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$Mod(k)$ VERTEX MAGIC LABELING OF QUADRILATERAL SNAKE GRAPHS

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Abstract

Let G be a simple, undirected and non trivial graph with p vertices and q edges. For any integer $k \geq 2, l \in Z_k$, there exists an injective function $f : V(G) \to \{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right] + l, \left[\frac{k}{2}\right] + l + 1, \left[\frac{k}{2}\right] + k, \left[\frac{k}{2}\right] + k + l, \left[\frac{k}{2}\right] + k + l + 1, \ldots, \left[\frac{k}{2}\right] + k(p-1)\}$ such that for every edge $(e = uv) \in E(G)$, the mapping $f^* : E(G) \to Z_k$ defined by $f^*(e = uv) = (f(u + f(v)))(\text{mod } K) = l$ is a constant mapping. The function f is called a $Mod(k)$ vertex magic labeling of G. A graph G is called $Mod(k)$ vertex magic graph if it admits a $Mod(k)$ vertex magic labeling. In this paper, we prove that Quadrilateral Snakes, Double Quadrilateral Snakes are $Mod(k)$ vertex magic graphs.

1. Introduction

In this paper, all the graphs are simple, connected and undirected graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and edge set $E(G)$ and $|V(G)| = p$ and $|E(G)| = q$.

The concept of graph labeling introduced by A. Rosa in 1967. A labeling of a graph G is a mapping that carries a set of graph elements usually vertices

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and edges into a set of numbers, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [2]. The concept of labeling of graphs has gained a lot of popularity in the area of graph theory. This graph labeling is very useful in mathematical models for a wide range of applications such as X-ray, Crystallography, Coding theory, Cryptography, Astronomy, Circuit design, Radar, Communication networks design, and Database Management [2].

In P. Sumathi and B. Fathima [3] defined as a $Mod(k)$ vertex magic labeling and $Mod(k)$ vertex magic graph. Let G be a simple, undirected and non trivial graph containing p vertices and q edges.

For any integer $k \geq 2, l \in Z_k$, there exists an injective mapping $f: V(G) \to \left\{\left\lceil \frac{k}{2} \right\rceil, \left\lceil \frac{k}{2} \right\rceil + l, \left\lceil \frac{k}{2} \right\rceil + l + 1, \left\lceil \frac{k}{2} \right\rceil + k, \left\lceil \frac{k}{2} \right\rceil + k + l, \left\lceil \frac{k}{2} \right\rceil + k + l + 1, \ldots, \left\lceil \frac{k}{2} \right\rceil + k(p-1)\right\}$ such that for every edge $(e = uv) \in E(G)$, the mapping $f^*: E(G) \to Z_k$ defined by $f^*(uv) = (f(u) + f(v))(\bmod k) = l$ is a constant mapping. The function f is said to be a $Mod(k)$ vertex magic labeling $(MVML(k))$ of G. A graph G is called $Mod(k)$ vertex magic graph if it admits a $Mod(k)$ vertex magic labeling.

In this paper, we study the $Mod(k)$ vertex magic labeling of Quadrilateral Snakes, Double Quadrilateral Snakes.

2. Preliminaries

In this section, we provide the basic definitions and notations related to this paper.

Definition 2.1. A quadrilateral snake Q_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} to new vertices v_i and w_i respectively and adding edges $v_i w_i$ for $i = 1, 2, \ldots, n-1$. That is every edge of a path is replaced by a cycle C_4[7].

Definition 2.2. A double quadrilateral snake $D(Q_n)$ is obtained from two quadrilateral snakes that have a common path [7].

3. Main Results

In this section, the existence of $Mod(k)$ vertex magic labeling of Quadrilateral Snake graphs are established.

Theorem 3.1. *Quadrilateral Snake Q_n admits $MVML(k)$ for all $n \geq 2$.*

Proof. Let $G = (V(G), E(G))$ be a Quadrilateral Snake graph Q_n where $V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n-1\} \cup \{w_i : 1 \leq i \leq n-1\}$ and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n-1\} \cup \{u_{i+1} w_i : 1 \leq i \leq n-1\}$ $\cup \{v_i w_i : 1 \leq i \leq n-1\}$. It has $3n-2$ vertices and $4n-4$ edges.

Case (i). k is odd.

Define the function $f : V(G) \to \{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right] + l + 1, \left[\frac{k}{2}\right] + k, \left[\frac{k}{2}\right] + k + l + 1, \ldots, \left[\frac{k}{2}\right] + k(3n-1)\}$ in the following table.

Table 3.1.1. Vertex labeling of Q_n when k is odd.

$f(u_i)$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-3)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-4) + l + 1$
	$l = k-1$	$1 \leq i \leq n$	$\left[\frac{k}{2}\right] + k(3i-3)$
$f(v_i)$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-3) + l + 1$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-2)$
	$l = k-1$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-2)$

$f(w_i)$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-1)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-2) + l + 1$
	$l = k-1$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(3i-1)$

Therefore, f is an injective mapping. The sum of the labels of the vertices incident with $(e = uv) \in E(G)$ are computed in the following table.

Table 3.1.2. The sum of the labels of the vertices in Q_n which are incident with an edge $(e = uv)$ when k is odd.

$f(u)+f(v)$	$0 \leq l \leq k-2$	$u = u_i, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(3i-1)+l$
		$u = u_i, v = v_i$	$1 \leq i \leq n-1$	$k(3i-2)+l$
		$u = u_{i+1}, v = w_i$	$1 \leq i \leq n-1$	$k(3i)+l$
		$u = v_i, v = w_i$	$1 \leq i \leq n-1$	$k(3i-1)+l$
	$l = k-1$	$u = u_i, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(6i-3)+k-1$
		$u = u_i, v = v_i$	$1 \leq i \leq n-1$	$k(6i-5)+k-1$
		$u = u_{i+1}, v = w_i$	$1 \leq i \leq n-1$	$k(6i-1)+k-1$
		$u = v_i, v = w_i$	$1 \leq i \leq n-1$	$k(6i-3)+k-1$

Hence $f^*(uv) = (f(u) + f(v))(\text{mod } k) = l$, the induced mapping f^* is a constant mapping. Thus f is a $MVML(k) \cdot Q_n$ admits $MVML(k)$ if k is odd.

Case (ii). k is even.

Define the function $f : V(G) \to \left\{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right] + l, \left[\frac{k}{2}\right] + k, \left[\frac{k}{2}\right] + k + \right.$

$l, \ldots, \left[\frac{k}{2}\right] + k(3n-1)\}$ in the following table.

Table 3.1.3. Vertex labeling of Q_n when k is even.

$f(u_i)$	$l = 0$	$1 \leq i \leq n$	$\left[\frac{k}{2}\right] + k(3i-3)$
	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-3)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-4) + l$
$f(v_i)$	$l = 0$	$1 \leq l \leq n-1$	$\left[\frac{k}{2}\right] + k(3i-2)$
	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-3) + l$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-2)$
$f(w_i)$	$l = 0$	$1 \leq l \leq n-1$	$\left[\frac{k}{2}\right] + k(3i-1)$
	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-1)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(3i-2) + l$

Hence f is an one-to-one mapping. The sum of the labels of the vertices incident with $(e = uv) \in E(G)$ are computed in the following table.

Table 3.1.4. The sum of the labels of the vertices in Q_n which are incident with an edge $(e = uv)$ when k is even.

$f(u)+f(v)$	$l=0$	$u=u_i, v=u_{i+1}$	$1 \le i \le n-1$	$k(6i-2)$
		$u=u_i, v=v_i$	$1 \le i \le n-1$	$k(6i-4)$
		$u=u_{i+1}, v=w_i$	$1 \le i \le n-1$	$k(6i)$
		$u=v_i, v=w_i$	$1 \le i \le n-1$	$k(6i-2)$
	$0 \le l \le k-2$	$u=u_i, v=u_{i+1}$	$1 \le i \le n-1$	$k(3i-1)+l$
		$u=u_i, v=v_i$	$1 \le i \le n-1$	$k(3i-2)+l$
		$u=u_{i+1}, v=w_i$	$1 \le i \le n-1$	$k(3i-1)+l$
		$u=v_i, v=w_i$	$1 \le i \le n-1$	$k(3i-1)+l$

According to the definition of $MVML(k)$, the induced mapping f^* is a constant mapping.

Thus f is a $MVML(k)$ for G. Q_n admits $MVML(k)$ if k is even.

Hence Quadrilateral Snake Q_n admits $MVML(k)$ for all $n \geq 2$.

Illustration 1. Q_6 is a $Mod(3)$ vertex magic graph for $l = 1$ is shown in figure 1.

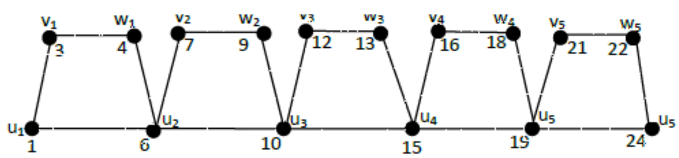

Figure 1.

Theorem 3.2. *Double Quadrilateral Snake* $D(Q_n)$ *admits* $MVML(k)$ *for all* $n \geq 2$.

Proof. Let $G = (V(G), E(G))$ be a Double Quadrilateral Snake $D(Q_n)$ where $V(G) = \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n-1\} \cup \{w_i : 1 \le i \le n-1\} \cup \{v'_i : 1 \le i \le n-1\}$ $\cup \{w'_i : 1 \le i \le n-1\}$ and $E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n-1\} \cup \{u_{i+1} w_i, 1 \le i \le n-1\} \cup \{v_i w_i : 1 \le i \le n-1\} \cup \{u_i v'_i : 1 \le i \le n-1\} \cup \{u_{i+1} w'_{i+1} : 1 \le i \le n-1\} \cup \{v'_i w'_i : 1 \le i \le n-1\}$. It has $5n-4$ vertices and $7n-7$ edges.

Case (i). k is odd.

Define the function $f : V(G) \to \{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right] + l + 1, \left[\frac{k}{2}\right] + k, \left[\frac{k}{2}\right] + k + l + 1, \ldots, \left[\frac{k}{2}\right] + k(5n-3)\}$ in the following table

Table 3.2.1. Vertex labeling of $D(Q_n)$ when k is odd.

$f(u_i)$	$0 \le l \le k-1$	$i = 1$	$\left[\frac{k}{2}\right]$
	$0 \le l \le k-2$	i is odd $i \ne 1$	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-7)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-8) + l + 1$
	$l = k-1$	$1 \le i \le n$	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-7)$
$f(v_i)$	$0 \le l \le k-2$	i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-5) + l + 1$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-4)$
	$l = k-1$	$1 \le i \le n-1$	$\left[\frac{k}{2}\right] + k(5i-4)$

$f(w_i)$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-3)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-4) + l + 1$
	$l = k-1$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i-3)$
$f(v_i')$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-1) + l + 1$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i)$
	$l = k-1$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i)$
$f(w_i')$	$0 \leq l \leq k-2$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-1)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-2) + l + 1$
	$l = k-1$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i-1)$

Therefore f is an injective mapping. The sum of the labels of the vertices incident with $(e = uv) \in E(G)$ are computed in the following table.

Table 3.2.2. The sum of the labels of the vertices in $D(Q_n)$ which are incident with an edge $(e = uv)$ when k is odd.

$f(u) + f(v)$		$u = u_i, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(5i-4) + l$
		$u = u_i, v = v_i$	$1 \leq i \leq n-1$	$k(5i-5) + l$

	$0 \leq l \leq k-2$	$u = u_{i+1},\, v = w_i$	$1 \leq i \leq n-1$	$k(5i-2)+l$
		$u = v_i,\, v = w_i$	$1 \leq i \leq n-1$	$k(5i-4)+l$
		$u = u_i,\, v = v_i'$	$1 \leq i \leq n-1$	$k(5i-3)+l$
		$u = u_{i+1},\, v = w_i'$	$1 \leq i \leq n-1$	$k(5i-2)+l$
		$u = v_i',\, v = w_i'$	$1 \leq i \leq n-1$	$k(5i-1)+l$
	$l = k-1$	$u = u_i,\, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(10i-9)+k-1$
		$u = u_i,\, v = v_i$	$1 \leq i \leq n-1$	$k(10i-11)+k-1$
		$u = u_{i+1},\, v = w_i$	$1 \leq i \leq n-1$	$k(10i-5)+k-1$
		$u = v_i,\, v = w_i$	$1 \leq i \leq n-1$	$k(10i-7)+k-1$
		$u = u_i,\, v = v_i'$	$1 \leq i \leq n-1$	$k(10i-7)+k-1$
		$u = u_{i+1},\, v = w_i'$	$1 \leq i \leq n-1$	$k(10i-3)+k-1$
		$u = v_i',\, v = w_i'$	$1 \leq i \leq n-1$	$k(10i-1)+k-1$

Hence $f^*(uv) = (f(u)+f(v))(\text{mod } k) = l,$ the induced mapping f^* is a constant mapping. Thus f is a $MVML(k)$ for G. $D(Q_n)$ admits $MVML(k)$ if k is odd.

Case (ii). k is even.

Define the function $f : V(G) \to \{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right]+l, \left[\frac{k}{2}\right]+k, \left[\frac{k}{2}\right]+k+l+, \ldots,$ $\left[\frac{k}{2}\right]+k(5n-3)\}$ in the following table.

Table 3.2.3 (i). Vertex labeling of $D(Q_n)$ when k is odd.

$f(u_i)$	$0 \leq l \leq k-1$	$i = 1$	$\left[\frac{k}{2}\right]$
	$l = 0$	$1 \leq i \leq n$	$\left[\frac{k}{2}\right] + k(5i-7)$
	$0 \leq l \leq k-1$	i is odd, $i \neq 1$	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-7)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-8) + l$
$f(v_i)$	$l = 0$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i-4)$
	$0 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-5) + l$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-4)$

Table 3.2.3 (ii). Vertex labeling of $D(Q_n)$ when k is odd.

$f(w_i)$	$l = 0$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i-3)$
	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-3)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-4) + l$
$f(v_i')$	$l = 0$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i)$

	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-1) + l$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i)$
$f(w_i')$	$l = 0$	$1 \leq i \leq n-1$	$\left[\frac{k}{2}\right] + k(5i-1)$
	$1 \leq l \leq k-1$	i is odd	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-1)$
		i is even	$\left[\frac{k}{2}\right] + \frac{k}{2}(5i-2) + l$

Hence f is an one-to-one mapping. The sum of the labels of the vertices incident with $(e = uv) \in E(G)$ are computed in the following table.

Table 3.2.4. The sum of the labels of the vertices in $D(Q_n)$ which are incident with an edge $(e = uv)$ when k is even.

$f(u) + f(v)$	$l = 0$	$u = u_i, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(10i-8)$
		$u = u_i, v = v_i$	$1 \leq i \leq n-1$	$k(10i-10)$
		$u = u_{i+1}, v = w_i$	$1 \leq i \leq n-1$	$k(10i-4)$
		$u = v_i, v = w_i$	$1 \leq i \leq n-1$	$k(10i-6)$
		$u = u_i, v = v_i'$	$1 \leq i \leq n-1$	$k(10i-6)$
		$u = u_{i+1}, v = w_i'$	$1 \leq i \leq n-1$	$k(10i-2)$
		$u = v_i', v = w_i'$	$1 \leq i \leq n-1$	$k(10i)$
		$u = u_i, v = u_{i+1}$	$1 \leq i \leq n-1$	$k(5i-4)+l$

		$u = u_i, v = v_i$	$1 \le i \le n-1$	$k(5i-5)+l$
	$1 \le l \le k-1$	$u = u_{i+1}, v = w_i$	$1 \le i \le n-1$	$k(5i-2)+l$
		$u = v_i, v = w_i$	$1 \le i \le n-1$	$k(5i-4)+l$
		$u = u_i, v = v_i'$	$1 \le i \le n-1$	$k(5i-3)+l$
		$u = u_{i+1}, v = w_i'$	$1 \le i \le n-1$	$k(5i-2)+l$
		$u = v_i, v = w_i'$	$1 \le i \le n-1$	$k(5i-1)+l$

According to the definition of $MVML(k)$, the induced mapping f^* is a constant mapping. Thus f is a $MVML(k)$ for G. $D(Q_n)$ admits $MVML(k)$ if k is even.

Hence Double Quadrilateral Snake $D(Q_n)$ admits $MVML(k)$ for all $n \ge 2$.

Illustration 2. A Double Quadrilateral Snake $D(Q_5)$ with $MVML(4)$ for $l = 2$ is shown figure 2.

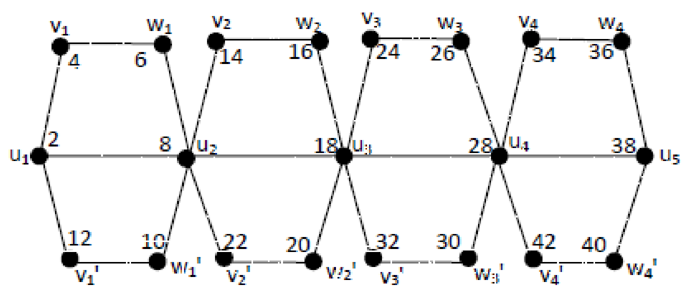

Figure 2.

4. Conclusion

In this paper we have discussed that Quadrilateral snake graphs are Mod(k) vertex magic graphs. Analogues work can be carried by us for other families also.

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