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MODELLING INTER-SPECIFIC COMPETITION BETWEEN ONE PREY AND TWO PREDATORS

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Abstract

Content in this paper depicts interaction between one prey and two predator species. Their interaction might be defined as a food pyramid, with the prey in the lowermost level of the pyramid, secondary predators in the middle, and primary predators in the upper of the pyramid. This model represents the competition between populations of two predator species and one prey species study including three types of interactions amongst them which eventually establishes that growth rates of population are non-sympathetic in direction.

Introduction

The prey predator association contains the relations between two species and their important embellishments on one another [6]. In the predator prey association, one animal variety is serving different species. The predator is the creature being taken care of and the prey species is the creature being

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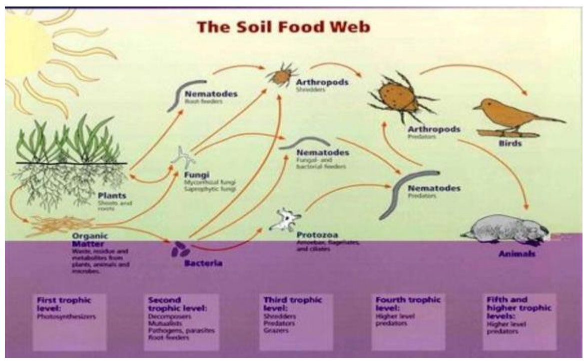
benefited from. The prey predator association develops after some time the same number old enough gathering of every species interrelates. In doing so, they have passionately affected the endurance and accomplishment of one another's animal groups [1]. The procedure of advancement chooses for versions which develop the appropriateness of every occupant. Scientists examining populace dynamic powers, or changes in populaces above time, have seen that prey predator relations stunningly have passionate effect on the populaces of every species, and that in light of the predator prey affiliation, these populace likenesses are associated [4].

Note that the prey masses developments when there are no predators; however the predator people decreases when there are no preys [5].



Almost all species in a given biological system are associated, to a degree that the loss of one animal group can affect others. From a wide perspective, the reliance can be arranged into advantageous connections and predator-prey connections [2]. It should not come as a surprise that the connection among predator and prey has a vital task to carry out with regards to environmental parity. A tilt on either side can trigger a domino impact on nature all in all. On the off chance that, for example, food gracefully is changed because of absence of prey, it will think about the number of inhabitants in savage species, as they will think that it is hard to recreate in the midst of food shortage. What’s more, similar to what we said before, if the number of inhabitants in predators descends, herbivores will run a mob in the environment [3].

Two-Predator, One-Prey Model

The one-prey, two-predator model is a difference on the basic Lotka-Volterra prey predator model that balance sheet for a state of affairs in which two predator populaces are existing and in cooperation predator on a single prey species as their primary food source [9]. A one-prey, two-predator system is self-possessed of three differential equations. Single differential equation signifies the change in populace extent above time for to each of the populaces [7].

For this situation, let us characterize the prey populace at time t as $p(t)$, the principal predator's populace simultaneously as $q(t)$, and the second predator's populace at time t as $r(t)$. The simplest system of equations modeling this type of behavior is as follows:

Equations:

In the absence of one Predator, the population of prey and predator species remains constant [8].

i.e. $$\frac{dp}{dt} = constant \tag{1}$$

In the absence of both predators, the population of prey species increases and this growth rate is proportional to the number of prey species.

i.e. $$\frac{dp}{dt} \propto p \tag{2}$$

In the absence of Prey, the population of both Predators species decreases and this decay rate is proportional to the number of Predator species.

i.e. $$\frac{dr}{dt} \propto -q \tag{3}$$

In the absence of Prey, the population of Predator species decreases and this decay rate is proportional to the number of Predator species.

i.e. $$\frac{dp}{dt} \propto -r \tag{4}$$

In the presence of one Predator, the population of Prey species decreases and this decay rate is proportion to the product of prey and Predator species at that time.

i.e. $$\frac{dp}{dt} \propto -pq \tag{5}$$

In the presence of one Predator, the population of Prey species decreases and this decay rate is proportion to the product of prey and Predator species at that time.

i.e. $$\frac{dp}{dt} \propto -pr \tag{6}$$

By assumptions (1), (4) and (5)

$$\frac{dp}{dt} = ap - bpq - cpr \tag{7}$$

In the presence of Prey, the population of Predator species decreases and this growth rate is proportion to the product of prey and Predator species at that time [11].

i.e. $$\frac{dp}{dt} \propto pq \tag{8}$$

By assumptions (2) and (7)

$$\frac{dp}{dt} = dpq - eq \tag{9}$$

In the presence of Prey, the population of Predator species decreases and this growth rate is proportional to the product of prey and Predator species at that time.

i.e. $$\frac{dp}{dt} \propto pr \tag{10}$$

By assumptions (3) and (9)

$$\frac{dr}{dt} = fpr - gr \tag{11}$$

For the equilibrium position $\frac{dp}{dt} 0 \Rightarrow ap - bpq - cpr = 0 \Rightarrow p =$ or $a - bq - cr = 0$

$$\frac{dp}{dt} = 0 \Rightarrow dpq - eq = 0 \Rightarrow q = 0 \text{ or } p = \frac{e}{d}$$

$$\frac{dr}{dt} = 0 \Rightarrow fpr - gr = 0 \Rightarrow r = 0 \text{ or } p = \frac{g}{f}$$

$$a - bq - cr = 0$$

If $q = 0 \Rightarrow r = \frac{a}{c}$ and if $r = 0 \Rightarrow q = \frac{a}{b}$

Now we have three equilibrium positions $(0, 0, 0), \left(\frac{e}{d}, 0, \frac{a}{c}\right), \left(\frac{g}{f}, \frac{a}{b}, 0\right)$.

So that if the initial population size is $\left(\frac{e}{d}, 0, \frac{a}{c}\right), \left(\frac{g}{f}, \frac{a}{b}, 0\right)$ then the population remains constant

If $p < \frac{e}{d} \Rightarrow dp - e < 0 \Rightarrow \frac{dq}{dt} < 0 \Rightarrow q$ is decreases

If $r < \frac{a}{c} \Rightarrow cr - a < 0 \Rightarrow \frac{dr}{dt} > 0 \Rightarrow p$ is increases and $q = 0$

If $p < \frac{g}{f} \Rightarrow fp - g < 0 \Rightarrow \frac{dr}{dt} < 0 \Rightarrow r$ is decreases

If $q < \frac{a}{b} \Rightarrow bq - a < 0 \Rightarrow \frac{dq}{dt} > 0 \Rightarrow q$ is increases and $q = 0$

Now from (5) and (6) $\frac{dp}{dt} = ap - bpq - cpr$ and $\frac{dq}{dt} = dpq - eq$

$$\frac{dp}{dq} = \frac{dp}{dt} \div \frac{dq}{dt} = \frac{dpq - eq}{ap - bpq - cpr} = \frac{q}{p} * \frac{dp - e}{a - bq - cr}$$

If $r < \frac{a}{c}$ then $\frac{dq}{dq} = \frac{q}{p} * \frac{dq - e}{-bq} = \frac{e - dp}{bp} = \frac{e}{bp} - \frac{d}{b}$

$$\Rightarrow dq = \frac{e}{bp} dp - \frac{d}{b} dp \Rightarrow q = \frac{e}{b} \log p - \frac{d}{b} p + c$$

$$\Rightarrow q - q_0 = \log p_b^e + \log p_b^e - \frac{d}{b} p - \frac{d}{b} p_0$$

$$\Rightarrow q = \log p_b^e + \log p_{0b}^{\,e} - \frac{d}{b}p - \frac{d}{b}p_0 + q_0$$

$$\therefore q = \frac{e}{b}(\log p + \log p_0) - \frac{d}{b}p - \frac{d}{b}p_0 + q_0$$

$$\therefore q = \frac{e}{b}\log pp_0 - \frac{d}{b}p - \frac{d}{b}p_0 + q_0$$

Now from (6) and (7) $\frac{dq}{dt} = dpq - eq$ and $\frac{dr}{dt} = fpr - gr$

$$\frac{dp}{dr} = \frac{dq}{dt} \div \frac{dr}{dt} = \frac{dpq - eq}{fpr - gr} = \frac{q}{r} * \frac{dp - e}{fp - g}$$

If $p < \frac{g}{f}$ then $\frac{dq}{dr} = \frac{q}{r} * \frac{dp - e}{fp - g} = \frac{q}{r} * \frac{dg - ef}{fg - ef} \Rightarrow \frac{dq}{q(dg - ef)} = \frac{dr}{r(fg - ef)}$

$$\Rightarrow \frac{\log\log q}{dg - ef} = \frac{\log\log r}{(fg - ef)} + c$$

$$\Rightarrow \frac{1}{q^{dg-ef}} - \frac{1}{r^{fg-ef}} = \frac{1}{q_0^{dg-ef}} + \frac{1}{r_0^{fg-ef}}$$

$$\therefore \frac{1}{q^{dg-ef}} = \frac{1}{q_0^{fg-ef}} - \frac{1}{r_0^{fg-ef}} + \frac{1}{r^{fg-ef}}$$

$$\frac{dp}{dt} = ap - bpq - cpr \Rightarrow \frac{dp}{b} = (a - bq - cr)dt \Rightarrow \log p = (a - bq - cr)t$$

$$\Rightarrow p = e^{(a-bq-cr)t}$$

$$\frac{dq}{dt} - dpq - eq \Rightarrow \frac{dq}{q} = (dp - e)dt \Rightarrow \log\log p = (dp - e)t$$

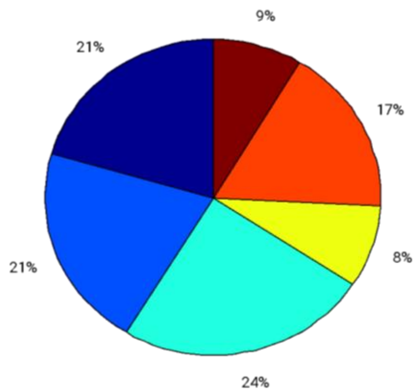
$$\Rightarrow p = e^{(dp-e)t}$$

MATLAB coding of graph

```
P = 10:5:100;

p0 = 1.5;

q0 = 0.5;

e = 0.1;

b = 0.25;
```

```
d = 0.7;
Q = [];
for i = 1:length(P)
q = q0 + (log(P(i)))^(e/b)+(log(p0))^(e/b)-(d/b)*P(i)-(d/b)*p0;
Q = [Q q];
end
plot(P,Q)
pie abs(Q)
```

Graph of Predator Species



Conclusion

In this paper, a three-dimensional isolated predator-prey model is considered and gained three types of equilibrium points. Steady conditions are found and the results are explained with appropriate parameter values. For instance, food assets of prey populaces are restricted and not just by predation and no predator can eat endless amounts of prey. Numerous different instances of rehashed connections among predator and prey populaces have been set up in the research center or experiential in nature, however all in all these are more beneficial fitting by models including terms that signify profound limits.

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