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INTERVAL VALUED FUZZY W-HAUSDORFF SPACES

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Abstract

In this paper the Interval Valued Fuzzy W-Hausdorff Space is defined by extending the definition of fuzzy Hausdorff space of Warren [4].

1. Introduction

Decision making is one of the most complex issues that need scientific analysis of various factors both tangibles and intangibles like attitude, belief, taste and preferences of people. Real world decision making problems with these factors are very often uncertain or vague in a number of ways. In 1965, Zadeh [14] introduced the concept of fuzzy set theory which provides us with an intuitively pleasing method of representing one form of uncertainty. In 1975, Zadeh [15] made an extension of the concept of a fuzzy set by an

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interval valued fuzzy set with an interval valued membership function. Mondal and Samantha [10] defined the topology of interval valued fuzzy sets and studied some of its properties. In Section 2 of this paper, preliminary definitions regarding fuzzy set, interval valued fuzzy set and interval valued fuzzy topological space are given. In Section 3 of this paper, the Hausdorff axiom of Gantner, Steinlage and Warren [4] is extended to interval valued fuzzy topological space and it is proved that the concept is hereditary and productive.

2. Preliminary Definitions

Definition 2.1 [14]**.** Let X be an arbitrary non empty set. Let $I = [0, 1]$. A fuzzy set in X is a mapping from X into I that is a fuzzy set is an element of I^X.

For any two fuzzy sets μ, λ in I^X,

(i) $\mu = \lambda \Leftrightarrow \mu(x) = \lambda(x)$, for every $x \in X$

(ii) $\mu \leq \lambda \Leftrightarrow \mu(x) \leq \lambda(x)$, for every $x \in X$ when $\mu \leq \lambda$, the fuzzy set λ is said to contain μ.

(iii) The union $\mu \vee \lambda$ and the intersection $\mu \wedge \lambda$ are defined, respectively, by

$$(\mu \vee \lambda)(x) = \max\{\mu(x), \lambda(x)\} \text{ and}$$

$$(\mu \wedge \lambda)(x) = \min\{\mu(x), \lambda(x)\}$$

(iv) The complement μ^c of a fuzzy set μ is defined by $\mu^c(x) = 1 - \mu(x)$ for every $x \in X$.

(v) For a family $\{\mu_j / j \in J\}$ of fuzzy sets defined on a set X, the union $\underset{j \in J}{\vee} \mu_j$ and the intersection $\underset{j \in J}{\wedge} \mu_j$ are defined, respectively, by

$$(\underset{j \in J}{\vee} \mu_j)(x) = \underset{j \in J}{\vee} (\mu_j(x)) \text{ and}$$

$$(\underset{j \in J}{\wedge} \mu_j)(x) = \underset{j \in J}{\wedge} (\mu_j(x))$$

(vi) For any $\alpha \in I$, the constant fuzzy set α in X is a fuzzy set in X defined by $\alpha(x) = \alpha$ for every $x \in X$.

Definition 2.2 [14]**.** Let μ and λ be fuzzy sets on X and Y respectively. The *Cartesian product* $\mu * \lambda$ of μ and λ is a fuzzy set on $X \times Y$ defined by

$$(\mu * \lambda)(x, y) = \min(\mu(x), \lambda(y)) \text{ for each } (x, y) \in X \times Y.$$

Definition 2.3 [2]**.** Let X be a non empty set. A subset $\tau \subset I^X$ is called a *fuzzy topology* on X iff τ satisfies the following requirements.

(i) The constant fuzzy sets 0 and 1 belong to τ.

(ii) $\mu_j \in \tau$ for each $j \in J$ implies $\vee_{j \in J} \mu_j \in \tau$.

(iii) $\mu, \lambda \in \tau$ implies $\mu \wedge \lambda \in \tau$.

The pair (X, τ) is called a *fuzzy topological space*.

Definition 2.4 [13]**.** Let (X, τ_1) and (Y, τ_2) be two fuzzy topological spaces. Then the *product topology* $\tau_1 \times \tau_2$ on $X \times Y$ is the fuzzy topology having the collection $\{\mu * \lambda / \mu \in \tau_1, \lambda \in \tau_2\}$ as a basis.

Definition 2.5 [13]**.** Let $\{(X_j, \tau_j)/j \in J\}$ be a family of fuzzy topological spaces and $X = \prod_{j \in J} X_j$. The product topology on X is the one with basic fuzzy open sets of the form $\prod_{j \in J} \mu_j$ where $\mu_j \in \tau_j$ and $\mu_j = 1$, except the finitely many j's.

Definition 2.6 [4]**.** A fuzzy topological space (X, τ) is said to be *fuzzy* W-*Hausdorff* if for every $x, y \in X, x \neq y$, there exist $\mu, \lambda \in \tau$ such that $\mu(x) = 1, \lambda(y) = 1$ and $\mu \wedge \lambda = 0$.

Definition 2.7 [5], [6]**.** Let X be a nonempty set. A function $\hat{\mu} : X \to [I]$ is called an *interval valued fuzzy set* in X, where $[I]$ is the set of all closed subintervals of $[0, 1]$. $[I]^X$ denotes the collection of all interval valued fuzzy sets in X.

For every $\hat{\mu} \in [I]^X$ and $x \in X$, $\hat{\mu}(x) = [\mu^-(x), \mu^+(x)]$ is called the degree of membership of an element x to $\hat{\mu}$, where $\mu^- : X \to I$ and $\mu^+ : X \to I$ are called the lower fuzzy set and upper fuzzy set in X respectively, $\hat{\mu}$ is denoted as $\hat{\mu} = [\mu^-, \mu^+]$.

For any two interval valued fuzzy sets $\hat{\mu}, \hat{\lambda}$ in $[I]^X$

(i) $\hat{\mu} \,\hat{\subseteq}\, \hat{\lambda}$ iff $\mu^-(x) \leq \lambda^-(x)$ and

$$\mu^+(x) \leq \lambda^+(x) \text{ for every } x \in X$$

(ii) $\hat{\mu} = \hat{\lambda}$ iff $\hat{\mu} \subseteq \hat{\lambda}$ and $\hat{\lambda} \subseteq \hat{\mu}$.

(iii) The *union* $\hat{\mu} \,\hat{\cup}\, \hat{\lambda}$ and *intersection* $\hat{\mu} \,\hat{\cap}\, \hat{\lambda}$ are defined respectively as

$$\hat{\mu} \,\hat{\cup}\, \hat{\lambda} = [\mu^- \vee \lambda^-, \mu^+ \vee \lambda^+]$$

$$\hat{\mu} \,\hat{\cap}\, \hat{\lambda} = [\mu^- \wedge \lambda^-, \mu^+ \wedge \lambda^+]$$

(iv) The *complement* of $\hat{\mu}$, denoted by $\hat{\mu}^c$ is defined $\hat{\mu}^c(x) = [1 - \mu^+(x), 1 - \mu^-(x)]$ for every $x \in X$.

(v) For a family $\{\hat{\mu}_j / j \in J\}$ of interval valued fuzzy sets on a set X, the *union* $\hat{\cup}_{j \in J} \hat{\mu}_j$ and the *intersection* $\hat{\cap}_{j \in J} \hat{\mu}_j$ are defined, respectively, as

$$\hat{\bigcup_{j \in J}} \hat{\mu}_j = [\bigvee_{j \in J} (\mu_j^-), \bigvee_{j \in J} (\mu_j^+)]$$

$$\hat{\bigcap_{j \in J}} \hat{\mu}_j = [\bigwedge_{j \in J} (\mu_j^-), \bigwedge_{j \in J} (\mu_j^+)]$$

(vi) The constant interval valued fuzzy sets zero and one are denoted as $\hat{0}$ and $\hat{1}$ which are defined, respectively, by

$$\hat{0} = [\mathbf{0}, \mathbf{0}], \hat{1} = [\mathbf{1}, \mathbf{1}]$$

Definition 2.8 [10]**.** Let X be a nonempty set. A subset $\hat{\tau} \subset [I^I]^X$ is called an *interval valued fuzzy topology* (*Chang*) on X iff $\hat{\tau}$ satisfies the following axioms.

(i) $\hat{0}, \hat{1} \in \hat{\tau}$

(ii) $\hat{\mu}, \hat{\lambda} \in q$ implies $\hat{\mu} \hat{\cap} \hat{\lambda} \in \hat{\tau}$

(iii) $\hat{\mu}_j \in \hat{\tau}$ for each $j \in J$ implies $(\hat{\cup}_{j \in J} \hat{\mu}_j) \in \hat{\tau}$.

The pair $(X, \hat{\tau})$ is called an *interval valued fuzzy topological space*.

3. Interval Valued Fuzzy W-Hausdorff Spaces

Definition 3.1. An interval valued fuzzy topological space $(X, \hat{\tau})$ is said to be an *interval valued fuzzy* W-*Hausdorff or interval valued fuzzy* $W - T_2$, if for all pair of disjoint points $x, y \in X$, there exists two interval valued fuzzy open sets $\hat{\mu} = [\mu^-, \mu^+] \in \hat{\tau}$ and $\hat{\lambda} = [\lambda^-, \lambda^+] \in \hat{\tau}$ such that $\mu^-(x) = 1$, $\mu^+(x) = 1$, $\lambda^-(y) = 1$, $\lambda^+(y) = 1$ and $\hat{\mu} \hat{\cap} \hat{\lambda} = \hat{0}$.

Definition 3.2. Let $(X, \hat{\tau})$ be an interval valued fuzzy topological space. Let $Y \subseteq X$. Let $\hat{\mu} \in \hat{\tau}$. Define $\hat{\mu}/Y = [\mu^-/Y, \mu^+/Y]$ as follows.

$(\mu^-/Y)(z) = \mu^-(z)$

$(\mu^+/Y)(z) = \mu^+(z)$ if $z \in Y$.

Define $(\hat{\tau}/Y) = \{(\hat{\mu}/Y)/\hat{\mu} \in \hat{\tau}\}$, Then $(\hat{\tau}/Y)$ is called the *interval valued fuzzy subspace topology* of Y and $(Y, \hat{\tau}/Y)$ is called an *interval valued fuzzy subspace of* $(X, \hat{\tau})$.

Definition 3.3. Let $\hat{\mu} = [\mu^-, \mu^+]$ and $\hat{\lambda} = [\lambda^-, \lambda^+]$ be interval valued fuzzy sets on X and Y respectively. The *Cartesian product* of $\hat{\mu}$ and $\hat{\lambda}$ is an interval valued fuzzy sets on $X \times Y$ denoted as $\hat{\mu} \hat{*} \hat{\lambda}$ and is defined as

$(\hat{\mu} \hat{*} \hat{\lambda}) = [(\mu^- * \lambda^-), (\mu^+ * \lambda^+)]$, where

$(\mu^- * \lambda^-)(x, y) = \min(\mu^-(x), \lambda^-(y))$ and

$(\mu^{-} * \lambda^{-})(x, y) = \min(\mu^{+}(x), \lambda^{+}(y))$, for all $(x, y) \in X \times Y$

Definition 3.4. Let $(X, \hat{\tau}_1)$ and $(Y, \hat{\tau}_2)$ be two interval valued fuzzy topological spaces. Then the *interval valued fuzzy product topology* $\hat{\tau}_1 \times \hat{\tau}_2$ on $X \times Y$ is the interval valued fuzzy topology having the collection $\{\hat{\mu} \hat{*} \hat{\lambda} / \hat{\mu} \in \hat{\tau}_1, \hat{\lambda} \in \hat{\tau}_2\}$ as a basis.

Theorem 3.5. *Subspace of an interval valued fuzzy W-Hausdorff space is an interval valued fuzzy W-Hausdorff space.*

Proof. Let $(X, \hat{\tau})$ be an interval valued fuzzy W-Hausdorff space. Let Y be a nonempty subset of X.

Let $(Y, \hat{\tau}/Y)$ be an interval valued fuzzy W-Hausdorff space.

Consider $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Then $y_1, y_2 \in X$. Since $(X, \hat{\tau})$ is an interval valued fuzzy W-Hausdorff space, there exists $\hat{\mu} = [\mu^{-}, \mu^{+}] \in \hat{\tau}, \hat{\lambda} = [\lambda^{-}, \lambda^{+}] \in \hat{\tau}$ such that $\mu^{-}(y_1) = 1, \mu^{+}(y_1) = 1, \lambda^{-}(y_2) = 1, \lambda^{+}(y_2) = 1$ and $\hat{\mu} \hat{\cap} \hat{\lambda} = \hat{0}$.

Therefore $(\hat{\mu}/Y) = [\mu^{-}/Y, \mu^{+}/Y]$,

$(\hat{\lambda}/Y) = [\lambda^{-}/Y, \lambda^{+}/Y] \in \hat{\tau}/Y$

Also,

$$(\mu^{-}/Y)(y_1) = \mu^{-}(y_1) \text{ if } y_1 \in Y$$
$$= 1$$
$$(\mu^{+}/Y)(y_1) = \mu^{+}(y_1) \text{ if } y_1 \in Y$$
$$= 1$$
$$(\lambda^{-}/Y)(y_2) = \lambda^{-}(y_2) \text{ if } y_2 \in Y$$
$$= 1$$
$$(\lambda^{+}/Y)(y_2) = \lambda^{+}(y_2) \text{ if } y_2 \in Y$$
$$= 1$$

$$(\widehat{\mu}/Y) \widehat{\cap} (\widehat{\lambda}/Y) = [(\mu^{-}/Y \wedge \lambda^{-}/Y), (\mu^{+}/Y \wedge \lambda^{+}/Y)]$$

Consider for $y \in Y \subseteq X$

$$(\mu^{-}/Y \wedge \lambda^{-}/Y)(y) = (\mu^{-}/Y)(y) \wedge (\lambda^{-}/Y)(y)$$

$$= (\mu^{-}(y) \wedge \lambda^{-})(y)$$

$$= (\mu^{-} \wedge \lambda^{-})(y)$$

$$\Rightarrow \mu^{-}/Y \wedge \lambda^{-}/Y = \mu^{-} \wedge \lambda^{-}$$

$$(\mu^{+}/Y \wedge \lambda^{+}/Y)(y) = (\mu^{+}/Y) \wedge (\lambda^{+}/Y)(y)$$

$$= (\mu^{+}(y) \wedge \lambda^{+})(y)$$

$$= (\mu^{+} \wedge \lambda^{+})(y)$$

$$\Rightarrow \mu^{+}/Y \wedge \lambda^{+}/Y = \mu^{+} \wedge \lambda^{+}$$

Therefore $(\widehat{\mu}/Y) \widehat{\cap} (\widehat{\lambda}/Y) = [\mu^{-} \wedge \lambda^{-}, \mu^{+} \wedge \lambda^{+}]$

$$= \widehat{\mu} \widehat{\cap} \widehat{\lambda}$$

$$= \widehat{0}$$

Hence $(Y, \widehat{\tau}/Y)$ is an interval valued fuzzy W-Hausdorff space.

Theorem 3.6. *Product of two interval valued fuzzy W-Hausdorff spaces is an interval valued fuzzy W-Hausdorff space.*

Proof. Let $(X, \widehat{\tau}_1)$ and $(Y, \widehat{\tau}_2)$ be two intervals valued fuzzy W-Hausdorff spaces.

To prove $(X \times Y, \widehat{\tau} \times \widehat{\tau}_2)$ is an interval valued fuzzy W-Hausdorff space. Consider two distinct points $(x_1, y_1), (x_2, y_2) \in X \times Y$. Either $x_1 \neq x_2$ or $y_1 \neq y_2$. Assume $x_1 \neq x_2$. Therefore there exists two interval valued fuzzy open sets $\widehat{\mu} = [\mu^{-}, \mu^{+}] \in \widehat{\tau}$, $\widehat{\lambda} = [\lambda^{-}, \lambda^{+}] \in \widehat{\tau}$, such that $\mu^{-}(x_1) = 1$, $\mu^{+}(x_1) = 1$, $\lambda^{-}(x_2) = 1$, $\lambda^{+}(x_2) = 1$ and $\widehat{\mu} \widehat{\cap} \widehat{\lambda} = \widehat{0}$.

$\hat{\mu}, \hat{\lambda} \in \hat{\tau}$ implies $\hat{\mu} \hat{*} \hat{1} = \hat{\tau}_1 \times \hat{\tau}_2$, and $\hat{\lambda} \hat{*} \hat{1} = \hat{\tau}_1 \times \hat{\tau}_2$,

Where,

$\hat{\mu} \hat{*} \hat{1} = [\mu^- * 1^-, \mu^+ * 1^+]$ and

$\hat{\lambda} \hat{*} \hat{1} = [\lambda^- * 1^-, \lambda^+ * 1^+]$

Consider for all $(x_1, y_1) \in X \times Y$

$$(\mu^- * 1^-)(x_1, y_1) = \min(\mu^-(x_1), 1^-(y_1))$$
$$= \min(1, 1)$$
$$= 1$$
$$(\mu^+ * 1^+)(x_1, y_1) = \min(\mu^+(x_1), 1^+(y_1))$$
$$= \min(1, 1)$$
$$= 1$$

Consider for all $(x_2, y_2) \in X \times Y$

$$(\lambda^- * 1^-)(x_2, y_2) = \min(\lambda^-(x_2), 1^-(y_2))$$
$$= \min(1, 1)$$
$$= 1$$
$$(\lambda^+ * 1^+)(x_2, y_2) = \min(\lambda^+(x_2), 1^+(y_2))$$
$$= \min(1, 1)$$
$$= 1$$

Also,

$\hat{\mu} \hat{\cap} \hat{\lambda} = \hat{0}$

$\Rightarrow [\mu^- \wedge \lambda^-, \mu^+ \wedge \lambda^+] = [0, 0]$

$\Rightarrow (\mu^- \wedge \lambda^-)(x) = 0, (\mu^+ \wedge \lambda^+)(x) = 0,$ for all $x \in X.$

$\Rightarrow \mu^{-}(x) \wedge \lambda^{-}(x) = 0,\ \mu^{+}(x) \wedge \lambda^{+}(x) = 0,$ for all $x \in X$.

$\Rightarrow \mu^{-}(x) = 0$ and $\lambda^{-}(x) = 0,\ \mu^{+}(x) = 0$ and $\lambda^{+}(x) = 0,$ for all $x \in X$.

$\Rightarrow \mu^{-}(x) \wedge 1^{-}(y) = 0$ and $\lambda^{-}(x) \wedge 1^{-}(y) = 0,$

$\mu^{+}(x) \wedge 1^{+}(y) = 0$ and $\lambda^{+}(x) \wedge 1^{+}(y) = 0,$ for all $x \in X$ and $y \in Y$.

$\Rightarrow (\mu^{-} * 1^{-})(x, y) = 0$ and $(\lambda^{-} * 1^{-})(x, y) = 0,\ (\mu^{+} * 1^{+})(x, y) = 0,$

$(\lambda^{+} * 1^{+})(x, y) = 0,$ for all $(x, y) \in X \times Y$

$\Rightarrow ((\mu^{-} * 1^{-}) \wedge (\lambda^{-} * 1^{-}))(x, y) = 0,\ ((\mu^{+} * 1^{+}) \wedge (\lambda^{-} * 1^{-}))(x, y) = 0,$

for all $(x, y) \in X \times Y$

$\Rightarrow (\widehat{\mu} \,\widehat{*}\, \widehat{1}) \,\widehat{\cap}\, (\widehat{\lambda} \,\widehat{*}\, 1) = \widehat{0}$

Hence $(X \times Y, \widehat{\tau}_1 \times \widehat{\tau}_2)$ is an interval valued fuzzy W-Hausdorff space.

Definition 3.7. Let $\{(X_j, \widehat{\tau}_j)/j \in J\}$ be a family of interval valued fuzzy topological spaces and $X = \prod_{j \in J} X_j$. The interval valued fuzzy product topology on X is the one with basic interval valued fuzzy open sets of the form $\prod_{j \in J} \widehat{\mu}_j [\prod_{j \in J} \mu_j^{-}, \prod_{j \in J} \mu_j^{+}]$, where $\widehat{\mu}_j \in \widehat{\tau}_j$ and $\widehat{\mu}_j = \widehat{1}$ except for finitely many j's. Here $(\prod_{j \in J} \mu_j^{-})(x_j)_{j \in J} = \underset{j \in J}{\wedge} \mu_j^{-}(x_j), (\prod_{j \in j} \mu_j^{+})(x_j)_{j \in l}$ $= \underset{j \in J}{\wedge} \mu_j^{+}(x_j),$ for every $(x_j)_{j \in J} \in \prod_{j \in J} X_j$.

Theorem 3.8. *Arbitrary product of interval valued fuzzy W-Hausdorff spaces is an interval valued fuzzy W-Hausdorff space.*

Proof. Let $\{(X_j, \widehat{\tau}_j)/ j \in J\}$ be a collection of interval valued fuzzy W-Hausdorff spaces. Consider $X = \prod_{j \in J} X_j$. Consider two distinct points $(X_j)_{j \in J}, (y_i)_{j \in J} \in \prod_{j \in J} X_j$. Therefore $x_k \neq y_k$ for some $k \in J$. Therefore there exists two interval valued fuzzy open sets $\widehat{\mu}_k = [\mu_k^{-}, \mu_k^{+}] \in \widehat{\tau}_k$ and

$\widehat{\lambda}_k = [\lambda_k^-, \lambda_k^+] \in \widehat{\tau}_k$ such that $\mu_k^-(x_k) = 1$, $\mu_k^+(x_k) = 1$, $\lambda_k^-(x_k) = 1$, $\lambda_k^+(x_k) = 1$ and $\widehat{\mu}_k \cap \widehat{\lambda}_k = \widehat{0}$. Let $\widehat{\mu} = \prod_{j \in J} \widehat{\mu}_j$, where $\widehat{\mu}_j = \widehat{I}_j$ for $j \neq k$ and $\widehat{\lambda} = \prod_{j \in J} \widehat{\lambda}_j$, where $\widehat{\lambda}_j = \widehat{1}_j$ for $j \neq k$

Then $\widehat{\mu}, \widehat{\lambda} \in \prod_{j \in J} \widehat{\tau}_j$,

$$\widehat{\mu} = \prod_{j \in J} \widehat{\mu}_j = [\prod_{j \in J} \mu_j^-, \prod_{j \in J} \mu_j^+]$$

$$\begin{aligned} \prod_{j \in J} (\mu_j^-)(x_j)_{j \in J} &= \min((\mu_j^-)(x_j)_{j \in J}) \\ &= (\mu_k^-)(x_k), \text{ for some } k \in J \\ &= 1 \end{aligned}$$

$$\begin{aligned} \prod_{j \in J} (\mu_j^+)(x_j)_{j \in J} &= \min((\mu_j^+)(x_j)_{j \in J}) \\ &= (\mu_k^+)(x_k), \text{ for some } k \in J \\ &= 1 \end{aligned}$$

$$\widehat{\lambda} = \prod_{j \in J} \widehat{\lambda}_j = [\prod_{j \in J} \lambda_j^-, \prod_{j \in J} \lambda_j^+]$$

$$\begin{aligned} \prod_{j \in J} (\lambda_j^-)(y_i)_{j \in J} &= \min((\lambda_j^-)(y_i)_{j \in J}) \\ &= (\lambda_k^-)(x_k), \text{ for some } k \in J \\ &= 1 \end{aligned}$$

$$\begin{aligned} \prod_{j \in J} (\lambda_j^+)(y_i)_{j \in J} &= \min((\lambda_j^+)(y_i)_{j \in J}) \\ &= (\lambda_k^+)(x_k), \text{ for some } k \in J \\ &= 1 \end{aligned}$$

Consider

$$\hat{\mu} \mathbin{\hat{\cap}} \hat{\lambda} = \prod_{j \in J} \hat{\mu}_j \mathbin{\hat{\cap}} \prod_{j \in J} \hat{\lambda}_j$$

$$= [(\prod_{j \in J} \mu_j^- \wedge \prod_{j \in J} \lambda_j^-), (\prod_{j \in J} \mu_j^+ \wedge \prod_{j \in J} \lambda_j^+)]$$

Then

$$(\prod_{j \in J} \mu_j^- \wedge \prod_{j \in J} \lambda_j^-)\,(x_j)_{j \in J} = (\prod_{j \in J} \mu_j^-(x_j)_{j \in J}) \wedge (\prod_{j \in J} \lambda_j^-(x_j)_{j \in J})$$

$$= \min((\mu_j^-)(x_j)_{j \in J}) \wedge \min((\lambda_j^-)(x_j)_{j \in J})$$

$$= (\mu_k^-)(x_k) \wedge (\lambda_k^-)(y_k), \text{ for some } k \in J$$

$$= (\mu_k^- \wedge \lambda_k^-) = 0$$

$$(\prod_{j \in J} \mu_j^+ \wedge \prod_{j \in J} \lambda_j^+)\,(x_j)_{j \in J} = (\prod_{j \in J} \mu_j^+(x_j)_{j \in J}) \wedge (\prod_{j \in J} \lambda_j^+(x_j)_{j \in J})$$

$$= \min((\mu_j^+)(x_j)_{j \in J}) \wedge \min((\lambda_j^+)(x_j)_{j \in J})$$

$$= (\mu_k^+)(x_k) \wedge (\lambda_k^+)(x_k), \text{ for some } k \in J$$

$$= (\mu_k^+ \wedge \lambda_k^+) = 0$$

Therefore $\hat{\mu} \mathbin{\hat{\cap}} \hat{\lambda} = \prod_{j \in J} \hat{\mu}_j \mathbin{\hat{\cap}} \prod_{j \in J} \hat{\lambda}_j$

$$= \hat{0}$$

Hence arbitrary product of interval valued fuzzy W-Hausdorff spaces is an interval valued fuzzy W-Hausdorff space.

4. Conclusion

In this paper the concept of interval valued fuzzy W-Hausdorff space is introduced and some basic properties of this concept are proved.

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