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NEUTROSOPHIC CRISP SUPRA SEMI-α LOCALLY CLOSED SETS

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Abstract

In this article we aimed to introduce the notations of neutrosophic crisp supra semi-α locally closed sets, we verify the relation between neutrosophic crisp supra semi-α locally closed sets to other locally closed sets. Also we introduce neutrosophic crisp supra dense set and submaximal spaces, neutrosophic crisp supra locally continuous, irresolute maps and investigate some of its properties.

1. Introduction

The idea of degree of membership and the concept of fuzzy set [18] was introduced by Zadeh in [18]. In 1983 generalization of fuzzy set intutionistic fuzzy set was introduced by K. Atanassov [3] as a beyond the degree of membership and the degree of non membership of each element. Neutrosophic set is a generalization of intutionistic fuzzy set. The idea of "neutrosophic set" was proposed by Smarandache [14, 15]. Neutrosophic operations have been developed by Salama et al. [12, 13, 17, 9, 8, 10, 11, 1, 5, 6, 7]. Salama and Albowi [17] define neutrosophic crisp topological space and established some of its properties. Salama and Smarandache [16, 7, 14, 17] introduced the concept of neutrosophic crisp sets; neutrosophic crisp operators have been investigated. Bourbaki introduced the concept of locally closed sets in topological space 1966 [4]. Amarendra Babu and Rajasekhar [2] introduced the concept of neutrosophic crisp supra topology and neutrosophic

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crisp supra semi-α closed sets in 2020. Now in this paper we extend the concept of locally closed sets in topological spaces to neutrosophic crisp supra semi-α locally closed sets in neutrosophic crisp supra topology and we study the notations of neutrosophic crisp supra semi-α locally closed sets and we verify the relation between neutrosophic crisp supra semi-α locally closed sets to other locally closed sets in neutrosophic crisp supra topology. Also we introduced neutrosophic crisp supra dense set and submaximal spaces, neutrosophic crisp supra locally continuous, irresolute maps and we investigate some of its properties. Throughout this paper $(X, \tau^{\mu}), (Y, \sigma^{\mu}), (Z, \eta^{\mu})$ or X, Y, Z represents nonempty neutrosophic crisp supra topological spaces.

2. Preliminaries

The definitions of neutrosophic crisp set (NCS for short), neutrosophic crisp types of φ_N and X_N, neutrosophic crisp union and intersection, neutrosophic crisp subsets, neutrosophic crisp complement and family of union and intersection of neutrosophic crisp sets was introduced by the author [16] The definitions of neutrosophic crisp supra semi open (closed) set (NCS-S-OS, NCS-S-CS) , neutrosophic crisp supra pre open (closed) set (NCS-P-OS, NCS-P-CS) neutrosophic crisp supra semi-pre open (closed) set and neutrosophic crisp supra α-closed (open) sets (NCS-α-OS, NCS-α-CS) are introduced by the authors [2].

Definition 2.1 [2]**.** A neutrosophic crisp supra topology (NCST for short) on a nonempty set X is a family τ^{μ} of neutrosophic crisp subset in X if satisfying

(a) $\varphi_N, X_N \in \tau^{\mu}$

(b) $\cup E_i \in \tau^{\mu}\ \forall \{E_i : i \in I\} \subseteq \tau^{\mu}$.

Then (X, τ^{μ}) is said to be a neutrosophic crisp supra topological space (NCSTS), elements in τ^{μ} are called neutrosophic crisp supra open sets (NCSOS) and the complement of τ^{μ} are called neutrosophic crisp supra closed sets (NCSCS). Throughout this paper (X, τ^{μ}) represent NCSTS.

Definition 2.2 [2]. Let (X, τ^{μ}) be NCSTS and E is a NC-subset in X. Then E is called neutrosophic crisp supra semi-α-closed set (NCS-Sα-CS for short) if ∃ a NCS-α-CS H in (X, τ^{μ}) such that NCS-int $(H) \subseteq E \subseteq H$. Here the family of all NCS-Sα-CS denoted by NCS-Sα-CS(X).

Definition 2.3 [2]. A neutrosophic crisp subset E is called neutrosophic crisp supra semi-α-open set (NCS-Sα-OS for short) if and only if E^{C} is a NCS-Sα-CS.

3. Neutrosophic Crisp Supra semi-α Locally Closed Sets

Definition 3.1. Let E be any subset of NCSTS (X, τ^{μ}). Then E is called neutrosophic crisp supra locally closed set (NCSLCS for short) if $E = K \cap K^{*}$, where K is open and K^{*} is closed set in (X, τ^{μ}).

Example 3.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}, \tau^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$ $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$. Then (X, τ^{μ}) is a NCSTS. Hence $\langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ and $\langle\varphi, \varphi, X\rangle$, are NCSLCSs.

Definition 3.3. Let E be any subset of NCSTS (X, τ^{μ}). Then E is called

(a) Neutrosophic crisp supra pre-locally closed set (NCS-P-LCS for short) if $E = K \cap K^{*}$, where K is NCS-P-OS and K^{*} is NCS-P-CS in (X, τ^{μ}).

(b) Neutrosophic crisp supra semi-locally closed set (NCS-S-LCS for short) if $E = K \cap K^{*}$, where K is NCS-S-OS and K^{*} is NCS-S-CS in (X, τ^{μ}).

(c) Neutrosophic crisp supra-α-locally closed set (NCS-α-LCS for short) if $E = K \cap K^{*}$, where K is NCS-α-OS and K^{*} is NCS-α-CS in (X, τ^{μ}).

(d) Neutrosophic crisp supra-semi pre-locally closed set (NCS-SP-LCS for short) if $E = K \cap K^{*}$, where K is NCS-SP-OS and K^{*} is NCS-SP-CS in (X, τ^{μ}).

(e) Neutrosophic crisp supra-semi-α-locally closed set (NCS-Sα-LCS for short) if $E = K \cap K^*$, where K is NCS-Sα-OS and K^* is NCS-Sα-CS in (X, τ^{μ}).

Example 3.4.

(a) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$ then (X, τ^{μ}) is NCSTS. If $E = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, then E is NCS-P-OS and $E^c = \langle\{\delta_1, \psi_3\}, \varphi, \{\eta_2\}\rangle$, is a NCS-P-CS. Hence $E \cap E^c = \langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is a NCS-P-LCS.

(b) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$, $Q = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then (X, τ^{μ}) is NCSTS. If $E = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$ then E is NCS-S-OS and $E^C = \langle\{\psi_3, \sigma_4\}, \varphi, \{\eta_2\}\rangle$ is a NCS-S-CS. Hence $E \cap E^C = \langle\varphi, \varphi, \{\eta_2, \psi_3, \sigma_4\}\rangle$ is a NCS-S-LCS.

(c) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$, $Q = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then (X, τ^{μ}) is NCSTS. If $E = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then E is NCS-α-OS and $E^C = \langle\{\sigma_4\}, \{\psi_3\}, \{\delta_1, \eta_2\}\rangle$ is a NCS-α-CS. Hence $E \cap E^C = \langle\varphi, \{\psi_3\}, \{\delta_1, \eta_2, \sigma_4\}\rangle$ is a NCS-α-LCS.

(d) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$ then is NCSTS. If $E = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$ then E is NCS-SP-OS and $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$ then is NCSTS. If $E = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$ then E is a NCS-SP-OS and $E^C = \langle\{\psi_3, \sigma_4\}, \varphi, \{\eta_2\}\rangle$ is a NCS-SP-CS. Hence $E \cap E^C = \langle\varphi, \varphi, \{\eta_2, \psi_3, \sigma_4\}\rangle$ is a NCS-SP-LCS.

(e) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $R = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$ then (X, τ^{μ}) is NCSTS. Let $H = \langle\{\delta_1\}, \varphi, \{\sigma_4\}\rangle$ and $E = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ hence E is NCS-Sα-CS and $E^C = \langle\{\delta_1, \sigma_4\}, \varphi, \varphi\rangle$ is NCS-Sα-OS. Hence $E \cap E^C = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-LCS.

Definition 3.5. Let E be any subset of NCSTS (X, τ^{μ})

(a) Then E is said to be a neutrosophic crisp supra semi-α-locally closed set (NCS-Sα-LC∗S for short) if $E = K \cap K^*$, where K is NCS-Sα-OS and K^* is NCSCS in (X, τ^{μ}).

(b) Then E is said to be a neutrosophic crisp supra semi-α-locally closed∗∗ set (NCS-Sα-LC ∗∗S for short) if $E = K \cap K^*$, where K is NCSOS and K^* is NCS-Sα-CS in (X, τ^{μ}).

Example 3.6.

(a) From 3.4 (e) $E^C = \langle\{\delta_1, \sigma_4\}, \varphi, \varphi\rangle$ is NCS-Sα-OS and $P^C = \langle\{\psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ is a NCSCS. Hence $E^C \cap P^C = \langle\varphi, \varphi, \{\delta_1, \eta_2\}\rangle$ is a NCS-Sα-LC ∗S.

(b) From 3.4 (e) $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$ is NCSOS and $E = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-CS. Hence $Q \cap E = \langle\varphi, \varphi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is a NCS-Sα-LC ∗∗S.

Remark 3.7.

(a) The class of all NCS-Sα-LCS is denoted by NCS-Sα-LCS (X, τ^{μ}).

(b) The class of all NCS-Sα-LC ∗S is denoted by NCS-Sα-LC ∗S (X, τ^{μ}).

(c) The class of all NCS-Sα-LC ∗∗S is denoted by NCS-Sα-LC ∗∗S (X, τ^{μ}).

Theorem 3.8. *In a NCSTS* (X, τ^{μ})

(a) $E \in$ NCS-Sα-LC $*$S $(X, \tau^{\mu}) \Rightarrow E \in$ NCS-Sα-LC S (X, τ^{μ})

(b) $E \in$ NCS-Sα-LC $**$S $(X, \tau^{\mu}) \Rightarrow E \in$ NCS-Sα-LCS (X, τ^{μ}).

Proof. (a) Since E is NCS-Sα-LC$*$S. Then we have $E = K \cap K^*$, where K is NCS-Sα-OS and K^* is NCSCS in (X, τ^{μ}). Since every NCSCS is NCS-Sα-CS in (X, τ^{μ}), $E = K \cap K^*$, where K is NCS-Sα-OS and K^* is NCS-Sα-CS in (X, τ^{μ}). Therefore $E \in$ NCS-Sα-LC S (X, τ^{μ}).

(b) Since E is NCS-Sα-LC$**$S. Then we have $E = K \cap K^*$, where K is NCSOS and K^* is NCS-Sα-CS in (X, τ^{μ}). Since every NCSOS is NCS-Sα-OS in (X, τ^{μ}), $E = K \cap K^*$, where K is NCS-Sα-OS and K^* is NCS-Sα-CS in (X, τ^{μ}). Therefore $E \in$ NCS-Sα-LC S (X, τ^{μ}).

Remark 3.9. Converse of 3.8 need not be true from 3.10.

Example 3.10. From 3.4 (e) $\langle \varphi, \varphi, \{\delta_1, \sigma_4\} \rangle$ is a NCS-Sα-LCS but not NCS-Sα-LC$*$S and NCS-Sα-LC$**$S in (X, τ^{μ}).

Remark 3.11. Since every NCS-Sα-LCS is the intersection of NCS-Sα-OS and NCS-Sα-CS then we can conclude the following.

Theorem 3.12. *A subset E of* (X, τ^{μ}) *is NCS-S*α*-LCS if and only if EC is the union of NCS-S*α*-OS and NCS-S*α*-CS.*

Theorem 3.13. *In a NCSTS* (X, τ^{μ})

(a) *If a subset E of* (X, τ^{μ}) *is NCSLCS then it is NCS-*α*-LCS.*

(b) *If a subset E of* (X, τ^{μ}) *is NCS-*α*-LCS then it is NCS-S*α*-LCS.*

(c) *If a subset E of* (X, τ^{μ}) *is NCSLCS then it is NCS-S*α*-LCS.*

(d) *If a subset E of* (X, τ^{μ}) *is NCS-*α*-LCS then it is NCS-P-LCS.*

(e) *If a subset E of* (X, τ^{μ}) *is NCS-S-LCS then it is NCS-SP-LCS.*

Proof.

(a) Since E is NCSLCS then $E = K \cap K^*$, where K is NCSOS and K^* is NCSCS. Since every NCSOS is NCS-α-OS, every NCSCS is NCS-α-CS. Therefore E is NCS-α-LCS.

(b) Since E is NCS-α-LCS then $E = K \cap K^*$, where K is NCS-α-OS and K^* is NCS-α-CS. Since every NCS-α-OS is NCS-Sα-OS, every NCS-α-CS is NCS-Sα-CS. Therefore E is NCS-Sα-LCS.

(c) Since E is NCSLCS then $E = K \cap K^*$, where K is NCSOS and K^* is NCSCS. Since every NCSOS is NCS-Sα-OS, every NCSCS is NCS-Sα-CS. Therefore E is NCS-Sα-LCS.

(d) Since E is NCS-α-LCS then $E = K \cap K^*$, where K is NCS-α-OS and K^* is NCS-α-CS. Since every NCS-α-OS is NCS-P-OS, every NCS-α-CS is NCS-P-CS. Therefore E is NCS-P-LCS.

(e) Since E is NCS-S-LCS then $E = K \cap K^*$, where K is NCS-S-OS and K^* is NCS-S-CS. Since every NCS-S-OS is NCS-SP-OS, every NCS-S-CS is NCS-SP-CS. Therefore E is NCS-SP-LCS.

Example 3.14. The converse of 3.13 need not be true.

(a) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$, $Q = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then (X, τ^μ) is NCSTS. If $E = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4\}\rangle$ then E is NCS-α-OS and $E^C = \langle\{\sigma_4\}, \varphi, \{\delta_1, \eta_2\}\rangle$ is a NCS-α-CS. Hence $E \cap E^C = \langle\varphi, \varphi, \{\delta_1, \eta_2, \sigma_4\}\rangle$ is a NCS-α-LCS but not NCSLCS.

(b) From 3.4 (e) $E \cap E^C = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-LCS but not NCS-Sα-LCS.

(c) From 3.4 (e) $E \cap E^C = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-LCS but not NCSLCS.

(d) Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$ then (X, τ^μ) is NCSTS. If $E = \langle\varphi, \varphi, \{\delta_1\}\rangle$ then E is NCS-P-OS and $E^C = \langle\{\delta_1\}, \varphi, \varphi\rangle$ is a NCS-P-CS. Hence $E \cap E^C = \langle\varphi, \varphi, \{\delta_1\}\rangle$ is a NCS-P-LCS but not NCS-α-LCS.

(e) From 3.4 (d) $E \cap E^C = \langle\varphi, \varphi, \{\eta_2, \psi_3, \sigma_4\}\rangle$ is a NCS-SP-LCS but not NCS-S-LCS.

4. Neutrosophic Crisp Supra Dense Sets and Submaximal Spaces

Definition 4.1. A subset E of a NCSTS (X, τ^μ) is called neutrosophic crisp supra dense (NCSDS for short) if NCS-cl $(E) = X_N$.

Example 4.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle, Q = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle, R = \langle\{\eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then (X, τ^μ) is NCSTS. If $E = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then E is NCSDS.

Definition 4.3. A NCSTS (X, τ^μ) is called neutrosophic crisp supra submaximal space (NCSSMS for short) if every dense subset in it is open in (X, τ^μ).

Example 4.4. From 4.2. E is a NCSDS and also E is open in (X, τ^μ). Hence E is NCSSMS.

Definition 4.5. A NCSTS (X, τ^μ) is called NCS-Sα-submaximal space (NCS-Sα-SMS for short) if every dense subset in it is NCS-Sα-OS in (X, τ^μ).

Example 4.6. From example 3.4 (e). If $E = \langle\{\delta_1, \sigma_4\}, \varphi, \varphi\rangle$. Then E is NCSDS and also it is NCS-Sα-OS in (X, τ^μ). Hence E is NCS-Sα-SMS.

Theorem 4.7. *Every NCSSMS is NCS-Sα-SMS.*

Proof. Let (X, τ^μ) be a NCSSMS and E be a neutrosophic crisp supra dense subset of (X, τ^μ). Then E is neutrosophic crisp supra open. Since every neutrosophic crisp supra open set is a NCS-Sα-OS. Hence E is NCS-Sα-OS. Therefore (X, τ^{μ}) is NCS-Sα-SMS.

The converse of 4.8 need not be true as seen from 4.8.

Example 4.8. From 4.6. E is NCS-Sα-SMS but not NCSSMS because E is not open in (X, τ^{μ}).

5. NCSLC-continuous and NCS-Sα-LC-continuous Functions in NCSTS

Definition 5.1. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCSLC-continuous if $\omega^{-1}(\xi) \in$ NCSLCS in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}).

Example 5.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1, \eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle, Q = \langle\{\eta_2\}, \varphi, \{\psi_3, \sigma_4\}\rangle, R = \langle\{\eta_2\}, \{\psi_3\}, \{\sigma_4\}\rangle$ then (X, τ^{μ}) is NCSTS and let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}, \sigma^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $L = \langle\varphi, \{\delta_1^*\}, \{\psi_3^*\}\rangle, M = \langle\varphi, \{\delta_1^*\}, \{\sigma_4^*, \psi_3^*\}\rangle, N = \langle\varphi, \{\delta_1^*\}, \{\eta_2^*, \psi_3^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \sigma_4^*, \omega(\eta_2) = \eta_2^*, \omega(\psi_3) = \delta_1^*$, $\omega(\sigma_4) = \psi_3^*$. N is open in (Y, σ^{μ}) then $\omega^{-1}(N) = \langle\varphi, \{\psi_3\}, \{\eta_2, \sigma_4\}\rangle = R \cap R^C$ is NCSLCS in (X, τ^{μ}). Hence the map is NCSLC-continuous.

Definition 5.3. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC-continuous if $\omega^{-1}(\xi) \in$ NCS-Sα-LCS in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}).

Example 5.4. From 3.4 (e) (X, τ^{μ}) is a NCSTS and $\langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-LCS. Let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}$, $\sigma^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\varphi, \varphi, \{\delta_1^*, \eta_2^*\}\rangle$, $M = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*\}\rangle$, $N = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*, \psi_3^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \delta_1^*, \omega(\eta_2) = \psi_3^*, \omega(\psi_3) = \sigma_4^*$, $\omega(\sigma_4) = \eta_2^*$. L is open in (Y, σ^{μ}) then $\omega^{-1}(L) = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-LCS in (X, τ^{μ}). Hence the map is NCS-Sα-LC-continuous.

Definition 5.5. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC$*$-continuous if $\omega^{-1}(\xi) \in$ NCS-Sα-LC$*$S in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}).

Example 5.6. From 3.4(e) (X, τ^{μ}) is a NCSTS, $E^C = \langle\{\delta_1, \sigma_4\}, \varphi, \varphi\rangle$ is a NCS-Sα-OS and Q^C is a NCSCS. Hence $E^C \cap Q^C = \langle\{\sigma_4\}, \varphi, \{\delta_1, \eta_2\}\rangle$ is a NCS-Sα-LC$*$S. Let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}$, $\sigma^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\{\delta_1^*\}, \varphi, \{\psi_3^*\}\rangle$, $M = \langle\varphi, \varphi, \{\psi_3^*\}\rangle$, $N = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*, \psi_3^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \eta_2^*, \omega(\eta_2) = \psi_3^*, \omega(\psi_3) = \sigma_4^*$, $\omega(\sigma_4) = \delta_1^*$. N is open in (Y, σ^{μ}) then $\omega^{-1}(N) = \langle\{\sigma_4\}, \varphi, \{\delta_1, \eta_2\}\rangle$ is a NCS-Sα-LC$*$S in (X, τ^{μ}). Hence the map is NCS-Sα-LC$*$-continuous.

Definition 5.7. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC$**$-continuous if $\omega^{-1}(\xi) \in$ NCS-Sα-LC$**$ S in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}).

Example 5.8. From 3.4(e) (X, τ^{μ}) is a NCSTS, $E = \langle\varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is a NCS-Sα-CS and P is a NCSOS. Hence $E \cap P = \langle\varphi, \varphi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is a NCS-Sα-LC$**$S. Let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}$, $\sigma^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\varphi, \varphi, \{\delta_1^*, \eta_2^*, \psi_3^*\}\rangle$, $M = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*\}\rangle$, $N = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*, \sigma_4^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \delta_1^*, \omega(\eta_2) = \sigma_4^*, \omega(\psi_3) = \eta_2^*$, $\omega(\sigma_4) = \psi_3^*$. L is open in (Y, σ^{μ}) then $\omega^{-1}(L) = \langle\varphi, \varphi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is a NCS-Sα-LC$**$S in (X, τ^{μ}). Hence the map is NCS-Sα-LC$**$-continuous.

Theorem 5.9. *Let* $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ *be a map then the following*

(a) *If* ω *is NCSLC- continuous, then* ω *is NCS-S*α*-LC-continuous.*

(b) *If* ω *is NCS-S*α*-LC$*$-continuous, then* ω *is NCS-S*α*-LC-continuous.*

(c) *If* ω *is NCS-S*α*-LC$**$-continuous, then* ω *is NCS-S*α*-LC-continuous.*

Proof.

(a) Assume that ω is NCSLC-continuous, then from the definition we have $\omega^{-1}(\xi) \in$ NCSLCS in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}). Since every NCSLCS is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-continuous.

(b) Assume that ω is NCS-Sα-LC∗-continuous, then from the definition we have $\omega^{-1}(\xi) \in$ NCS-Sα-LC∗S in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}). Since every NCS-Sα-LC∗S is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-continuous.

(c) Assume that ω is NCS-Sα-LC∗∗-continuous, then from the definition we have $\omega^{-1}(\xi) \in$ NCS-Sα-LC∗S in (X, τ^{μ}) for each NCSOS ξ of (Y, σ^{μ}). Since every NCS-Sα-LC∗∗S is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-continuous.

The converse of 5.9 need not be true as seen from 5.10.

Example 5.10. From 5.4 the map $\omega : X \to Y$ is NCS-Sα-LC-continuous but not NCSLC-continuous, NCS-Sα-LC∗-continuous, NCS-Sα-LC∗∗-continuous.

6. NCSLC-irresolute and NCS-Sα-LC-irresolute Functions in NCSTS

Definition 6.1. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCSLC-irresolute if $\omega^{-1}(\xi) \in$ NCSLCS in (X, τ^{μ}) for each $\xi \in$ NCSLC in (Y, σ^{μ}).

Example 6.2. From 3.4(e) (X, τ^{μ}) is a NCSTS and $P \cap P^{C} = \langle \varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is NCSLCS. From 5.8 (Y, σ^{μ}) is a NCSTS and $L \cap L^{C} = \langle \varphi, \varphi, \{\delta_1^{*}, \eta_2^{*}, \psi_3^{*}\}\rangle$ is a NCSLCS. Define an identity map $\omega : X \to Y$. Then $\omega^{-1}(L \cap L^{C}) = (P \cap P^{C})$. Hence the map is NCSLC-irresolute map.

Definition 6.3. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC-irresolute if $\omega^{-1}(\xi) \in$ NCS-Sα-LCS in (X, τ^{μ}) for each $\xi \in$ NCS-Sα-LC in (Y, σ^{μ}).

Example 6.4. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, P, Q, R\}$, where $P = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\psi_3, \sigma_4\}\rangle$, $R = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$ then (X, τ^{μ}) is NCSTS. Let $H = \langle\{\eta_2\}, \varphi, \{\delta_1\}\rangle$ and $E = \langle\varphi, \varphi, \{\delta_1, \psi_3\}\rangle$. Hence $E \cap E^C$ say $U, U = \langle\varphi, \varphi, \{\delta_1, \psi_3\}\rangle$ is a NCS-Sα-LCS. Let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}$, $\sigma^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\{\delta_1^*, \eta_2^*\}, \varphi, \{\psi_3^*\}\rangle$, $M = \langle\{\delta_1^*, \eta_2^*\}, \varphi, \{\psi_3^*, \sigma_4^*\}\rangle$, $N = \langle\{\eta_2^*\}, \varphi, \{\delta_1^*, \psi_3^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Let $H = \langle\{\delta_1^*\}, \varphi, \{\sigma_4^*\}\rangle$ and $E = \langle\varphi, \varphi, \{\delta_1^*, \sigma_4^*\}\rangle$. Hence $E \cap E^C$ say $U^*, U^* = \langle\varphi, \varphi, \{\delta_1^*, \sigma_4^*\}\rangle$ is a NCS-Sα-LCS. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \delta_1^*, \omega(\eta_2) = \eta_2^*$, $\omega(\psi_3) = \sigma_4^*, \omega(\sigma_4) = \psi_3^*$. Then $\omega^{-1}(U^*) = U$. Hence the map is NCS-Sα-LC-irresolute map.

Definition 6.5. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC∗-irresolute if $\omega^{-1}(\xi) \in$ NCS-Sα-LC∗S in (X, τ^{μ}) for each $\xi \in$ NCS-Sα-LC∗S in (Y, σ^{μ}).

Example 6.6. From 3.4 (e) (X, τ^{μ}) is a NCSTS, $E^C = \langle\{\delta_1, \sigma_4\}, \varphi, \varphi\rangle$ is NCS-Sα-OS and R^C is NCSLCS. Hence $E^C \cap R^C = \langle\{\delta_1\}, \varphi, \{\eta_2\}\rangle$ is NCS-Sα-LC∗S. Let $Y = \{\delta_1^*, \eta_2^*, \psi_3^*, \sigma_4^*\}$, $\sigma^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle\{\delta_1^*\}, \varphi, \{\psi_3^*\}\rangle > M = \langle\{\delta_1^*\}, \varphi, \{\psi_3^*, \sigma_4^*\}\rangle$, $N = \langle\{\delta_1^*\}, \varphi, \{\eta_2^*, \psi_3^*\}\rangle$ then (Y, σ^{μ}) is NCSTS. Let $H = \langle\varphi, \varphi, \{\delta_1^*\}\rangle$ and $E = \langle\varphi, \varphi, \{\delta_1, \psi_3\}\rangle$ then E is NCS-Sα-CS, $E^C = \langle\{\delta_1^*, \psi_3^*\}, \varphi, \varphi\rangle$ is NCS-Sα-OS and $L^C = \langle\{\psi_3^*\}, \varphi, \{\delta_1^*\}\rangle$ is NCSLCS. Hence $E^C \cap L^C = \langle\{\psi_3^*\}, \varphi, \{\delta_1^*\}\rangle$ is NCS-Sα-LC∗S. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \psi_3^*$, $\omega(\eta_2) = \delta_1^*$, $\omega(\psi_3) = \sigma_4^*$, $\omega(\sigma_4) = \eta_3^*$. Then $\omega^{-1}(E^C \cap L^C) = E^C \cap R^C$. Hence the map is NCS-S$\alpha$-LC∗ irresolute map.

Definition 6.7. A map $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ is said to be a NCS-Sα-LC∗∗-irresolute if $\omega^{-1}(\xi) \in$ NCS-Sα-LC ∗∗S in (X, τ^{μ}) for each $\xi \in$ NCS-Sα-LC ∗∗S in (Y, σ^{μ}).

Example 6.8. From 3.4(e) (X, τ^{μ}) is a NCSTS, $E = \langle \varphi, \varphi, \{\delta_1, \sigma_4\}\rangle$ is NCS-Sα-CS and P is NCSOS. Hence $E \cap P = \langle \varphi, \varphi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is NCS-Sα-LC∗∗S. From 6.6 (Y, σ^{μ}) is NCSTS $E = \langle \varphi, \varphi, \{\delta_1^*, \psi_3^*\}\rangle$ is NCS-Sα-CS and N is NCSOS. Then $E \cap N = \langle \varphi, \varphi, \{\delta_1^*, \eta_2^*, \psi_3^*\}\rangle$ is NCS-Sα-LC∗∗S. Define a map $\omega : X \to Y$ by $\omega(\delta_1) = \delta_1^*, \omega(\eta_2) = \sigma_4^*, \omega(\psi_3) = \eta_2^*, \omega(\sigma_4) = \psi_3^*$. Then $\omega^{-1}(E \cap N) = E \cap P$. Hence the map is NCS-Sα-LC∗∗ irresolute map.

Theorem 6.9. *Let* $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ *be a map then the following*

(a) *If* ω *is NCSLC-irresolute, then* ω *is NCS-Sα-LC-irresolute.*

(b) *If* ω *is NCS-Sα-LC∗-irresolute, then* ω *is NCS-Sα-LC-irresolute.*

(c) *If* ω *is NCS-Sα-LC∗∗-irresolute, then* ω *is NCS-Sα-LC-irresolute.*

Proof.

(a) Assume that ω is NCSLC-irresolute, then from the definition we have $\omega^{-1}(\xi) \in$ NCSLCS in (X, τ^{μ}) for each $\xi \in$ NCSLC in (Y, σ^{μ}). Since every NCSLCS is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-irresolute.

(b) Assume that ω is NCS-Sα-LC∗-irresolute, then from the definition we have $\omega^{-1}(\xi) \in$ NCS-Sα-LC∗S in (X, τ^{μ}) for each $\xi \in$ NCS-Sα-LC∗S in (Y, σ^{μ}). Since every NCS-Sα-LC∗S is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-irresolute.

(c) Assume that ω is NCS-Sα-LC∗∗-irresolute, then from the definition we have $\omega^{-1}(\xi) \in$ NCS-Sα-LC∗∗S in (X, τ^{μ}) for each $\xi \in$ NCS-Sα-LC∗∗S in (Y, σ^{μ}). Since every NCS-Sα-LC∗∗S is a NCS-Sα-LCS it follows that ω is NCS-Sα-LC-irresolute maps.

The converse of 6.9 need not be true as seen from 6.10.

Example 6.10. From 6.4 the map $\omega : X \to Y$ is NCS-Sα-LC-irresolute but not NCSLC-irresolute, NCS-Sα-LC∗∗-irresolute, NCS-Sα-LC∗∗-irresolute maps.

Theorem 6.11. *Let* $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ *and* $\vartheta : (Y, \tau^{\mu}) \to (Z, \eta^{\mu})$ *be any two maps then*

(a) $\vartheta \circ \omega$ *is NCS-S*α*-LC irresolute if* ω *and* ϑ *are NCS-S*α*-LC irresolute.*

(b) $\vartheta \circ \omega$ *is NCS-S*α*-LC∗ irresolute if* ω *and* ϑ *are NCS-S*α*-LC∗ irresolute.*

(c) $\vartheta \circ \omega$ *is NCS-S*α*-LC∗∗ irresolute if* ω *and* ϑ *are NCS-S*α*-LC∗∗ irresolute.*

Proof. Let ξ be any NCS-Sα-LC in (Z, η^{μ}). Since ϑ is NCS-Sα-LC irresolute, then $\vartheta^{-1}(\xi)$ is NCS-Sα-LC in (Y, σ^{μ}). Since ω is NCS-Sα-LC irresolute then $\omega^{-1}(\vartheta^{-1}(\xi))$ is NCS-Sα-LC in (X, τ^{μ}). Therefore $(\vartheta \circ \omega)^{-1}(\xi)$ is NCS-Sα-LC in (X, τ^{μ}). Hence $\vartheta \circ \omega$ is NCS-Sα-LC irresolute.

(b) and (c) are similar to (a).

Theorem 6.12. *Let* $\omega : (X, \tau^{\mu}) \to (Y, \sigma^{\mu})$ *and* $\vartheta : (Y, \sigma^{\mu}) \to (Z, \eta^{\mu})$ *be any two maps then*

(a) $\vartheta \circ \omega$ *is NCS-S*α*-LC continuous if* ω *is NCS-S*α*-LC irresolute and* ϑ *is NCS-S*α*-LC continuous.*

(b) $\vartheta \circ \omega$ *is NCS-S*α*-LC∗ continuous if* ω *is NCS-S*α*-LC∗ irresolute and* ϑ *is NCS-S*α*-LC∗ continuous.*

(c) $\vartheta \circ \omega$ *is NCS-S*α*-LC∗∗ continuous if* ω *is NCS-S*α*-LC∗∗ irresolute and* ϑ *is NCS-S*α*-LC∗∗ continuous.*

Proof.

(a) Let ξ be a NCSOS in (Z, η^{μ}). Since ϑ is NCS-Sα-LC continuous then $\vartheta^{-1}(\xi)$ is NCS-Sα-LC in (Y, σ^{μ}). Since ω is NCS-Sα-LC irresolute then $\omega^{-1}(\vartheta^{-1}(\xi))$ is NCS-Sα-LC in (X, τ^{μ}). Therefore $(\vartheta \circ \omega)^{-1}(\xi)$ is NCS-Sα-LC in (X, τ^{μ}). Hence $\vartheta \circ \omega$ is NCS-Sα-LC continuous.

(b) and (c) are similar to (a).

7. Conclusion and Future Work

The definitions of NCS-Sα-LCS, some other locally closed sets with examples are introduced in this paper and the relation between these sets are investigated. Also their continuity and irresolute maps introduced and verify its relation. Finally these concepts going to pave the way of new types of open, closed sets and their continuity in neutrosophic crisp supra topology.

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