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# ON JOHAN COLORING AND JOHAN CHROMATIC CORE SUBGRAPH OF LINE GRAPH OF STAR GRAPH AND PRODUCT OF LINE GRAPH OF STAR GRAPH

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## Abstract

Johan coloring concept is motivated by the newly introduced invariant called the rainbow neighbourhood number of a graph G. In this paper, we investigated Johan Chromatic Number (denoted $\chi_J(G)$ and Johan Chromatic Core Subgraph (denoted J-CCS) of Line graph of Star related graphs, Star related graphs and Product of Line graph of star related graph.

## 1. Introductions

All graphs $G = (V, E)$ considered in this paper are finite, undirected simple graphs. For general notation and concepts in graphs and digraphs, we refer J. A. Bondy, F. Harary and D. B. West [1, 4, 10]. In graph colouring, vertices or edges or both assigned colours subject to conditions. Johan colouring and Chromatic Core Subgraph (denoted by CCS) were introduced by Kok. J (2018) [6, 8]. We have investigated Johan Chromatic Number (denoted $\chi_J(G)$) and introduce the Johan Chromatic Core Subgraph (denoted J-CCS) of Line graph of Star related graphs and Product of Line graph of star related graph.

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## 2. Basic Definitions

**Definition 2.1.** For a finite, undirected simple graph $G$ of order $\upsilon(G) = n \geq 1,$ a chromatic core subgraph $H$ is the smallest induced subgraph $H$ (smallest in respect of $\mathrm{si}(H)$) such that, $X(H) = X(G)$. For a graph $G$, its structural size is measured by its structural index denoted and defined as, $si\,(G) = \upsilon(G) + \varepsilon(G)$.

**Definition 2.2.** A proper $k$-colouring $C$ of a graph $G$ is called Johan colouring if $C$ is the maximal colouring such that every vertex of $G$ belongs to a rainbow neighbourhood of $G$ [7]. A graph $G$ is $J$-colourable if it admits $J$-colouring. The J-colouring number of a graph $G$, denoted by $J(G)$ is the maximum number of colours in a $J$-colouring of $G$.

**Definition 2.3.** Let $V(G) = \{v_i : 1 \leq i \leq n\}$ and $V(H) = \{u_j : 1 \leq j \leq m\}$. The node set of the Cartesian Product $V(G) \times V(H)$ has nodes such that $(v_i, u_j)$ and $(v_n, u_m)$ are adjacent (adj, for brevity) if and only if, $v_i = v_n$ and $u_j$ *adj* $u_m$ or $v_i$ *adj* $v_n$ and $u_j = u_m$.

## 3. Johan Chromatic Number Of Line Graph Of Star Related Graph.

In this section ,we initiate the Johan coloring chromatic number $\chi_J(L(SG))$ of line of Star related Graphs namely, $\lfloor L(P_2 \circ K_1), L(B_{n,n}), L(B_{2,n,W_i}), L(C(m, n))$, and $\mathrm{L}(K_{1,n-1})\rfloor$

**Proposition 3.1.** *Let* $L(P_2 \circ K_1)$ *be a line graph of Comb Graph. Then,* $\chi_J(L(P_2 \circ K_1) = \Delta L(P_2 \circ K_1)$.

**Note 3.2:** Let $L(P_2 \circ K_1), n \geq 3$ be a line graph of Comb Graph. Then Johan coloring does not exist.

**Theorem 3.3.** *Let* $L(K_{1,n-1})$ *be a Line graph of Star graph. Then,* $\chi_J(L(K_{1,n-1})) = \Delta[L(K_{1,n-1}) + 1]$.

**Proof.** Let $\{x_1, x_2, x_3, \ldots, x_n\}$ be the vertices of line graph of star graph and $\{y_1, y_2, y_3, \ldots, y_n\}$, where $\{y_i = x_i, x_{i+1} : 1 \leq i \leq n\}$ be the edges of line graph of star graph has a regular graph (i,e) every vertices have the same degree.

Now, we allocate the Johan coloring to those vertices as follows,

$$C(v_i) = \begin{cases} c_1 & \text{if } i = 1 \\ c_2 & \text{if i = 2} \\ c_3 & \text{if i = 3} \\ c_4 & \text{if i = 4} \\ & \vdots \\ c_n & \text{if i = n} \end{cases}$$

Based on the above coloring pattern, the graph $L(K_{1, n-1})$ is Johan coloring with $\Delta [L(K_{1, n-1}) + 1]$ colors. Hence, the Johan Chromatic number is $\chi_J(L(K_{1, n-1})) = \Delta[L(K_{1, n-1}) + 1]$.

**Theorem 3.4.** *Let* $L(B_{n, n})$ *be a line graph of Bi-star graph then,* $\chi_J(L(B_{n, n})) = \Delta[(B_{n, n})]$.

**Proof.** Let $\{x_1, y_2, y_3, \ldots, y_n\}$, be the vertices of line graph of Bi-Star graph and $\{y_1, y_2, y_3, \ldots, y_n\}$, where $\{y_i = x_i, x_{i+1} : 1 \leq i \leq n\}$ the edges of line graph of Bi- Star graph. When *I* take line graph for Bi-Star graph, we get complete graphs. There is usually one point for both. New we allocate the Johan coloring to these vertices as follows,

$$C(v_i) = \begin{cases} 1 & \text{if } i = 1, 2n + 1 \\ 2 & \text{if } i = 2, 2\text{n} \\ 3 & \text{if } i = 3, 2n - 1 \\ 4 & \text{if } i = 4, 2n - 2 \\ & \quad \vdots \\ n & \text{if } i = n + 1 \end{cases}$$

Based on the above coloring pattern the graph $L(B_{n, n})$ is Johan colouring with $\Delta[(B_{n, n})]$ colours. Hence, the Johan Chromatic number is $\chi_J(L(B_{n, n})) = \Delta[(B_{n, n})]$.

**Note 3.5.** Subdivision of Bi-star graph is not a Johan colourable.

**Theorem 3.6.** *Let $L(C(m, n))$ be a Line graph of Coconut graph then,*

$$\chi_J(L(C(m, n))) = \begin{cases} \Delta(L(C(m,n))+1) & \text{if } m = 1 \text{ and } n \geq 1 \\ \Delta(L(C(m, n))) & \text{if } m = 1 \text{ and } n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

**Proof.** Let $\{x_1, x_2, x_3, \ldots, x_{n+1}\}$ be the vertices of line graph of coconut graph and $\{y_1, y_2, y_3, \ldots, y_n\}$ where $\{y_i = x_i, x_{i+1} : 1 \leq i \leq n\}$ the edge set of line graph of coconut graph be. By applying the definition of Johan coloring of $L(C(m, n))$ is as follows,

**Case** (i)**:** Suppose $m = 1$ and $n \geq 1$ the line graph of coconut graph has a regular graph of coconut graph has a regular graph every vertices $s$ as the same degree.

Now, we allocate the Johan coloring of these vertices as follows,

$$C(v_i) = \begin{cases} c_1 & \text{if } i = 1 \\ c_2 & \text{if } i = 2 \\ c_3 & \text{if } i = 3 \\ c_4 & \text{if } i = 4 \\ & \vdots \\ c_n & \text{if } i = n \end{cases}$$

Based on the above coloring pattern, the graph $(L(C(m, n)))$ is Johan coloring with $\Delta(L(C(m, n))+1)$ colors. Hence the Johan chromatic number $\Delta(L(C(m, n))+1)$ if $m = 1$ and $n \geq 1$.

**Case** (ii) Suppose $m \geq 1$ and $n = 1$ the line graph of coconut graph has acyclic graph.

Now we allocate the Johan coloring of this graph as follows,

$$C(v_i) = \begin{cases} c_1 & \text{if } i = 1, 3, 5, 7, 9, \ldots \\ c_2 & \text{if } i = 2, 4, 6, 8, 10, \ldots \end{cases}$$

Based on the above coloring pattern, the graph $(L(C(m, n)))$ is Johan coloring with $\Delta(L(C(m, n)))$ colors. Hence the Johan chromatic number $\Delta(L(C(m, n)))$ if $m \geq 1$ and $n = 1$.

## 4. Johan Chromatic Core SubGraph of Product of Line Graph of Star Graph

In this section, we found the Johan Chromatic Core Subgraph of line graph of Star related Graphs.

**Proposition 4.1.** *Let* $L(P_2 \circ K_1) \circ L(P_2 \circ K_1)$ *be a corona product of Line graph of Comb graph for* $n = 2$ *then,* J-CCS *of* $L(P_2 \circ K_1) \circ L(P_2 \circ K_1)$ *is* $K_3$.

**Note 4.2.** Let $L(P_n \circ K_1) \circ L(P_n \circ K_1)$ be a corona product of line graph of Comb graph for $n \geq 3$ then, J-CCS as does not exist.

**Theorem 4.3.** Let $L(K_{1,m-1}) \circ L(K_{1,n-1})$ *be a corona product of line graph of Star graph then* J-CCS of $L(K_{1,m-1}) \circ L(K_{1,n-1})$ is $K_n$.

**Proof.** Let $L(K_{1,m-1})$ and $L(K_{1,n-1})$ be a two set of line graph of star graph where the vertex and edge set it's denoted by, Let $\{x_1, x_2, x_3, \ldots, x_{n-1}\}$ be the vertices of line graph of star graph and $\{y_1, y_2, y_3, y_i\}$ where $\{y_i = x_i, x_{i+1} : 1 \leq i \leq n\}$ the edges of line graph of star graph.

The corona graph if a $L(K_{1,m-1}) \circ L(K_{1,n-1})$ admits Johan coloring and $\chi_J(L(K_{1,m-1}) \circ L(K_{1,n-1})) = n \cdot L(K_{1,m-1}) \circ L(K_{1,n-1})$ has $K_n$ as an induced subgraph and it's $\chi_J(L(K_{1,m-1}) \circ L(K_{1,n-1})) = n.$ Hence J-CCS of $(K_{1,m-1}) \circ L(K_{1,n-1})$ is $K_n$.

## 5. Conclusion

In this paper, we have established to Johan Coloring and Johan Chromatic Core Subgraph of the Line graph Star graph, Corona product of Line Star related graphs. This work has a future scope to find Central, Middle graph of Star related graph and Product of Star related graphs.

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