Advances and Applications in Mathematical Sciences
Volume 22, Issue 7, May 2023, Pages 1679-1687
ISSN 0974-6803


# ON SOLVING FULLY FUZZY NEUTROSOPHIC LINEAR COMPLEMENTARITY PROBLEM

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## Abstract

This paper presents a new approach to solve Fully Fuzzy Neutrosophic Linear Complementarity Problem (FFNLCP). The neutrosophic linear complementarity problem models with their parameters represented by trapezoidal neutrosophic numbers are presented here. Two ranking functions are introduced according to the problem type, for converting neutrosophic linear complementarity problem to crisp linear complementarity problem. The effectiveness of the proposed model is applied to both maximization and minimization problems.

## 1. Introduction

The concept of neutrosophic set (NS) was first introduced by Smarandache [5] which is a generalisation of classical sets, fuzzy set, intuitionistic fuzzy set etc. The linear complementarity problem (LCP) is a well-known problem in mathematical programming and it has been studied

2020 Mathematics Subject Classification: 90C05, 90C29, 90C33, 90C70.

Keywords: linear complementarity problem, neutrosophic linear complementarity programming problem, ranking function.

Received April 2, 2022; Accepted May 24, 2022

by many researchers. In 1968, Lemke [8] proposed a complementary pivoting algorithm for solving linear complementarity problem and Katta. G. Murthy in [7]. Fuzzy systems and intuitionistic fuzzy systems cannot successfully deal with a situation where the conclusion is adequate, unacceptable and decision maker declaration is uncertain et al. The neutrosophic sets reflect on the truth membership, indeterminacy membership and falsity membership concurrently, which is more practical and adequate than fuzzy sets and Intuitionistic fuzzy sets [3]. Single valued neutrosophic sets are an extension of neutrosophic sets which were introduced by Wang et al. [6], Ye [9, 11] which is simplified neutrosophic sets, Peng et al. [6] defined their novel operations as aggregation operators and Abdel-Basset [12, 13] their novel method of fully neutrosophic linear programming problems. Linear programming problems in the fuzzy environment have classified into two groups which are symmetric and non-symmetric problems according to Zimmermann [12].

The paper is organized as follows. The important concepts of neutrosophic set arithmetic are presented in Section 2. An algorithm for solving a fully neutrosophic linear complementarity problem [14] is given in Section 3. The proposed method for solving a fuzzy neutrosophic linear complementarity problem is presented in Section 4. Finally in Section 5, the effectiveness of the proposed method is illustrated by numerical examples.

## 2. Preliminaries

**Definition 2.1** [3]**.** Let $E$ be a universe. A neutrosophic set $A$ over $E$ is defined by $A = \{\langle x, (T_A(x), I(x), F_A(x)) : x \in E\}$, where $T_A(x), I(x), F_A(x)$ are called the truth-membership function, indeterminacy membership fun $T_A : E \to ]^-0, 1^+[$, $I_A : E \to ]^-0, 1^+[$, $F_A : E \to ]^-0, 1^+[$, ction and falsity membership function respectively. They are respectively defined by such that $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

**Definition 2.2**[3]**.** A single valued trapezoidal neutrosophic number (SVTN) $\widetilde{A} = \langle (a, b, c, d); w_{\widetilde{a}}, u_{\widetilde{a}}, y_{\widetilde{a}} \rangle$ is a special neutrosophic set on the real number set $R$, whose truth-membership, indeterminacy-membership and a falsity membership are given as follows:

$$T_{\widetilde{a}}(x)=\begin{cases}\dfrac{(x-a)w_{\widetilde{a}}}{(b-a)}, & a\le x\le b\\ w_{\widetilde{a}}, & b\le x\le c\\ \dfrac{(d-x)w_{\widetilde{a}}}{(d-c)}, & c\le x\le d\\ 0, & \text{otherwise}\end{cases}$$

$$I_{\widetilde{a}}(x)=\begin{cases}\dfrac{(b-x+u_{\widetilde{a}}(x-a))}{(b-a)}, & a\le x\le b\\ u_{\widetilde{a}}, & b\le x\le c\\ \dfrac{(x-c+u_{\widetilde{a}}(d-x))}{(d-c)}, & c\le x\le d\\ 0, & \text{otherwise}\end{cases}$$

$$F_{a}(x)=\begin{cases}\dfrac{(b-x+y_{\widetilde{a}}(x-a))}{(b-a)}, & a\le x\le b\\ y_{\widetilde{a}}, & b\le x\le c\\ \dfrac{(x-c+u_{\widetilde{a}}(d-c))}{(d-c)}, & c\le x\le d\\ 0, & \text{otherwise}\end{cases}$$

### 2.3 Arithmetic Operations on Neutrosophic Trapezoidal Numbers

Let $\widetilde{a}=\langle(a_1, b_1, c_1, d_1); w_{\widetilde{a}}, u_{\widetilde{a}}, y_{\widetilde{a}}\rangle$ and $\widetilde{b}=\langle(a_2, b_2, c_2, d_2); w_{\widetilde{b}}, u_{\widetilde{b}}, y_{\widetilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $\gamma\neq 0$.

$$\widetilde{a}+\widetilde{b}=\langle(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{a}}\vee y_{\widetilde{b}}\rangle$$

$$\widetilde{a}+\widetilde{b}=\langle(a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{a}}\vee y_{\widetilde{b}}\rangle$$

$$\widetilde{a}\widetilde{b}=\begin{cases}\langle(a_1a_2, b_1b_2, c_1c_2, d_1d_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{b}}\vee y_{\widetilde{b}}\rangle, (d_1>0, d_2>0)\\ \langle(a_1d_2, b_1c_2, c_1b_2, d_1a_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{b}}\vee y_{\widetilde{b}}\rangle, (d_1<0, d_2>0)\\ \langle(d_1d_2, c_1c_2, b_1b_2, a_1a_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{b}}\vee y_{\widetilde{b}}\rangle, (d_1<0, d_2<0)\end{cases}$$

$$\widetilde{a}/\widetilde{b}=\begin{cases}\langle(a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{a}}\vee y_{\widetilde{b}}\rangle, (d_1>0, d_2>0)\\ \langle(a_1/d_2, b_1/c_2, c_1/b_2, d_1/d_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{a}}\vee y_{\widetilde{b}}\rangle, (d_1<0, d_2>0)\\ \langle(d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); w_{\widetilde{a}}\wedge w_{\widetilde{b}}, u_{\widetilde{a}}\vee u_{\widetilde{b}}, y_{\widetilde{a}}\vee y_{\widetilde{b}}\rangle, (d_1<0, d_2<0)\end{cases}$$

$$\gamma\widetilde{a}=\begin{cases}\langle(\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); w_{\widetilde{a}}, u_{\widetilde{a}}, y_{\widetilde{a}}\rangle, (\gamma>0)\\ \langle(\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); w_{\widetilde{a}}, u_{\widetilde{a}}, y_{\widetilde{a}}\rangle, (\gamma<0)\end{cases}$$

$$\tilde{a}^{-1} = \left\langle \left( \frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \right\rangle, (\tilde{a} \neq 0).$$

## 3. Algorithm for Ranking Neutrosophic Numbers in Linear Complementarity Problem [Abdel-Basset, (13)]

**Step 1.** Let decision makers insert their neutrosophic linear complementarity problem with trapezoidal neutrosophic numbers, because we always want to maximize truth degree, minimize indeterminacy and falsity degree of information $m$ and then inform decision makers to apply this concept when entering trapezoidal neutrosophic numbers of neutrosophic linear complementarity model.

**Step 2.** Regarding to definition 2.2 deals with a ranking function for trapezoidal neutrosophic numbers.

**Step 3.** Let $\tilde{a} = \langle (a^l, a^{m1}, a^{m2}, a^u); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle$ be a trapezoidal neutrosophic number, where $a^l, a^{m1}, a^{m2}, a^u$ are lower bound, first, second median value and upper bound for trapezoidal neutrosophic number, respectively. Also $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$ are the truth-membership, indeterminacy-membership and a falsity membership.

**Step 4.** If neutrosophic linear complementarity problem is a maximization problem, then the ranking function for this trapezoidal neutrosophic number is as follows: $R(\tilde{a}) = \left( \frac{a^l + a^u + 2(a^{m1} + a^{m2}}{2} \right)$ confirmation degree.

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left( \frac{a^l + a^u + 2(a^{m1} + a^{m2}}{2} \right) \tag{3.1}$$

If neutrosophic linear complementarity problem is a minimization problem, then the ranking function for this trapezoidal neutrosophic number is as follows: $R(\tilde{a}) = \left( \frac{a^l + a^u + 3(a^{m1} + a^{m2})}{2} \right) +$ confirmation degree.

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u + 2(a^{m1} + a^{m2})}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \tag{3.2}$$

**Step 5.** According to the type of NLCP, apply the suitable ranking function to convert each trapezoidal neutrosophic number to its equivalent crisp value. This leads to convert NLP problem to its crisp model. Solve the crisp model by using the standard method and obtain the optimal solution of problem.

## 4. Neutrosophic Linear Complementarity Problem [3] (NLCP)

Then, the following neutrosophic linear complementarity problem can be obtained by replacing crisp parameters with neutrosophic numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \tag{4.1}$$

$$\tilde{W} \geq 0,\ Z_j \geq 0,\ j = 1, 2, 3, \ldots, n \tag{4.2}$$

$$\tilde{W}_j\tilde{Z}_j \geq 0,\ j = 1, 2, 3, \ldots, n \tag{4.3}$$

### 4.1 Algorithm for neutrosophic linear complementarity problem

Consider the neutrosophic linear complementarity problem $(q, M)$, where the neutrosophic fuzzy matrix $M_j$ is a positive semi definite matrix of order $n$. The original table for this version of the algorithm is:

| w | z | $z_0$ | |
|---|---|---|---|
| i | -M | -d | q |

This method deals only with neutrosophic complementary basic vectors, beginning with $w = (w_1, w_2, \ldots, w_n)$ as the initial neutrosophic complementary basic vector.

If $\tilde{q} \geq 0,$ then we have the solution satisfying (4.1)-(4.3), by letting $w = q$ and $z = 0.$ If $\tilde{q} < 0,$ we will consider the following system

$$w - MZ - ez_0 = q \tag{4.4}$$

$$w_j \geq 0,\ z_j \geq 0,\ j = 1, 2, 3, \ldots, n \tag{4.5}$$

$$w_i z_j = 0,\ j = 1, 2, 3, \ldots, n \tag{4.6}$$

Where $Z_0$ is an artificial neutrosophic fuzzy variable and $\widetilde{e}$ is an $n$-vector with all components equal to any constant. Letting $z_0 =$ maximum$\{q_i/1 \leq i \leq n\}$, $z = 0$ and $W = q + ez_0$. We obtain a starting solution to the system (4.4)-(4.6). Through a sequence of pivots, we attempt to drive the neutrosophic fuzzy artificial variable $z_0$ to level zero, thus obtaining a solution to the neutrosophic linear complementarity problem (NLCP).

**Step 1.** Introduce the neutrosophic artificial variable $z_0$ and consider the system (4.4)-(4.6).

If $q \geq 0$ stop; then $(w, z) = (q, 0)$ is a neutrosophic complementary basic feasible solution. If $q < 0$ display the system (4.4), (4.5) as given in the simplex method.

**Step 2.** Let $d_s$ be the updated column in the current table under the variable $y_s$. If $d_s \leq 0$, go to step 5, otherwise determine the index $r$ by the following minimum ratio test, $\frac{\overline{q}}{d_{rs}} = \min\left\{\frac{\overline{q}_i}{d_{is}}, d_{is} > 0\right\}$, where $\overline{q}$ is the updated right-hand side column denoting the values of the neutrosophic basic variables. If the neutrosophic basic variable at row $r$ is $z_0$ go to step 4, otherwise, go to step 3.

**Step 3.** The neutrosophic basic variable at row $r$ is either $w_l$ or $z_l$ for some $l \neq s$. The variable $y_s$ enters the basis and the table is updated by pivoting at row $r$ and $y_s$ the column. If $w_l$ leaves the basis, then let $y_s = z_l$; and if $z$ leaves the basis, then let $y_s = w_l$; return to step 2.

**Step 4.** Here $y_s$ enters the basis, and $z_0$ leaves the basis. Pivot at the $y_s$ column and the $z_0$ row, producing a neutrosophic complementary basic feasible solution stop.

**Step 5.** Stop with ray termination. A ray $R = \{(w, z, z_0) + \lambda d/\lambda \geq 0\}$ is found such that every point in $R$ satisfying (4.4), (4.5) and (4.6), where $(w, z, z_0)$ is the almost neutrosophic complementary basic feasible solution, and $d$ is an extreme direction of the set defined by (4.4) and (4.5) having a $\widetilde{1}$

in the row corresponding to $y_s, -d_s$ in the rows of the current basic variables and zeros everywhere else.

## 5. Numerical Examples

The difference between fuzzy set and neutrosophic set is that fuzzy set takes into consideration the truth degree only. But neutrosophic set takes into consideration the truth, indeterminacy and falsity degree. The decision makers and problem solver always seek to maximize the truth degree, minimize indeterminacy and falsity degree. Then, in the following example, we consider truth degree $(T) = 1$, indeterminacy (I) and falsity (F) degree = 0, as follows (1, 0, 0) for each trapezoidal neutrosophic number and this called the confirmation degree of each trapezoidal neutrosophic number. The trapezoidal number is symmetric with the following form:

$$\tilde{a} = \langle (a^l, a^u, \alpha, \alpha); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle,$$

where $a^l, a^u, \alpha, \alpha$ represented the lower bound, upper bound and first, second median value of trapezoidal neutrosophic number, respectively.

**Example 5.1.**

Maximize

$$Z = \langle (13, 15, 2, 2); 1, 0, 0 \rangle x_1 + \langle (12, 14, 3, 3); 1, 0, 0 \rangle x_2 + \langle (15, 17, 3, 3); 1, 0, 0 \rangle x_3$$

Subject to constraint $x_1 + 3x_3 \leq -3, -x_1 + 2x_2 + 5x_3 \leq -2, 2x_1 + x_2 + 2x_3 \leq -1$ and $x_1, x_2, x_3 \geq 0.$

The neutrosophic linear complementarity problem is maximization problem, then by using equation (3.1) each trapezoidal number will convert to its equivalent crisp number. Remember that confirmation degree of each trapezoidal number is (1, 0, 0) according to decision maker opinion and ranking function for this trapezoidal neutrosophic number is as follows:

$$R(\tilde{a}) = \left( \frac{a^l + a^u + 3(a^{m1} + a^{m2}}{2} \right) + \text{confirmation degree.}$$

Maximize $\tilde{Z} = 19x_1 + 20x_2 + 21x_3$

Subject to constraint $x_1 + 3x_3 \leq -3, -x_1 + 2x_2 + 5x_3 \leq -2, 2x_1 + x_2 + 2x_3 \leq -1$ and $x_1, x_2, x_3 \geq 0$

A linear complementary problem is solved by the proposed algorithm and the results are $z_1 = 0.75, z_2 = 1.25, z_0 = 3.75$.

The obtained solution is $x_1 = 0.75$ and $x_2 = 1.25$

Maximization $R(\widetilde{z}) = 39.25$.

**Example 5.2.**

Minimize

$$\widetilde{Z}\langle(13, 15, 2, 2); 1, 0, 0\rangle x_1 + \langle(12, 14, 3, 3); 1, 0, 0\rangle x_2 + \langle(15, 17, 3, 3); 1, 0, 0\rangle x_3$$

Subject to constraint $x_1 + 3x_3 \leq -3, -x_1 + 2x_2 + 5x_3 \leq -2, 2x_1 + x_2 + 2x_3 \leq -1$ and $x_1, x_2, x_3 \geq 0$.

The neutrosophic linear complementarity problem is minimization problem, then by using equation (3.2) each trapezoidal number will convert to its equivalent crisp number. Remember that confirmation degree of each trapezoidal number is (1, 0, 0) according to decision maker opinion and ranking function for this trapezoidal neutrosophic number is as follows:

Minimize $\widetilde{Z} = 9x_1 + 8x_2 + 11x_3$

Subject to constraint $x_1 + 3x_3 \leq -3, x_1 + 2x_2 + 5x_3 \leq -2, 2x_1 + x_2 + 2x_3 \leq -1$ and $x_1, x_2, x_3 \geq 0$.

Proceeding in this way, a linear complementary problem is solved by the proposed algorithm and the results are $z_1 = 0.75, z_2 = 1.25, z_0 = 3.75$.

The obtained solution is $x_1 = 0.75$ and $x_2 = 1.25$, Minimization $R(\widetilde{z}) = 16.75$.

## 6. Conclusion

In this research paper, a new methodology is proposed to solve a fully fuzzy linear complementarity problem by converting the trapezoidal neutrosophic coefficients to its equivalent crisp value with the help of ranking functions. A numerical example is also provided to verify the proposed method.

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