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INTUITIONISTIC FUZZY SOFT CUBIC RELATIONS

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Abstract

We propose the definition of intuitionistic fuzzy soft cubic relations and its properties are studied.

1. Introduction

In 1999, the conception of a soft set was proposed by Moldstov [2]. Zadeh [6] deals with the hypothesis of a fuzzy set in 1965. He ventilates the theory of an interval valued fuzzy sets [7]. K. Attansov [1, 3, 4] suggested the hypothesis of an intuitionistic fuzzy set and also introduces the theory of an interval valued intuitionistic fuzzy sets. The concept of fuzzy soft set was initiated by Maji et al. [8] and he enlarged the idea of soft sets to intuitionistic fuzzy system [9]. The notion of cubic set was initiated by Jun et al. [5].

2. Intuitionistic Fuzzy Soft Cubic Relations

Definition 2. Let U be an initial universal set and (P, E_1) and (Q, E_2) be two intuitionistic fuzzy soft cubic sets (IFSCS). Then the relation between (P, E_1) and (Q, E_2) is defined as $(R, E_1 \times E_2)$ where R is a mapping given

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by $R : E_1 \times E_2 \rightarrow IFSCS(U)$. This is called an intuitionistic fuzzy soft cubic set relation (IVIFSSR).

Remark 2.2. Let U be an initial universal set and $(P_1, E_1), (P_2, E_2), \ldots, (P_n, E_n)$ be the number of IFSCS over U. Then a relation Υ between them is defined as a pair $(R, E_1XE_2, \ldots, XE_n)$, where R is a mapping given by $R : E_1XE_2, \ldots, XE_2 \rightarrow IFSCS(U)$.

Example 3.2. Let us consider an IFSCS (P, E_1). Let the universal set $U = \{s_1, s_2, s_3, s_4\}$ and the set of parameters $E_1 = \{e_1, e_3\}$ then representation of IFSCS (P, E_1) is given below:

U	e_1	e_3
s_1	([0.4,0.5],[0.3,0.5])[0.3,0.5]	([0.3,0.6],[0.3,0.4])[0.4,0.3]
s_2	([0.5,0.7],[0.2,0.3])[0.2,0.6]	([0.5,0.6],[0.1,0.4])[0.5,0.4]
s_3	([0.4,0.6],[0.1,0.3])[0.6,0.2]	([0.5,0.7],[0.2,0.3])[0.7,0.2]
s_4	([0.2,0.4],[0.5,0.6])[0.4,0.5]	([0.6,0.7],[0.2,0.3])[0.3,0.6]

(ii) Let us consider an IFSCS (Q, E_2). Let the universal set $U = \{s_1, s_2, s_3, s_4\}$ and the set of parameters $E_2 = \{e_2, e_4\}$. Then the representation of IFSCS (Q, E_2) is given below:

U	e_2	e_4
s_1	([0.5,0.8],[0.1,0.2])[0.4,0.6]	([0.5,0.6],[0.2,0.4])[0.3,0.5]
s_2	([0.3,0.6],[0.2,0.3])[0.6,0.2]	([0.2,0.6],[0.3,0.4])[0.1,0.6]
s_3	([0.4,0.7],[0.1,0.3])[0.3,0.6]	([0.4,0.6],[0.3,0.4])[0.8,0.1]
s_4	([0.4,0.5],[0.3,0.4])[0.5,0.4]	([0.3,0.7],[0.1,0.2])[0.4,0.4]

Then the IFSCS relation $\mathcal{R} = (R, E_1XE_2)$ is given below:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
s_1	([0.4,0.5],[0.3,0.5]) [0.3,0.6]	([0.4,0.5],[0.3,0.5]) [0.3,0.5]	([0.3,0.6],[0.3,0.4]) [0.4,0.6]	([0.3,0.6],[0.3,0.4]) [0.3,0.5]
s_2	([0.3,0.6],[0.2,0.3]) [0.2,0.6]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]	([0.3,0.6],[0.2,0.4]) [0.5,0.4]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]
s_3	([0.4,0.6],[0.1,0.3]) [0.3,0.6]	([0.4,0.6],[0.3,0.4]) [0.6,0.2]	([0.4,0.7],[0.2,0.3]) [0.3,0.6]	([0.4,0.6],[0.3,0.4]) [0.7,0.2]
s_4	([0.2,0.4],[0.5,0.6]) [0.4,0.5]	([0.2,0.4],[0.5,0.6]) [0.4,0.5]	([0.4,0.5],[0.3,0.4]) [0.3,0.6]	([0.3,0.7],[0.2,0.3]) [0.3,0.6]

Definition 2.4. The order of the relational matrix is (ρ, σ) where $\rho =$ number of points in universal set and $\sigma =$ number of parameters. In the above example, the relational matrix $\mathcal{R}$ is of order (4, 4). If $= \sigma$, then the relational matrix is called square matrix.

Definition 2.5. Let $\mathcal{R}, \mathcal{S} \in \gamma_U(E_1XE_2), \mathcal{R} = (R, E_1XE_2), \mathcal{S} = (S, E_1XE_2)$, and the order of relational matrices are same. Then we define

(i) $\mathcal{R} \cup \mathcal{S} = (R \blacksquare S,\ E_1XE_2)$, where $R \blacksquare S : E_1XE_2 \to IVIFCS(U)$ and is defined as $(R \blacksquare S)(e_ie_j) \vee S(e_ie_j)$ for $(e_ie_j) \in E_1XE_2$, where $\vee$ denotes the intuitionistic fuzzy soft cubic union.

(ii) $\mathcal{R} \cap \mathcal{S} = (R \diamond S,\ E_1XE_2)$ where $R \diamond S : E_1XE_2 \to IVIFSCS(U)$ and is defined as $(\mathcal{R} \cap \mathcal{S})(e_i,\ e_j) \vee S(e_i,\ e_j)$ for $(e_i,\ e_j) \in E_1XE_2$, where $\wedge$ denotes the intuitionistic fuzzy soft cubic intersection.

(iii) $\mathcal{R}^c = (\sim R,\ E_1XE_2)$ where $R : E_1XE_2 \to IVFSS(U)$ and is defined as $(\sim R)(e_i,\ e_j)R(e_i,\ e_j)^c$ for $(e_i,\ e_j) \in E_1XE_2$, where c denotes the intuitionistic fuzzy soft cubic complement.

Example 2.6. Consider the IFSCS relations $\mathcal{R} = (R,\ E_1XE_2)$ and $\mathcal{S} = (S,\ E_1XE_2)$ as given below $\mathcal{R} = (R,\ E_1XE_2)$.

U	$(e_1,\ e_2)$	$(e_1,\ e_4)$	$(e_3,\ e_2)$	$(e_3,\ e_4)$
s_1	([0.4,0.5],[0.3,0.5]) [0.3,0.6]	([0.4,0.5],[0.3,0.5]) [0.3,0.5]	([0.3,0.5],[0.3,0.4]) [0.4,0.6]	([0.3,0.4],[0.3,0.5]) [0.3,0.5]
s_2	([0.3,0.4],[0.4,0.5]) [0.2,0.6]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]	([0.3,0.6],[0.2,0.4]) [0.5,0.4]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]
s_3	([0.4,0.6],[0.1,0.4]) [0.3,0.6]	([0.4,0.6],[0.3,0.4]) [0.6,0.2]	([0.4,0.7],[0.2,0.3]) [0.3,0.6]	([0.4,0.5],[0.3,0.5]) [0.7,0.2]
s_4	([0.2,0.3],[0.5,0.6]) [0.4,0.5]	([0.2,0.4],[0.5,0.6]) [0.4,0.5]	([0.6,0.7],[0.1,0.2]) [0.3,0.6]	([0.3,0.7],[0.2,0.3]) [0.3,0.6]

$\mathcal{S} = (S,\ E_1XE_2)$

U	$(e_1,\ e_2)$	$(e_1,\ e_4)$	$(e_3,\ e_2)$	$(e_3,\ e_4)$
s_1	([0.4,0.7],[0.2,0.3]) [0.2,0.7]	([0.3,0.4],[0.5,0.6]) [0.7,0.5]	([0.2,0.4],[0.3,0.6]) [0.4,0.2]	([0.3,0.5],[0,0.3]) [0.3,0.2]
s_2	([0.5,0.6],[0,04]) [0.3,0.4]	([0.5,0.6],[0.3,0.4]) [0.6,0.4]	([0.4,0.6],[0.2,0.4]) [0,0.5]	([0.5,0.6],[0.2,0.3]) [0.3,0.5]
s_3	([0.3,0.4],[0.2,0.4]) [0.6,0.4]	([0.4,0.6],[0,0.3]) [0.5,0.2]	([0.8,0.9],[0,0.1]) [0.7,0.3]	([0.4,0.6],[0,0.2]) [0.5,0.2]
s_4	([0.4,0.8],[0,0.2]) [0.5,0.4]	([0.4,0.6],[0.3,0.4]) [0.6,0.3]	([0.4,0.6],[0.2,0.4]) [0.5,0.4]	([0.4,0.6],[0.2,0.3]) [0.8,0.1]

Then $\mathcal{R} \cup \mathcal{S}$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
s_1	([0.4,0.7],[0.2,0.3]) 0.3,0.6]	([0.4,0.5],[0.3,0.5]) [0.7,0.5]	([0.3,0.5],[0.3,0.4]) [0.4,0.2]	([0.3,0.5],[0,0.3]) [0.3,0.2]
s_2	([0.5,0.6],[0,0.4]) [0.3,0.6]	([0.5,0.6],[0.3,0.4]) [0.6,0.4]	([0.4,0.6],[0.2,0.4]) [0.5,0.4]	([0.5,0.6],[0.2,0.3]) [0.3,0.5]
s_3	([0.4,0.6],[0.1,0.4]) 0.6,0.4]	([0.4,0.6],[0,0.3]) [0.6,0.2]	([0.8,0.9],[0,0.1]) [0.7,0.3]	([0.4,0.6],[0,0.2]) [0.7,0.2]
s_4	([0.4,0.8],[0,0.2]) [0.5,0.4]	([0.4,0.6],[0.3,0.4]) [0.6,0.3]	([0.6,0.7],[0.1,0.2]) [0.5,0.4]	([0.4,0.7],[0.2,0.3]) [0.8,0.1]

$\mathcal{R} \cap \mathcal{S}$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
s_1	([0.4,0.5],[0.3,0.5]) [0.2,0.7]	([0.3,0.4],[0.5,0.6]) [0.3,0.5]	([0.2,0.4],[0.3,0.6]) [0.4,0.6]	([0.3,0.4],[0.3,0.5]) [0.3,0.5]
s_2	([0.3,0.4],[0.4,0.5]) [0.2,0.6]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]	([0.3,0.6],[0.2,0.4]) [0,0.5]	([0.2,0.6],[0.3,0.4]) [0.1,0.6]
s_3	([0.3,0.4],[0.2,0.4]) [0.3,0.6]	([0.4,0.6],[0.3,0.4]) [0.5,0.2]	([0.4,0.7],[0.2,0.3]) [0.3,0.6]	([0.4,0.5],[0.3,0.5]) [0.3,0.5]
s_4	([0.2,0.3],[0.5,0.6]) [0.4,0.5]	([0.2,0.4],[0.5,0.6]) [0.4,0.5]	([0.4,0.6],[0.2,0.4]) [0.3,0.6]	([0.3,0.6],[0.2,0.3]) [0.3,0.6]

$\mathcal{R}^c$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
s_1	([0.3,0.5],[0.4,0.5]) [0.6,0.3]	([0.3,0.5],[0.4,0.5]) [0.5,0.3]	([0.3,0.4],[0.3,0.5]) [0.6,0.4]	([0.3,0.5],[0.3,0.4]) [0.5,0.3]
s_2	([0.4,0.5],[0.3,0.4]) [0.6,0.2]	([0.3,0.4],[0.2,0.6]) [0.6,0.1]	([0.2,0.4],[0.3,0.6]) [0.4,0.5]	([0.3,0.4],[0.2,0.6]) [0.6,0.1]
s_3	([0.1,0.4],[0.4,0.6]) [0.6,0.3]	([0.3,0.4],[0.4,0.6]) [0.2,0.6]	([0.2,0.3],[0.4,0.7]) [0.6,0.3]	([0.3,0.5],[0.4,0.5]) [0.2,0.7]
s_4	([0.5,0.6],[0.2,0.3]) [0.5,0.4]	([0.5,0.6],[0.2,0.4]) [0.5,0.4]	([0.1,0.2],[0.6,0.7]) [0.6,0.3]	([0.2,0.3],[0.3,0.7]) [0.6,0.3]

Theorem 2.7. *Let* $\mathcal{R}, \mathcal{S}, \mathcal{T} \in \gamma_U(E_1 X E_2)$ *and the order of the relational matrices are same. Then the following statements hold*:

(i) $(\mathcal{R} \cup \mathcal{S})^c = \mathcal{R}^c \cap \mathcal{S}^c$

(ii) $(\mathcal{R} \cap \mathcal{S})^c = \mathcal{R}^c \cup \mathcal{S}^c$

(iii) $\mathcal{R} \cup (\mathcal{S} \cup \mathcal{T}) = (\mathcal{R} \cup \mathcal{S}) \cup \mathcal{T}$

(iv) $\mathcal{R} \cap (\mathcal{S} \cap \mathcal{T}) = (\mathcal{R} \cap \mathcal{S}) \cap \mathcal{T}$

(v) $\mathcal{R} \cap (\mathcal{S} \cup \mathcal{T}) = (\mathcal{R} \cap \mathcal{S}) \cup (\mathcal{R} \cap \mathcal{T})$

(vi) $\mathcal{R} \cup (\mathcal{S} \cap \mathcal{T}) = (\mathcal{R} \cup \mathcal{S}) \cap (\mathcal{R} \cup \mathcal{T})$

Proof. (i) Let $\mathcal{R} = (R, E_1 X E_2)$, $\mathcal{S} = (S, E_1 X E_2)$ then $\mathcal{R} \cup \mathcal{S} = (R \blacksquare S, E_1 X E_2)$ where $R \blacksquare S : E_1 X E_2 \to IFSCS(U)$ and is defined as $(R \blacksquare S)(e_i, e_j) = R(e_i, e_j) \vee S(e_i, e_j)$ for $(e_i, e_j) \in E_1 X E_2$.

Then $(\mathcal{R} \cup \mathcal{S})^c = [R(e_i, e_j) \vee S(e_i, e_j)]^c = [\{\langle r_k, (T_{R(e_i, e_j)}(r_k)), (F_{R(e_i, e_j)}(r_k)),$
$[\mu_{R(e_i, e_j)}(r_k), \vartheta_{R(e_i, e_j)}(r_k)]\rangle\} \vee \{r_k, (T_{S(e_i, e_j)}(r_k), F_{S(e_i, e_j)}(r_k)), [\mu_{S(e_i, e_j)}(r_k),$
$\vartheta_{S(e_i, e_j)}(r_k)]\rangle\}\} : r_k \in U]^c = \{\langle r_k,$
$\max(\inf(T_{R(e_i, e_j)}(r_k), \inf(T_{R(e_i, e_j)}(r_k)), \max(\sup(T_{R(e_i, e_j)}(r_k),$
$\sup(T_{S(e_i, e_j)}(r_k))], [\min(\inf(F_{R(e_i, e_j)}(r_k), \inf(F_{S(e_i, e_j)}(r_k)),$
$\min(\sup(F_{R(e_i, e_j)}(r_k), \sup(F_{S(e_i, e_j)}(r_k))], [\max(\mu_{R(e_i, e_j)}(r_k),$
$\mu_{S(e_i, e_j)}(r_k), \min(\vartheta_{R(e_i, e_j)}(r_k), \vartheta_{R(e_i, e_j)}(r_k))] >: r_k \in U\}^c$
$= \{\langle r_k, [\min(\inf(F_{R(e_i, e_j)}(r_k), \inf(F_{S(e_i, e_j)}(r_k)), \min(\sup((r_k),$
$\sup(F_{S(e_i, e_j)}(r_k))], [\max(\inf(T_{R(e_i, e_j)}(r_k), \inf(T_{S(e_i, e_j)}(r_k)),$
$\max(\sup(T_{R(e_i, e_j)}(r_k), \sup(T_{S(e_i, e_j)}(r_k))], [\min(\vartheta_{R(e_i, e_j)}(r_k),$
$\vartheta_{R(e_i, e_j)}(r_k)), \max(\mu_{R(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k)]\rangle : r_k \in U\}.$

Now, $\mathcal{R}^c \cap \mathcal{S}^c = (\sim, R, E_1 X E_2) \cap (\sim S, E_1 X E_2)$ where $\sim R, \sim S : E_1$ $XE_2 \to IVIFSCS(U)$ are defined as $\sim R(e_i, e_j) = (R(e_i, e_j))^c$ and $\sim S(e_i, e_j)$

$= (S(e_i, e_j))^c$ for $(e_i, e_j) \in E_1XE_2$, we have $(\sim R, E_1XE_2) \cap (\sim S, E_1XE_2)$ $= (\sim R \lozenge \sim S, E_1XE_2)(e_i, e_j)$.

Now for $(e_i, e_j) \in E_1XE_2$, $(\sim R \lozenge \sim S)(e_i, e_j) = \sim R(e_i, e_j) \wedge \sim S(e_i, e_j) = \{\langle r_k, [F_{R(e_i, e_j)}(r_k), T_{R(e_i, e_j)}(r_k)], [\vartheta_{R(e_i, e_j)}(r_k), \mu_{R(e_i, e_j)}(r_k)]\} \wedge \{\langle r_k, [F_{S(e_i, e_j)}(r_k), T_{S(e_i, e_j)}(r_k)], [\vartheta_{S(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k)]\} : r_k \in U\} = \{\langle r_k, [\min(\inf(F_{R(e_i, e_j)}(r_k), \inf(F_{S(e_i, e_j)}(r_k)), \min(\sup(F_{R(e_i, e_j)}(r_k), \sup(F_{S(e_i, e_j)}(r_k))], \max(\inf(T_{R(e_i, e_j)}(r_k), \inf(T_{S(e_i, e_j)}(r_k)), \max(\sup(T_{R(e_i, e_j)}(r_k), \sup(T_{S(e_i, e_j)}(r_k))], [\min(\vartheta_{R(e_i, e_j)}(r_k), \vartheta_{S(e_i, e_j)}(r_k)), \max(\mu_{R(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k))]\rangle : r_k \in U\}$.

Then $(\mathcal{R} \cup \mathcal{S})^c = \mathcal{R}^c \cap \mathcal{S}^c$

(ii) Let $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$ then $\mathcal{R} \cap \mathcal{S} = (R \lozenge S, E_1XE_2)$ where $R \lozenge S : E_1XE_2 \to IFSCS(U)$ and is defined as $(R \lozenge S)(e_i, e_j)$ $= R(e_i, e_j) \wedge S(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Then $(\mathcal{R} \cap \mathcal{S})^c = [R(e_i, e_j) \wedge S(e_i, e_j)]^c$

$= [\{\langle r_k, T_{R(e_i,e_j)}(r_k), F_{R(e_i,e_j)}(r_k)), [\mu_{R(e_i,e_j)}(r_k), \vartheta_{R(e_i,e_j)}(r_k)]\rangle\} \wedge [\{\langle r_k, T_{S(e_i,e_j)}(r_k), F_{S(e_i,e_j)}(r_k), [\mu_{S(e_i,e_j)}(r_k), \vartheta_{S(e_i,e_j)}(r_k)]\rangle\} : r_k \in U]^c = \{\langle r_k, [\min(\inf(T_{R(e_i, e_j)}(r_k), \inf(T_{S(e_i, e_j)}(r_k)), \min(\sup(T_{R(e_i, e_j)}(r_k), \sup(T_{S(e_i, e_j)}(r_k))], [\max(\inf(F_{R(e_i, e_j)}(r_k), \inf(F_{S(e_i, e_j)}(r_k)), \max(\sup(F_{R(e_i, e_j)}(r_k), \sup(F_{S(e_i, e_j)}(r_k))], [\min(\mu_{R(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k), \max(\vartheta_{R(e_i, e_j)}(r_k), \vartheta_{S(e_i, e_j)}(r_k))], r_k \in U\}^c = \{\langle r_k, \max(\inf(F_{R(e_i, e_j)}(r_k), \inf(F_{S(e_i, e_j)}(r_k)), \max(\sup(F_{R(e_i, e_j)}(r_k), \sup(F_{S(e_i, e_j)}(r_k))], \min(\inf(T_{R(e_i, e_j)}(r_k), \inf(T_{S(e_i, e_j)}(r_k)), \min(\sup(T_{R(e_i, e_j)}(r_k), \sup(T_{S(e_i, e_j)}(r_k))], \max(\mu_{R(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k), \min(\vartheta_{R(e_i, e_j)}(r_k), \vartheta_{S(e_i, e_j)}(r_k))], r_k \in U\}$.

Now, $\mathcal{R}^c \cup \mathcal{S}^c = (\sim R, E_1XE_2) \cup (\sim S, E_1XE_2)$ where $\sim R, \sim S : E_1 XE_2 \to IVIFCS(U)$ are defined as $\sim R(e_i, e_j) = (R(e_i, e_j))^c$ and $\sim S(e_i, e_j)$

$= (S(e_i, e_j))^c$ for $(e_i, e_j) \in E_1XE_2$, we have $(\sim R, E_1XE_2) \cup (\sim S, E_1XE_2)$ $= (\sim R \blacksquare \sim S, E_1XE_2)(e_i, e_j)$

Now for $(e_i, e_j) \in E_1XE_2$, $(\sim R \blacksquare \sim S)(e_i, e_j) = \sim R(e_i, e_j) \cup \sim S(e_i, e_j) =$ $[\{\langle r_k, F_{R(e_i, e_j)}(r_k), T_{R(e_i, e_j)}(r_k), [\vartheta_{R(e_i, e_j)}(r_k), \mu_{R(e_i, e_j)}(r_k)\rangle\}] \vee \{\langle r_k, F_{S(e_i, e_j)}(r_k),$ $T_{S(e_i, e_j)}(r_k), [\vartheta_{S(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k)]\rangle\} : r_k \in U = \{\langle r_k, [\max(\inf(F_{R(e_i, e_j)}(r_k),$ $\inf(F_{S(e_i, e_j)}(r_k)), \max(\sup(F_{R(e_i, e_j)}(r_k), \sup(F_{S(e_i, e_j)}(r_k))], [\min(\inf(T_{R(e_i, e_j)}$ $(r_k), \inf(T_{S(e_i, e_j)}(r_k)), \min(\sup(T_{R(e_i, e_j)}(r_k), \sup(T_{S(e_i, e_j)}(r_k))],$ $\max(\mu_{R(e_i, e_j)}(r_k), \mu_{S(e_i, e_j)}(r_k), \min(\vartheta_{R(e_i, e_j)}(r_k), \vartheta_{S(e_i, e_j)}(r_k))], r_k \in U\}$.

Then $(\mathcal{R} \cup \mathcal{S})^c = \mathcal{R}^c \cap \mathcal{S}^c$.

(iii) Let $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$, $\mathcal{T} = (T, E_1XE_2)$ then $\mathcal{R} \cup \mathcal{S}$ $= (R \blacksquare S, E_1XE_2)$, where $R \blacksquare S : E_1XE_2 \to IVIFCS(U)$ and is defined as $(R \blacksquare S)(e_i, e_j) = R(e_i, e_j) \vee S(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Then $(\mathcal{R} \cup \mathcal{S}) \cup \mathcal{T} = ((R \blacksquare S) \blacksquare T, E_1XE_2)$ where $(R \blacksquare S) \blacksquare T : E_1XE_2$ $\to IVIFCS(U)$ and is defined as $(R \blacksquare S) \blacksquare T(e_i, e_j) = R(e_i, e_j) \vee S(e_i, e_j)$ $\vee T(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Now $(R(e_i, e_j) \vee S(e_i, e_j)) \vee T(e_i, e_j) = R(e_i, e_j) \vee (S(e_i, e_j) \vee T(e_i, e_j))$.

Therefore $((R \blacksquare S) \blacksquare T)(e_i, e_j) = (R \blacksquare (S \blacksquare T))(e_i, e_j)$.

Also $(\mathcal{R} \cup \mathcal{S}) \cup \mathcal{T} = \mathcal{R} \cup (\mathcal{S} \cup \mathcal{T}) = (\mathcal{R} \blacksquare (\mathcal{S} \blacksquare \mathcal{T})), E_1XE_2)$.

Therefore $(\mathcal{R} \cup \mathcal{S}) \cup \mathcal{T} = \mathcal{R} \cup (\mathcal{S} \cup \mathcal{T})$.

(iv) Let $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$, $\mathcal{T} = (T, E_1XE_2)$ then $\mathcal{R} \cap \mathcal{S}$ $= (R \Diamond S, E_1XE_2)$ where $R \Diamond S : E_1XE_2 \to IFCS(U)$ and is defined as $(R \Diamond S)(e_i, e_j) = R(e_i, e_j) \wedge S(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Then $(\mathcal{R} \cap \mathcal{S}) \cap \mathcal{T} = ((R \Diamond S) \Diamond T, E_1XE_2)$ where $(R \Diamond S) \Diamond T : E_1X$ $E_2 \to IVIFCS(U)$ and is defined as $(R \Diamond S) \Diamond T(e_i, e_j) = R(e_i, e_j)$

$\vee S(e_i, e_j) \wedge T(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Now $(R(e_i, e_j) \wedge S(e_i, e_j) \wedge T(e_i, e_j) = R(e_i, e_j)(S(e_i, e_j)) \wedge T(e_i, e_j)$.

Therefore $((R \Diamond S) \Diamond T)(e_i, e_j) = (R \Diamond (S \Diamond T))(e_i, e_j)$.

Also $(\mathcal{R} \cap \mathcal{S}) \cap \mathcal{T} = \mathcal{R} \cap (\mathcal{S} \cap \mathcal{T}) = (R \Diamond (S \Diamond T)), E_1XE_2)$.

Therefore $(\mathcal{R} \cap \mathcal{S}) \cap \mathcal{T} = \mathcal{R} \cap (\mathcal{S} \cap \mathcal{T})$.

(v) Let $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$, $\mathcal{T} = (T, E_1XE_2)$ then $\mathcal{S} \cup \mathcal{T} = (S \blacksquare T, E_1XE_2)$ where $S \blacksquare T : E_1XE_2 \to IVIFSCS(U)$ and is defined as $(S \blacksquare T)(e_i, e_j) = S(e_i, e_j) \vee T(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Then $\mathcal{R} \cap (\mathcal{S} \cup \mathcal{T}) = (R \Diamond (S \blacksquare T), E_1XE_2)$ where $(R \Diamond (S \blacksquare T)) : E_1XE_2 \to$ $IFSCS(U)$ and is defined as $R \Diamond (S \blacksquare T)(e_i, e_j) = R(e_i, e_j) \wedge (S(e_i, e_j)$ $\vee T(e_i, e_j))$ for $(e_i, e_j) \in E_1XE_2$.

Now $(R(e_i, e_j) \wedge S(e_i, e_j)) \vee T(e_i, e_j)) = (R(e_i, e_j) \wedge S(e_i, e_j)) \vee R(e_i, e_j)$ $\wedge T(e_i, e_j)$.

We have $R \Diamond (S \blacksquare T)(e_i, e_j) = (R(e_i, e_j) \wedge S(e_i, e_j)) \vee (R(e_i, e_j) \wedge T(e_i, e_j))$.

Also $(\mathcal{R} \cap \mathcal{S}) \cup (\mathcal{R} \cap \mathcal{T}) = (R \Diamond S, E_1XE_2) \cup (R \Diamond T, E_1XE_2) = ((R \Diamond S) \blacksquare (R \Diamond T),$ $E_1XE_2)$.

Now for $(e_i, e_j) \in E_1XE_2, ((R \Diamond S) \blacksquare (R \Diamond T))(e_i, e_j) = ((R \Diamond S)(e_i, e_j) \vee (R \Diamond T)$ $(e_i, e_j) = (R(e_i, e_j) \wedge S(e_i, e_j)) \vee (R(e_i, e_j) \wedge T(e_i, e_j)) = R \Diamond (S \blacksquare T)(e_i, e_j)$.

Then $\mathcal{R} \cap (\mathcal{S} \cup \mathcal{T}) = (\mathcal{R} \cap \mathcal{S}) \cup (\mathcal{R} \cap \mathcal{T})$.

(vi) Let $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$, $\mathcal{T} = (T, E_1XE_2)$ then $\mathcal{S} \cap \mathcal{T}$ $= (S \Diamond E_1XE_2)$ where $\mathcal{S} \Diamond \mathcal{T} = T : E_1XE_2 \to IVIFSCS(U)$ and is defined as $(S \Diamond T)(e_i, e_j) = S(e_i, e_j) \wedge T(e_i, e_j)$ for $(e_i, e_j) \in E_1XE_2$.

Then $\mathcal{R} \cup (\mathcal{S} \cap \mathcal{T}) = (R \blacksquare (S \Diamond T), E_1XE_2)$ where $(R \blacksquare (S \Diamond T)) : E_1XE_2 \to$ $IVIFSCS(U)$ and is defined as $R \blacksquare (S \Diamond T)(s_i, s_j) = R(s_i, s_j) \vee S(s_i, s_j) \wedge T(s_i, s_j)$

for $(s_i, s_j) \in E_1XE_2$.

Now $(R(s_i, s_j)) \vee (S(s_i, s_j)) \wedge (T(s_i, s_j)) = (R(s_i, s_j) \vee S(s_i, s_j)) \wedge (R(s_i, s_j) \vee T(s_i, s_j))$.

We have $R \blacksquare (S \lozenge T)(s_i, s_j) = (R(s_i, s_j) \vee S(s_i, s_j)) \wedge (R(s_i, s_j) \vee T(s_i, s_j))$.

Also $(\mathcal{R} \cup \mathcal{S}) \cap (\mathcal{R} \cup \mathcal{T}) = (R \blacksquare S, E_1XE_2) \cap (R \blacksquare T, E_1XE_2) = ((R \blacksquare S) \lozenge (R \blacksquare T), E_1XE_2)$.

Now for $(e_i, e_j) \in E_1XE_2$, $((R \blacksquare S) \lozenge (R \blacksquare T))(e_i, e_j) = ((R \blacksquare S)(e_i, e_j) \wedge (R \blacksquare S))(e_i, e_j) = (R(e_i, e_j) \vee S(e_i, e_j)) \wedge (R(e_i, e_j) \vee T(e_i, e_j)) = R \blacksquare (S \lozenge T)(e_i, e_j)$.

Then $\mathcal{R} \cup (\mathcal{S} \cap \mathcal{T}) = (\mathcal{R} \cup \mathcal{S}) \cap (\mathcal{R} \cup \mathcal{T})$.

3. Some More Concepts

Definition 3.1. Let $\mathcal{R}, \mathcal{S} \in \gamma_U(E_1XE_2)$ and their relational matrices are of same order. Then $\mathcal{R} \subseteq \mathcal{S}$ if $R(s_i, s_j) \subseteq S(s_i, s_j)$ for $(s_i, s_j) \in (E_1XE_2)$ where $\mathcal{R} = (R, E_1XE_2)$, $\mathcal{S} = (S, E_1XE_2)$.

Definition 3.2. Let (R, E_1) and (S, E_2) be two intuitionistic fuzzy soft cubic sets. Then a null relation between (R, E_1) and (S, E_2) is denoted as ϕ_U and is defined as $\phi_U = (T_0, E_1XE_2)$ where $T_0(e_i, e_j) = \{\langle t_k, [0, 0], [1, 1], [0, 1]\rangle / t_k \in U\}$ for $(e_i, e_j) \in (E_1XE_2)$. Here U denotes the initial universe set.

Definition 3.3. Let (R, E_1) and (S, E_2) be two intuitionistic fuzzy soft cubic sets. Then an absolute relation between (R, E_1) and (S, E_2) is denoted as ρ_U and is defined as $\rho_U = (T_1, E_1XE_2)$ where $T_1(s_i, s_j) = \{\langle t_k, [1, 1], [0, 0](0, 1)\rangle / t_k \in U\}$ for $(s_i, s_j) \in (E_1XE_2)$. Here U denotes the initial universe set.

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