau)$ is an IF semi

$\gamma^{*}G$ connected between the two IFSs $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $B = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ as there exists no IF semi $\gamma^{*}GO$ $E$ such that $A \subseteq E$ and $E_{q^c}B$.

**Theorem 3.11.** *If an IFTS $(X, \tau)$ is an IF semi $\gamma^{*}G$ connected between two IFSs A and B, then it is $IFC_5$-connected between A and B but the converse may not be true in general.*

**Proof.** Suppose $(X, \tau)$ is not $IFC_5$-connected between $A$ and $B$, then there exists an IFOS E in $(X, \tau)$ such that $A \subseteq E$ and $E_{q^c}B$. Since every IFOS is an IF semi $\gamma^{*}GOS$, there exists an IF semi $\gamma^{*}GOS$ E in $(X, \tau)$ such that A ⊆ E and $E_{q^c}B$. This implies $(X, \tau)$ is not IF semi $\gamma^{*}G$ connected between $A$ and $B$. Thus we get a contradiction to our hypothesis. Therefore the IFTS $(X, \tau)$ must be $IFC_5$-connected between $A$ and $B$.

**Example 3.12.** Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on $X$, where $G = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Then $(X, \tau)$ is $IFC_5$-connected between the IFSs $A = \langle x, (0.1_a, 0.2_b), (0.7_a, 0.7_b)\rangle$ and $B = \langle x, (0.7_a, 0.7_b), (0.3_a, 0.3_b)\rangle$. Here,

$IFSO(X) = \{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], v_a \in [0, 1]/0.4 \le \mu_a \le 0.6,$
$0.3 \le \mu_b \le 0.7, v_a \le 0.4, v_b \le 0.3$ and $0 \le \mu_a + v_b \le 1$ and $0 \le \mu_b + v_b \le 1\}$,

and $(X, \tau)$ is not IF semi $\gamma^{*}G$ connected between $A$ and $B$, since the IFS $E = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.7_b)\rangle$ is an IF semi $\gamma^{*}GOS$ such that $A \subseteq E$ and $E \subseteq B^c$.

**Theorem 3.13.** *An IFTS $(X, \tau)$ is IF semi $\gamma^{*}G$ connected between two IFSs A and B if and only if there is no IF semi $\gamma^{*}GOS$ and IF semi $\gamma^{*}GCS$ $E$ in $(X, \tau)$ such that $A \subseteq E \subseteq B^c$.*

**Proof. Necessity:** Let $(X, \tau)$ be IF semi $\gamma^{*}G$ connected between two

IFSs $A$ and $B$. Suppose that there exists an IF semi $\gamma^*GOS$ and IF semi $\gamma^*GCS$ $E$ in $(X, \tau)$ such that $A \subseteq E \subseteq B^c$, then $E_{q^c}B$ and $A \subseteq E$. This implies $(X, \tau)$ is not IF semi $\gamma^*G$ connected between $A$ and $B$, a contradiction to our hypothesis. Therefore there is no IF semi $\gamma^*GOS$ and an IF semi $\gamma^*GCS$ $E$ in $(X, \tau)$ such that $A \subseteq E \subseteq B^c$.

**Sufficiency:** Suppose that $(X, \tau)$ is not IF semi $\gamma^*G$ connected between $A$ and $B$. Then there exists an IF semi $\gamma^*GOS$ $E$ in $(X, \tau)$ such that $A \subseteq E$ and E qc B. This implies that there is an IF semi $\gamma^*GOS$ $E$ in $(X, \tau)$ such that $A \subseteq E \subseteq B^c$. Hence $(X, \tau)$ is IF semi $\gamma^*G$ connected between $A$ and $B$.

**Theorem 3.14.** *If an IFTS $(X, \tau)$ is IF semi $\gamma^*G$ connected between A and B and $A \subseteq A_1$, $B \subseteq B_1$, then $(X, \tau)$ is an IF semi $\gamma^*G$ connected between $A_1$ and $B_1$.*

**Proof.** Suppose that $(X, \tau)$ is not IF semi $\gamma^*G$ connected between $A_1$ and $B_1$, then by Definition 3.9, there exists an IF semi $\gamma^*GOS$ $E$ in $(X, \tau)$ such that $A_1 \subseteq E$ and $Eq^cB_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$. That is $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1^c \subseteq E^c$. That is $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore $(X, \tau)$ is not IF semi $\gamma^*G$ connected between $A$ and $B$, which is a contradiction to our hypothesis. Hence $(X, \tau)$ must be IF semi $\gamma^*G$ connected between $A_1$ and $B_1$.

**Theorem 3.15.** *Let $(X, \tau)$ be an IFTS and A and B be IFSs in $(X, \tau)$. If $A_qcB$. then $(X, \tau)$ is IF semi $\gamma^*G$ connected between A and B.*

**Proof.** Suppose $(X, \tau)$ is not an IF semi $\gamma^*G$ connected between $A$ and $B$. Then there exists an IF semi $\gamma^*GOS$ $E$ in $(X, \tau)$ such that $A \subseteq E$ and E $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is $A_qcB$. But this is a contradiction to our hypothesis. Hence $(X, \tau)$ must be IF semi $\gamma^*G$ connected between $A$ and $B$.

**Theorem 3.16.** *An IFTS $(X, \tau)$ is an IF semi $\gamma^*G$ connected space if and only if there exists no non-zero IF semi $\gamma^*G$ open sets A and B in $(X, \tau)$ such that $B = A^c$, $B = (\gamma cl(A))^c$, $A = (\gamma cl(B))^c$.*

**Proof. Necessity:** Assume that there exist IFSs $A$ and $B$ such that $A \neq 0_\sim \neq B$, $B = A^c$, $B = (\gamma cl(A))^c$, $A = (\gamma cl(B))^c$. Since $(\gamma cl(A))^c$ and $(\gamma cl(B))^c$ are IF $\gamma$ open sets in $(X, \tau)$, $A$ and $B$ are IF semi $\gamma^*G$ open sets in $(X, \tau)$. This implies $(X, \tau)$ is not an IF semi $\gamma^*G$ connected space, which is a contradiction. Therefore there exists no non-zero IF semi $\gamma^*G$ open sets $A$ and $B$ in $(X, \tau)$ such that $B = A^c$, $B = (\gamma cl(A))^c$, $A = (\gamma cl(B))^c$.

**Sufficiency:** Let $A$ be both an IF semi $\gamma^*GOS$ and an IF semi $\gamma^*GCS$ in $(X, \tau)$ such that $1_\sim \neq A \neq 0_\sim$. Now by taking $B = A^c$ we obtain a contradiction to our hypothesis. Hence $(X, \tau)$ is an IF semi $\gamma^*G$ connected space.

**Definition 3.17.** An IFS $A$ in $(X, \tau)$ is called an intuitionistic fuzzy regular semi $\gamma^*$ generalized open set (IFR semi $\gamma^*GOS$) if $A = s\gamma^*\text{gint}(s\gamma^*gcl(A))$. The complement of an IFR semi $\gamma^*GOS$ is called an intuitionistic fuzzy regular semi $\gamma^*$ generalized closed set (IFR semi $\gamma^*GCS$) in $(X, \tau)$.

**Definition 3.18.** An IFTS $(X, \tau)$ is called an intuitionistic fuzzy semi $\gamma^*$ generalized (IF semi $\gamma^*G$) super connected space if there exists no proper IFR semi $\gamma^*GOS$ in $(X, \tau)$.

**Theorem 3.19.** *Let $(X, \tau)$ be an IFTS, then the following are equivalent*:

(1) $(X, \tau)$ *is an IF semi* $\gamma^* G$ *super connected space,*

(2) *For every non-zero IFR semi* $\gamma^* GOS$ $A$, $s\gamma^* gcl(A) = 1_{\sim}$.

(3) *For every IFR semi* $\gamma^* GCS$ $A$ *with* $A \neq 1_{\sim}$, $s\gamma^* gcl(A) = 1_{\sim}$.

(4) *There exists no IFR semi* $\gamma^* GCSs$ $A$ *and* $B$ *in* $(X, \tau)$ *such that* $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$.

(5) *There exists no IFR semi* $\gamma^* GCSs$ $A$ *and* $B$ *in* $(X, \tau)$ *such that* $A \neq 0_{\sim} \neq B$, $B = (s\gamma^* gcl(A))^c$, $A = (s\gamma^* gcl(B))^c$.

(6) *There exists no IFR semi* $\gamma^* GCSs$ $A$ *and* $B$ *in* $(X, \tau)$ *such that* $A \neq 1_{\sim} \neq B$, $B = (s\gamma^* g\, \text{int}(A))^c$, $A = (s\gamma^* gcl(B))^c$.

**Proof.** (1) $\Rightarrow$ (2) Let $A \neq 0_{\sim}$ be an IFR semi $\gamma^* GOS$ in $X$ and $B = s\gamma^* g\, \text{int}(s\gamma^* gcl(A))^c$. Now let $B = s\gamma^* \text{gint}(s\gamma^* gcl(A))^c$. Then $B$ is a proper IFR semi $\gamma^* GOS$ in $(X, \tau)$. But this is a contradiction to the fact that $(X, \tau)$ is an IF semi $\gamma^* G$ super connected space. Therefore $s\gamma^* gcl(A) = 1_{\sim}$.

(2) $\Rightarrow$ (3) Let $A \neq 1_{\sim}$ be an IFR semi $\gamma^* GCS$ in $(X, \tau)$. If $B = A^c$, then $B$ is an IFR semi $\gamma^* GOS$ in $(X, \tau)$ with $B \neq 0_{\sim}$. Hence $s\gamma^* gcl(B) = 1_{\sim}$, by hypothesis. This implies $(s\gamma^* gcl(B))^c = 0_{\sim}$. That is $s\gamma^* \text{gint}(B^c) = 0_{\sim}$. Hence $s\gamma^* \text{gint}(A) = 0_{\sim}$.

(3) $\Rightarrow$ (4) Suppose $A$ and $B$ be two IFR semi $\gamma^* GCSs$ in $(X, \tau)$ such that $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$. Since $B^c$ is an IFR semi $\gamma^* GCS$ in $(X, \tau)$ and $B \neq 0_{\sim}$ implies $B^c \neq 1_{\sim}$, $B^c = s\gamma^* gcl(s\gamma^* \text{gint}(B^c))$ and we have $s\gamma^* \text{gint}(B^c) = 0_{\sim}$. But $A \subseteq B^c$. Therefore $0_{\sim} \neq A = s\gamma^* \text{gint}(s\gamma^* gcl(A))$ $\subseteq s\gamma^* \text{gint}(s\gamma^* \text{gint}(B^c) = s\gamma^* \text{gint}(s\gamma^* gcl(s\gamma^* gcl(s\gamma^* \text{gint}(B^c)))) = s\gamma^* \text{gint}(s\gamma^* gcl$ $(s\gamma^* \text{gint}(B^c))) = s\gamma^* \text{gint}(B^c) = 0_{\sim}$. Hence it is a contradiction. Therefore (4) is

true.

(4) $\Rightarrow$ (1) Suppose $0_{\sim} \neq A \neq 1_{\sim}$ be an IFR semi $\gamma^{*}GCSs$ in $(X, \tau)$. If we take $B = (s\gamma^{*}gcl(A))^{c}$, then $B$ is an IFR semi $\gamma^{*}GOS$ since $s\gamma^{*}\text{gint}(s\gamma^{*}gcl(B)) = s\gamma^{*}gcl\,(s\gamma^{*}gcl(s\gamma^{*}gcl(A))^{c}) = s\gamma^{*}\text{gint}(s\gamma^{*}\text{gcl}\,(\text{A})))^{c}$ $= s\gamma^{*}\text{gint}(A^{c}) \; = (s\gamma^{*}gcl\,(A))^{c} = B$. Also we get $B \neq 0_{\sim}$, since otherwise, if $B = 0_{\sim}$, this implies $(s\gamma^{*}gcl(A))^{c} = 0_{\sim}$. That is $s\gamma^{*}gcl(A) = 1_{\sim}$. Hence $A = s\gamma^{*}\text{gint}(s\gamma^{*}gcl(A)) = s\gamma^{*}\text{gint}\,(1_{\sim}) = 1_{\sim}$, which is a contradiction. Therefore $B = 0_{\sim}$, and $A \subseteq B^{c}$. But this is a contradiction to (4). Therefore $(X, \tau)$ is an IF semi $\gamma^{*}G$ super connected space.

(1) $\Rightarrow$ (5) Suppose $A$ and $B$ are any two IFR semi $\gamma^{*}GCSs$ in $(X, \tau)$ such that $A \neq 0_{\sim} \neq B$, $B = (s\gamma^{*}gcl(A))^{c}$ and $A = (s\gamma^{*}gcl(B))^{c}$. Now we have $s\gamma^{*}\text{gint}(s\gamma^{*}gcl(A)) = s\gamma^{*}\text{gint}(B^{c}) = (s\gamma^{*}gcl(B))^{c} = A$, $A \neq 0_{\sim}$ and $A \neq 1_{\sim}$, since if $A \neq 1_{\sim}$, then $1_{\sim} = (s\gamma^{*}gcl(B))^{c}$. This implies $s\gamma^{*}gcl(B)$ $= 0_{\sim} \Rightarrow B = 0_{\sim}$. But $B = 0_{\sim}$, Therefore $A \neq 1_{\sim}$. Hence $A$ is a proper IFR semi $\gamma^{*}GOS$ in $(X, \tau)$, which is a contradiction to (1). Hence (5) is true.

(5) $\Rightarrow$ (1) Suppose $A$ is an IFR semi $\gamma^{*}GOS$ in $(X, \tau)$ such that $0_{\sim} \neq A \neq 1_{\sim}$. Now take $B = (s\gamma^{*}gcl(A))^{c}$. In this case we get $B = 0_{\sim}$ and $B$ is an IFR semi $\gamma^{*}GOS$ in $(X, \tau)$, $B = (s\gamma^{*}gcl(A))^{c}$ and $(s\gamma^{*}gcl(B))^{c}$ $= (s\gamma^{*}gcl(s\gamma^{*}gcl(A)^{c}) = s\gamma^{*}\text{gint}\,(s\gamma^{*}gcl(A)) = A$. But this is a contradiction to (5). Therefore $(X, \tau)$ is an IF semi $\gamma^{*}G$ super connected space.

(5) $\Rightarrow$ (6) Suppose $A$ and $B$ be two IFR semi $\gamma^{*}GCSs$ in $(X, \tau)$ such that $A \neq 1_{\sim} \neq B$, $B = (s\gamma^{*}\text{gint}(A))^{c}$ and $A = (s\gamma^{*}\text{gint}(B))^{c}$. Taking $C = A^{c}$ and $D = B^{c}$, $C$ and $D$ become IFR semi $\gamma^{*}GCSs$ in $(X, \tau)$ with $C \neq 0_{\sim} \neq D$, $D = (s\gamma^{*}gcl(C))^{c} = (s\gamma^{*}gcl(D))^{c}$, which is a contradiction to (5). Hence (6) is

true.

(6) $\Rightarrow$ (5) It can be proved easily by the similar way as in (5) $\Rightarrow$ (6).

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