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FORCING NONSPLIT GEODETIC DOMINATION NUMBER OF A GRAPH

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Abstract

We have described an ingenious parameter referred to as the forcing nonsplit geodetic domination number of a graph. We have investigated a few results regarding the forcing nonsplit geodetic domination number of a graph. In addition, we have determined the forcing nonsplit geodetic domination number of some graphs. Also we have obtained for integers a, b with $0 \leq a < b$, and $b > a + 1$, there exists a connected graph G such that $f_{\gamma nsg}(G) = a$ and $\gamma_{nsg}(G) = b$.

1. Introduction

By a graph $G = (V, E)$, we mean a simple graph with order $n \geq 2$. For basic graph theoretic terminology, (see [3], [7]). The neighborhood of a point v is the set $N(v)$ consisting of all points u which are adjacent with v. The closed neighborhood of a point v is the set $N[v] = N(v) \cup \{v\}$. A point v is an extreme point if the subgraph induced by its neighbor is complete. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. A set S of points is a geodetic set

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[9] if $I[S] = V(G)$ and the minimum cardinality of such set is geodetic number and is denoted as $g(G)$. A set of points D in a graph G is a dominating set [8] if each point of G is dominated by some point of D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. A subset S of $V(G)$ is a geodetic dominating set [5] of G if S is a geodetic set and a dominating set of G. The minimum cardinality of a geodetic dominating set is called the geodetic domination number of G and is denoted by $\gamma_g(G)$. A geodetic set S of a graph $G = (V, E)$ is a nonsplit geodetic set [12] if $\langle V - S\rangle$ is connected. The nonsplit geodetic number $g_{ns}(G)$ of G is the minimum cardinality of a nonsplit geodetic set of G. A set $S \subseteq V(G)$ is said to be a nonsplit geodetic dominating set [2] of G if S is both a nonsplit geodetic set and a dominating set of G ($\langle V - S\rangle$ is connected). The minimum cardinality of nonsplit geodetic dominating set of G is called the nonsplit geodetic domination number of G and is denoted by $\gamma_{nsg}(G)$.

We present some basic information in this area that help in the creation of the paper. In section 2, we defined and demonstrated the forcing nonsplit geodetic domination number of a graph. Section 3 contains the paper's conclusion. In the sequel, the following results are used.

Theorem 1.1 [2]**.** *If G is a connected graph with n points*, *then* $2 \leq g(G) \leq \gamma_{nsg}(G) \leq n - 1$.

Definition 1.2 [7]**.** The join of G and H is denoted as $G + H$ and is defined as the graph with $G \cup H$ and all edges joining $V(G)$ and $V(H)$.

2. Forcing Nonsplit Geodetic Domination Number of a Graph

In this section we define the forcing nonsplit geodetic domination number $f_{\gamma nsg}(G)$ of a graph and initiate a study of this parameter.

Definition 2.1. Let G be a connected graph and S be a minimum nonsplit geodetic dominating set of G. A subset $T \subseteq S$ is called a forcing subset of S, if S is the unique minimum nonsplit geodetic dominating set containing T. The forcing subset of S of minimum cardinality is the minimum forcing subset of S. The forcing nonsplit geodetic domination number of S denoted by

$f_{\gamma nsg}(S)$ is the cardinality of the minimum forcing subset of S and is given by $f_{\gamma nsg}(G) = \min\{f_{\gamma nsg}(S)\}$, where the minimum is taken over all minimum nonsplit geodetic dominating sets S of G.

Example 2.2. For the graph G given in Figure 1, $S_1 = \{d_1, d_3, d_6, d_7\}$ and $S_2 = \{d_2, d_4, d_6, d_7\}$ are the only two minimum nonsplit geodetic dominating sets of G such that $f_{\gamma nsg}(S_1) = 1$ and $f_{\gamma nsg}(S_2) = 1$. Hence $f_{\gamma nsg}(G) = 1$.

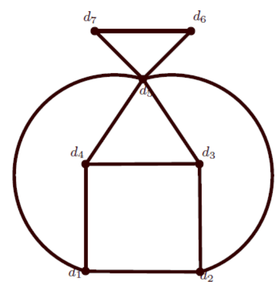

Figure 1. A graph G with $f_{\gamma nsg}(G) = 1$.

Theorem 2.3. *For any connected graph* G, $0 \le f_{\gamma nsg}(G) \le \gamma_{nsg}(G) \le n-1$.

Proof. It is clear from the definition of forcing nonsplit geodetic dominating set of G that, $f_{\gamma nsg}(G) \ge 0$. Let S be a minimum nonsplit geodetic dominating set of G. Since $f_{\gamma nsg}(G) \le \gamma_{nsg}(G)$ and $f_{\gamma nsg}(G) = \min\{f_{\gamma nsg}(S)\}$, it follows that $f_{\gamma nsg}(G) \le \gamma_{nsg}(G)$. Also from Theorem 1.1, $\gamma_{nsg}(G) \le n-1$. Therefore $0 \le f_{\gamma nsg}(G) \le \gamma_{nsg}(G) \le n-1$. □

Theorem 2.4. *Let G be a connected graph. Then*

1. $f_{\gamma nsg}(G) = 0$ *if and only if G has a unique nonsplit geodetic dominating set.*

2. $f_{\gamma nsg}(G) = 1$ *if and only if G has at least two nonsplit geodetic dominating set, there exists an element in one of the nonsplit geodetic dominating sets which is not in any other nonsplit geodetic dominating set.*

3. $f_{\gamma nsg}(G) = \gamma_{nsg}(G)$ *if and only if there is no unique nonsplit geodetic dominating set containing any of its proper subsets*.

Proof. 1. Assume that $f_{\gamma nsg}(G) = 0$. Then by definition 2.1, $f_{\gamma nsg}(S) = 0$ for some nonsplit geodetic dominating set S of G. So the empty set is the minimum forcing subset for S. Since the empty set is always a subset of any set, it follows that S is the unique nonsplit geodetic dominating set of G. Conversely, assume that S is the unique nonsplit geodetic dominating set. It is clear that ϕ is the forcing subset of S. Thus $f_{\gamma nsg}(G) = 0$.

2. $f_{\gamma nsg}(G) = 1$. Then by Theorem 2.4 (1), G has at least two minimum nonsplit geodetic dominating sets. There is a singleton subset S_1 of a minimum nonsplit geodetic dominating set S of G such that S_1 is not a subset of any other minimum nonsplit geodetic dominating set of G. Thus S is the unique minimum nonsplit geodetic dominating set containing one of its elements. Conversely, assume that G has at least two nonsplit geodetic dominating sets, there exists an element in one of the nonsplit geodetic dominating sets which is not in any other nonsplit geodetic dominating set. Clearly, $f_{\gamma nsg}(G) = 1$.

3. Let $f_{\gamma nsg}(G) = \gamma_{nsg}(G)$. Then $f_{\gamma gs}(S) = \gamma_{nsg}(G)$, for every minimum nonsplit geodetic dominating set S in G. Also by the Theorem 2.3, $\gamma_{nsg}(G) \geq 2$ and hence $f_{\gamma nsg}(G) \geq 2$. Then by Theorem 2.4 (1), G has at least two minimum nonsplit geodetic dominating sets, so that the empty set is not a forcing subset for any minimum nonsplit geodetic dominating set of G. Since $f_{\gamma nsg}(S) = \gamma_{nsg}(G)$, no proper subset of S is a forcing subset of S. Thus no minimum nonsplit geodetic dominating set of G is the unique minimum nonsplit geodetic dominating set containing any of its proper subsets.

Conversely, assume that no minimum nonsplit geodetic dominating set of G is the unique minimum nonsplit geodetic dominating set containing any of its proper subsets. To show that $f_{\gamma nsg}(G) = \gamma_{nsg}(G)$. By our assumption G contains more than one minimum nonsplit geodetic dominating set and no subset of any minimum nonsplit geodetic dominating set S other than S_0 is a forcing subset for S. Hence, $f_{\gamma nsg}(G) = \gamma_{nsg}(G)$. □

Definition 2.5. A vertex v of G is said to be nonsplit geodetic dominating point of G, if v belongs to every nonsplit geodetic dominating set of G.

Example 2.6. For the graph G given in Figure 1, d_6 and d_7 are the nonsplit geodetic dominating points of G.

Theorem 2.7. *Let G be a connected graph and W be the set of all nonsplit geodetic dominating points of G, then* $f_{\gamma nsg}(G) \leq \gamma_{nsg}(G) - |W|$.

Proof. Let S be a minimum nonsplit geodetic dominating set of G, then $\gamma_{nsg}(G) = |S|$, $W \subseteq S$ and S is the unique minimum nonsplit geodetic dominating set containing $S - W$. Then $f_{\gamma nsg}(G) \leq |S - W| = |S| - |W|$ $= \gamma_{nsg}(G) - |W|$. □

Corollary 2.8. *If G is a connected graph with k extreme points, then* $f_{\gamma nsg}(G) \leq \gamma_{nsg}(G) - k$.

Theorem 2.9. *For the path* $G = P_n (n \geq 5)$,

$$f_{\gamma nsg}(G) = \begin{cases} 0 & \text{if } n = 4, \\ \left\lfloor \dfrac{n-3}{2} \right\rfloor & \text{if } n \geq 5. \end{cases}$$

Proof. Case (i). Let $n = 4$. Let $P_4 : d_1, d_2, d_3, d_4$. Let $S = \{d_1, d_4\}$. It is clear that S is the unique minimum nonsplit geodetic dominating set of G. Hence it follows from Theorem 2.4 (1) that $f_{\gamma nsg}(G) = 0$.

Case (ii). Let $n \geq 5$.

Let $P_n : d_1, d_2, d_3, \ldots, d_n$. It is observed that $S_1 = \{d_1, d_2, \ldots, d_{n-3}, d_n\}$, $S_2 = \{d_1, d_2, \ldots, d_{n-4}, d_{n-1}, d_n\}$, $S_3 = \{d_1, d_2, \ldots, d_{n-5}, d_{n-1}, d_n\}, \ldots, S_{n-3} = \{d_1, d_4, d_5, \ldots, d_{n-1}, d_n\}$ are the $n-3$ minimum nonsplit geodetic dominating sets. Clearly no $\left\lfloor \frac{n-3}{2} \right\rfloor$ element subset is contained in any other minimum nonsplit geodetic dominating sets of P_n. Therefore, $f_{\gamma nsg}(G) = \left\lfloor \frac{n-3}{2} \right\rfloor$. □

Theorem 2.10. *For the cycle* $G = C_n (n \geq 4)$,

$$f_{\gamma nsg}(G) == \begin{cases} 3 & if\ n = 4, \\ 4 & if\ n = 5, \\ \left\lfloor \frac{n}{2} \right\rfloor & if\ n > 5. \end{cases}$$

Proof. Case (i). Let $n = 4$.

Let $C_4 : b_1, b_2, b_3, b_4, b_1$. Clearly C_4 has four minimum nonsplit geodetic dominating sets such as $S_1 = \{b_1, b_2, b_3\}$, $S_2 = \{b_2, b_3, b_4\}$, $S_3 = \{b_1, b_3, b_4\}$, and $S_4 = \{b_1, b_2, b_4\}$ in such a way that $f_{\gamma nsg}(S_1) = 3$, $f_{\gamma nsg}(S_2) = 3$, $f_{\gamma nsg}(S_3) = 3$ and $f_{\gamma nsg}(S_4) = 3$. Hence, $f_{\gamma nsg}(C_4) = 3$.

Case (ii)**.** Let $n = 5$.

Let $C_5 : b_1, b_2, b_3, b_4, b_5, b_1$. It is observed that C_5 has five minimum nonsplit geodetic dominating sets $S_1 = \{b_1, b_2, b_3, b_4\}$, $S_2 = \{b_2, b_3, b_4, b_5\}$, $S_3 = \{b_1, b_3, b_4, b_5\}$, $S_4 = \{b_1, b_2, b_4, b_5\}$ and $S_5 = \{b_1, b_2, b_3, b_5\}$ such that $f_{\gamma nsg}(S_1) = 4$, $f_{\gamma nsg}(S_2) = 4$, $f_{\gamma nsg}(S_3) = 4$, $f_{\gamma nsg}(S_4) = 4$ and $f_{\gamma nsg}(S_5) = 4$. Hence, $f_{\gamma nsg}(C_5) = 4$.

Case (iii). Let $n > 5$.

Let $C_n : b_1, b_2, \ldots, b_n, b_1$. Let S be any $n - 2$ consecutive points of C_n. Clearly S is a nonsplit geodetic dominating set of C_n. Every C_n has n minimum nonsplit geodetic dominating sets. Let $S = \{b_1, b_2, \ldots, b_{n-2}\}$. It is easily verified that S is the unique nonsplit geodetic dominating set of C_n containing $\{b_1, b_3, b_5, \ldots, b_{n-3}, b_{n-2}\}$ for n is even and $\{b_1, b_3, b_5, \ldots, b_{n-4}, b_{n-2}\}$ for n is odd. Therefore $f_{\gamma nsg}(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$. □

Theorem 2.11. *For the wheel* $W_n = K_1 + C_{n-1}$, $(n \geq 6)$,

$$f_{\gamma nsg}(W_n) = \begin{cases} 1 & if\ n\ is\ odd, \\ 2 & is\ n\ is\ even. \end{cases}$$

Proof. Case (i). Let n be odd.

Let $V(W_n) = \{x, y_1, y_2, \ldots, y_{n-1}\}$ be the points of W_n. It is verified that $S_1 = \{y_1, y_3, y_5, \ldots, y_{n-4}, y_{n-2}\}$ and $S_2 = \{y_2, y_4, y_6, \ldots, y_{n-3}, y_{n-1}\}$ are the only two minimum nonsplit geodetic dominating set of W_n so that $f_{\gamma nsg}(S_1) = 1$ and $f_{\gamma nsg}(S_2) = 1$. Hence $f_{\gamma nsg}(W_n) = 1$.

Case (ii). Let n be even.

Let $V(W_n) = \{x, y_1, y_2, \ldots, y_{n-1}\}$ be the points of W_n. It is observed that W_n has $n-1$ minimum nonsplit geodetic dominating sets. Also every singleton subset belongs to every other minimum nonsplit geodetic dominating sets of W_n. Therefore $f_{\gamma nsg}(W_n) \geq 1$. Let $u, v \in V(W_n)$, where u and v are adjacent points. Then these adjacent points u, v do not belong to any other minimum nonsplit geodetic dominating set of W_n. Hence it follows that $f_{\gamma nsg}(W_n) = 2$. □

Theorem 2.12. *For any two integers* $2 \leq m \leq n$,

$$f_{\gamma nsg}(K_{m,\, n}) = \begin{cases} 0 & \text{if } m = 1,\ n \geq 2, \\ 3 & \text{if } m = n = 2, \\ 1 & \text{if } m = 2,\ n \geq 3, \\ 4 & \text{if } 3 \leq m \leq n. \end{cases}$$

Proof. Let $U = \{u_1, u_2, \ldots, u_m\}$ and $V = \{v_1, v_2, \ldots, v_n\}$ be the two bipartite sets.

Case (i)**.** Let $m = 1,\ n \geq 2$.

Let $S = \{v_1, v_2, \ldots, v_n\}$. Clearly S is the unique minimum nonsplit geodetic dominating set of G. Hence it is follows from Theorem 2.4 (1) that $f_{\gamma nsg}(K_{m,\, n}) = 0$.

Case (ii)**.** Let $m = n = 2$.

Let $S_1 = \{v_1, v_2, u_1\}, S_2 = \{v_1, v_2, u_2\}, S_3 = \{v_1, u_1, u_2\}$ and $S_4 = \{v_2, v_1, u_2\}$. Clearly $S_i, 1 \leq i \leq 4$ are the minimum nonsplit geodetic dominating sets such that $f_{\gamma nsg}(S_1) = 3$, $f_{\gamma nsg}(S_2) = 3$, $f_{\gamma nsg}(S_3) = 3$ and $f_{\gamma nsg}(S_4) = 3$. Hence $f_{\gamma nsg}(K_{m,n}) = 3$.

Case (iii). Let $m = 2$, $n \geq 3$.

Let $S_1 = \{u_1, v_1, v_2, \ldots, v_n\}$ and $S_2 = \{u_2, v_1, v_2, \ldots, v_n\}$. It is clear that S_1 and S_2 be the minimum nonsplit geodetic dominating set such that $f_{\gamma nsg}(S_1) = 1$ and $f_{\gamma nsg}(S_2) = 1$. Therefore $f_{\gamma nsg}(K_{m,n}) = 1$.

Case (iv). Let $3 \leq m \leq n$.

Let $S = \{u_i, u_j, v_r, v_s\}$, $1 \leq i < j \leq m$ and $1 \leq r < s \leq n$. Clearly S is the minimum nonsplit geodetic dominating set such that 1-element, 2 element, 3-element subset belong to some other minimum nonsplit geodetic dominating sets of $K_{m,n}$. It is clear that the 4-element subset does not belong to any other subset of $K_{m,n}$. Therefore, $f_{\gamma nsg}(K_{m,n}) = 4$. □

Theorem 2.13. *For the graph* $G = K_1 + P_{n-1}$, $n \geq 6$,

$$f_{\gamma nsg}(G) = \begin{cases} 0 & \text{if } n \text{ is even,} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let x be a point of K_1 and $h_1, h_2, \ldots, h_{n-1}$ be the points of P_{n-1}.

Case (i). Let n be even.

Let $S = \{h_1, h_2, \ldots, h_{n-1}\}$. Clearly S is the unique minimum nonsplit geodetic dominating set of G. Hence it is follows from Theorem 2.4 (1) that $f_{\gamma nsg}(G) = 0$.

Case (ii). Let n be odd.

It is observed that, $S_1 = \{h_1, h_3, \ldots, h_{n-2}, h_{n-1}\}$, $S_2 = \{h_1, h_2, h_4, \ldots, h_{n-3}, h_{n-1}\}$, $S_3 = \{h_1, h_3, h_4, \ldots, h_{n-3}, h_{n-1}\}$, $\ldots$, $S_{\frac{n-1}{2}-1} = \{h_1, h_3, h_5, h_6, \ldots, h_{n-3}, h_{n-1}\}$, $S_{\frac{n-1}{2}} = \{h_1, h_3, h_5, \ldots, h_{n-3}, h_{n-1}\}$ are the $\frac{n-1}{2}$ minimum nonsplit geodetic dominating sets of G. Clearly, some of the singleton subsets of S_i's are forcing subsets of S_i's and some of the 2-element subsets of S_i's are forcing subsets of S_i's. Therefore, $f_{\gamma nsg}(G) = \min\{f_{\gamma nsg}(S_1), f_{\gamma nsg}(S_2), \ldots, f_{\gamma nsg}(S_{\frac{n-1}{2}})\} = \min\{1, 2\} = 1$. □

Theorem 2.14. *For integers a, b with $0 \leq a < b$, and $b > a + 1$, there exists a connected graph G such that $f_{\gamma nsg}(G) = a$ and $\gamma_{nsg}(G) = b$.*

Proof. Case 1. $f_{\gamma nsg}(G) = a = 0$, $\gamma_{nsg} = b$.

Let $P_5 : v_1, v_2, v_3, v_4, v_5$. Let H be the graph obtained from P_5 by adding 2 new points h_1 and h_2, and connecting them to each v_i, $(1 \leq i \leq 5)$. Let G be the graph obtained from H by adding $b - 1$ new points $l_1, l_2, \ldots, l_{b-1}$ and connect each l_i, $(1 \leq i \leq b - 1)$ to h_2. The resulting graph G is given in Figure 2.

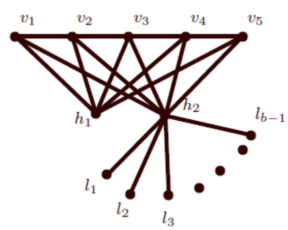

Figure 2. Graph G with $f_{\gamma nsg}(G) = a = 0$, $\gamma_{nsg} = b$.

Let $S = \{l_1, l_2, \ldots, l_{b-1}\}$ be the set of all end points of G. Clearly S is a subset of every nonsplit geodetic dominating set of G. But S is not a nonsplit geodetic dominating set of G. Let $S_1 = S \cup \{h_1\}$. Clearly S_1 is the unique minimum nonsplit geodetic dominating set of G. Now, it follows from Theorem 2.4, that $f_{\gamma nsg}(G) = 0$ and $\gamma_{nsg}(G) = b$.

Case 2. $f_{\gamma nsg}(G) = a < \gamma_{nsg} = b$.

Let $C_1 : d_i, h_i, r_i = d_{i+1}, y_i, d_i (1 \leq i \leq a)$ be a copy of cycle 4. Let H_1 be the graph obtained from C_i by adding $b - a$ new points $l_1, l_2, \ldots, l_{b-a-1}, x_0$ and joining each l_i, $(1 \leq i \leq b - a - 1)$ to r_a and joining x_0 to d_1. Let G be the graph obtained from H_1 by identifying the edges $h_i y_i$, $(1 \le i \le a)$. The resulting graph G is given in Figure 3.

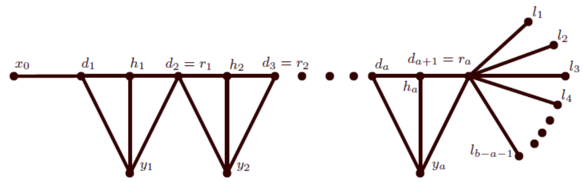

Figure 3. Graph G with $f_{\gamma nsg}(G) = a < \gamma_{nsg} = b$.

Let $S = \{l_1, l_2, \ldots, l_{b-a-1}, x_0\}$ be the set of all end points of G. First we show that $\gamma_{nsg} = b$. It is clear that S is a subset of every nonsplit geodetic dominating set of G, but S is not a nonsplit geodetic dominating set of G. Let $Q = \{d_1, y_1, y_2, \ldots, y_a, h_1, h_2, \ldots, h_a, r_a\}$. Let S_1 be a γ_{nsg}-set of G. Then it is easily observed that (i) nonsplit geodetic dominating set must contain 'a' points from Q. (ii) some pairs of Q such that $\{d_1, y_1\} \notin S$, $\{y_a, r_a\} \notin S$ and $\{y_i, h_i\} \notin S (1 \le i \le a)$. Thus $\gamma_{nsg}(G) \ge b - a + a = b$. On the other hand, since the set $S_1 = S \cup \{d_1, y_2, y_3, \ldots, y_a\}$ is a nonsplit geodetic dominating set of G, it follows that $\gamma_{nsg}(G) \le |S_1| = b$. Hence $\gamma_{nsg}(G) = b$. Next we show that $f_{\gamma nsg}(G) = a$. Every nonsplit geodetic dominating set of G contains S and so it follows from Theorem 2.7 that $f_{\gamma nsg}(G) \le \gamma_{nsg}(G) - |S| = b - (b - a) = a$. Now since $\gamma_{nsg}(G) = b$ and every nonsplit geodetic dominating set of G contains S. It is easily seen that every nonsplit geodetic dominating set S_1 is of the form $S \cup \{j_1, j_2, \ldots, j_a\}$ where $j_i \in Q$. Let T be any proper subset of S_1 with $|T| < a$. Clearly for some nonsplit geodetic dominating set S_1 such that $T \cap S_1 \ne \phi$. Therefore T is not a forcing subset of S_1. Clearly S_1 is the unique minimum nonsplit geodetic dominating set containing $\{j_1, j_2, \ldots, j_a\}$. Therefore $f_{\gamma nsg}(G) = a$.

3. Conclusion

This work initiates the study of forcing nonsplit geodetic domination number of a graph. The idea of forcing nonsplit geodetic domination number of a graph can be extended to study the forcing nonsplit edge geodetic domination number of a graph.

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