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APPLICATION OF MADGM PROBLEM USING LINEAR PROGRAMMING TECHNIQUES UNDER TRIANGULAR INTUITIONISTIC FUZZY MATRIX GAMES

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Abstract

The paper discusses about determining the unknown weights of the decision makers for the Multiple Attribute Group Decision Making (MAGDM) problems with Triangular Intuitionistic Fuzzy Numbers (TIFNs) by solving triangular intuitionistic fuzzy matrix games using linear programming technique. A new and improved method for ranking triangular intuitionistic fuzzy sets associated with membership function and non-membership function are proposed and utilized in forming the linear programming problem from the triangular intuitionistic fuzzy payoff matrix. A new algorithm for MAGDM problem with TIFNs utilizing the expert weights derived from triangular intuitionistic fuzzy matrix games is proposed and numerical illustration is given to justify the viability and effectiveness of the proposed method.

1. Introduction

In real-life situations, Multiple Attribute Group Decision Making (MAGDM) problems are more common. A MAGDM problem is to find the outcome from a restricted set of available alternatives analyzed for multiple attributes, which are normalized and restricted. The decision maker normally provides preference data in the form of numerical values in order to select a

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desirable solution. However, numerical values are recurrently unsatisfactory to model legit decision-making problems. Human verdicts, comprising preference information details, can indeed be expressed as intuitionistic fuzzy knowledge. As a result, the MAGDM problem with intuitionistic fuzzy nature is an intriguing research topic for new researchers. Details about the attribute's weights can be known, partially known, or completely unknown at times. A preset, restricted set of alternatives are anticipated for MAGDM problems. The MAGDM problem to be resolving entails filtering and ordering, and it may be seen of as another technique to integrate information into a decision matrix, as well as supplemental data from the decision maker, in order to arrive at a final ordering or selection from a set of alternatives. In many circumstances, decision makers have just a hazy understanding of alternative qualities.

The Fuzzy Set (FS) developed by Zadeh [21] is one of the ways for defining atypical circumstances in the field of uncertainty. Atanassov [2] developed the notion of Intuitionistic Fuzzy Sets (IFSs) to integrate the unpredictable level in the membership function of FS. The Triangular Intuitionistic Fuzzy Set (TIFS), which has its membership function and non-membership function is applied by many researchers in decision making theory. Robinson and Amirtharaj [13], [14], [15], [16] proposed correlation coefficient for various higher order IFS and applied them in MAGDM problems. Robinson and Amirtharaj [17] have given a MAGDM analysis for triangular and trapezoidal intuitionistic fuzzy sets. Robinson and Poovarasan [18] have introduced a robust MADGM method for TIFS.

Robinson et al. [19] have introduced an automated decision support system miner algorithm to solve MAGDM problem using fuzzy matrix games. Li [5], [7], Li and Yang [8], Li and Wan [9] and Li et al. [10] introduced some linear programming approaches to multi attribute decision making with IFS. Li [6] presents the decision-making process and game theory under IFS. Nan and Li [11] and an and Li [1] introduced linear programming approach to solve matrix games under IFS. Li [4] has presented a ratio ranking approach to rank triangular intuitionistic fuzzy numbers and has offered various arithmetic and logical operations on TIFS. Bhaumik et al. [3] has introduced a linear programming approach to solve triangular intuitionistic fuzzy matrix games using robust ranking method. The purpose of this paper is to propose some weight determining methods using Linear Programming Problems (LPP), propose a novel ranking method for TIFS, and utilize arithmetic aggregation operators including Triangular Intuitionistic Fuzzy Weighted Arithmetic Averaging (TIFWAA) operator and Triangular Intuitionistic Fuzzy Ordered Weighted Averaging (TIFOWA) operator, for the decision making problem.

2. The Road to a Novel Ranking Method for TIFNs Basic concepts and definitions

Definition 2.1 [4]. A Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ is a IF set on the real numbers set $\Re$, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a_1)u_{\tilde{A}}/(a_1 - a_1); & a_1 \leq x < a_2 \\ u_{\tilde{A}}; & x = a_2 \\ (a_3 - x)u_{\tilde{A}}/(a_3 - a_2); & a_2 < x \leq a_3 \\ 0; & \text{otherwise} \end{cases} \tag{1}$$

$$\gamma_{\tilde{A}}(x) = \begin{cases} [a_2 - x + (x - a_1)v_{\tilde{A}}]/(a_1 - a_1); & a_1 \leq x < a_2 \\ v_{\tilde{A}}; & x = a_2 \\ [x - a_2 + (a_3 - x)v_{\tilde{A}}]/(a_3 - a_2); & a_2 < x \leq a_3 \\ 1; & \text{otherwise} \end{cases} \tag{2}$$

The value $u_{\tilde{A}}$ represents the supremum of membership and the value $v_{\tilde{A}}$ represents the infimum of non-membership and they satisfy the conditions $0 \leq u_{\tilde{A}} \leq 1$, $0 \leq v_{\tilde{A}} \leq 1$ and $0 \leq u_{\tilde{A}} + v_{\tilde{A}} \leq 1$.

Definition 2.2 [4]. Consider a TIFN $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ where $\tilde{A}_{\alpha,\beta}\{x \mid \mu_{\tilde{A}} \geq \alpha, \gamma_{\tilde{A}} \leq \beta\}$ where $0 \leq \alpha \leq u_{\tilde{A}}, v_{\tilde{A}} \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$ is the (α, β)-cut set of A which is a subset of $\Re$.

Definition 2.3 [4]. Consider a TIFN $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ where $\tilde{A}_{\alpha,\beta}\{x \mid \mu_{\tilde{A}} \geq \alpha\}$, where $0 \leq \alpha \leq u_{\tilde{A}}$ is the α-cut set of A which is a closed interval $[L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)]$ of $\Re$, this can be determined by

$$[L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)] = \left[a_1 + \frac{\alpha(a_2 - a_1)}{u_{\tilde{A}}}, a_3 - \frac{\alpha(a_3 - a_2)}{u_{\tilde{A}}}\right]. \tag{3}$$

Definition 2.4 [4]**.** Consider a TIFN $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ where $\tilde{A}_\beta = \{x \mid \gamma_{\tilde{A}} \leq \beta\}$, where $v_{\tilde{A}} \leq \beta \leq 1$ is the β-cut set of A which is a closed interval $[L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)]$ of $\Re$, this can be determined by

$$[L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)] = \left[\frac{(1-\beta)a_2 + (\beta - v_{\tilde{A}})a_1}{1 - v_{\tilde{A}}}, \frac{(1-\beta)a_2 + (\beta - v_{\tilde{A}})a_3}{1 - v_{\tilde{A}}}\right]. \tag{4}$$

Robust Ranking Method for TIFN. In [3], Bhaumik has presented a robust ranking technique for TIFN $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ is defined as

$$R(\tilde{A}) = \int_{\alpha=0}^{1} \left(\frac{\sqrt{3}}{3}\right) \times (a_1 + \alpha(a_2 - a_1) + a_3 - \alpha(a_3 - a_2))d\alpha \tag{5}$$

The TIFNs' membership and non-membership degrees are significant in the intuitionistic fuzzy environment because they provide the data's entire preference information. The above mentioned robust ranking method [3] does not consider the membership and non-membership degrees for the TIFNs. Hence, the new ranking method associated with the membership and non-membership function is needed for a better ranking of TIFNs.

3. A Novel Ranking Method for TIFNs

We offer a novel ranking approach for the TIFNs associated with membership and non-membership function in this section.

Definition 3.1. The new ranking method called Li-Samuel ranking method associated with membership function $R_\mu(\tilde{A})$ for the TIFN $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ is defined as:

$$R_\mu(\tilde{A}) = \int_{\alpha=0}^{\mu} \left(\frac{\sqrt{3}}{3}\right) \times [L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)]d\alpha \tag{6}$$

where $[L_{\widetilde{A}}(\alpha)+R_{\widetilde{A}}(\alpha)]$ is the α-cut interval of the TIFN $\widetilde{A}=(a_1, a_2, a_3; u_{\widetilde{A}}, v_{\widetilde{A}})$ given in (3).

Hence, $$R_{\mu}(\widetilde{A})=\int_{\alpha=0}^{u_{\widetilde{A}}}\left(\frac{\sqrt{3}}{3}\right)\times\left[a_1+\frac{\alpha(a_2-a_1)}{u_{\widetilde{A}}}+a_3\frac{\alpha(a_3-a_2)}{u_{\widetilde{A}}}\right]d\alpha \tag{7}$$

By simplifying and integrating (7) with respect to α, we get,

$$R_{\mu}(\widetilde{A})=\left(\frac{\sqrt{3}}{3}\right)\times\left[(a_1-a_3)u_{\widetilde{A}}-\frac{(a_1-2a_2+a_3)u_{\widetilde{A}}}{2}\right]. \tag{8}$$

Definition 3.2. The new ranking method called Li-Samuel ranking method associated with non-membership function $R_{\gamma}(\widetilde{A})$ for the TIFN $\widetilde{A}=(a_1, a_2, a_3; u_{\widetilde{A}}, v_{\widetilde{A}})$ is defined as:

$$R_{\gamma}(\widetilde{A})=\int_{\beta=v_{\widetilde{A}}}^{1}\left(\frac{\sqrt{3}}{3}\right)\times[L_{\widetilde{A}}(\beta), R_{\widetilde{A}}(\beta)]d\beta \tag{9}$$

where $[L_{\widetilde{A}}(\alpha)+R_{\widetilde{A}}(\alpha)]$ is the β-cut interval of the TIFN A given in (4).

Hence,

$$R_{\gamma}(\widetilde{A})=\int_{\beta=v_{\widetilde{A}}}^{1}\left(\frac{\sqrt{3}}{3}\right)\times\left[\frac{(1-\beta)a_2+(\beta-v_{\widetilde{A}})a_1}{1-v_{\widetilde{A}}}+\frac{(1-\beta)a_2+(\beta-v_{\widetilde{A}})a_3}{1-v_{\widetilde{A}}}\right]d\beta \tag{10}$$

By simplifying and integrating (10) with respect to β, we get,

$$R_{\gamma}(\widetilde{A})=\left(\frac{\sqrt{3}}{3}\right)\times\left[(2a_2-(a_1+a_3)v_{\widetilde{A}})+\frac{(a_1-2a_2+a_3)(1-v_{\widetilde{A}})}{2}\right]. \tag{11}$$

Equations (8) and (11) are called the Li-Samuel ranking method associated with membership and non-membership function respectively. In the robust ranking method, the TIFNs with same triangular numbers but with different membership degrees and non-membership degrees are ranked with the same order. For example (4, 5, 8; 0.5, 0.2) and (4, 5, 8; 0.8, 0.1) have the same robust ranking 6.3509. By using Li-Samuel ranking method associated with membership function, we get the ranking 3.1754 and 5.0807, where the TIFN with the higher membership value gets the higher ranking. Likewise, by using Li-Samuel ranking method associated with non-membership function we get the ranking 5.0807 and 5.7158, where the TIFN with higher non-membership degree gets the least ranking.

The proposed Li-Samuel ranking method associated with membership and non-membership satisfies the axioms A_1, A_2, A_3 and A_5 from the axioms A_1 to A_7 proposed in Wang and Kerre [20]. In the following, two theorems are proved to show that the proposed Li-Samuel ranking method satisfies the condition of linearity.

Theorem 1. *The Li-Samuel ranking associated with membership function satisfies* $R_\mu(\tilde{A}+\tilde{B}) = R_\mu(\tilde{A}) + R_\mu(\tilde{B})$ *only if* $u_{\tilde{A}} = u_{\tilde{B}}$ *and* $v_{\tilde{A}} = v_{\tilde{B}}$ *in* $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ *and* $\tilde{A} = (b_1, b_2, b_3; u_{\tilde{B}}, v_{\tilde{B}})$.

Proof. We know that $\tilde{A}+\tilde{B} = (a_1, b_1, a_2 + b_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$. Using equation (8) we obtain,

$$R_\gamma(\tilde{A}+\tilde{B}) = \left(\frac{\sqrt{3}}{3}\right)\left[(a_1 + b_1 + a_3 + b_3)u_{\tilde{A}} - \frac{(a_1 + b_1 - 2(a_2 + b_2) + a_3 + b_3)u_{\tilde{A}}}{2}\right]$$

$$= R_\mu(\tilde{A}) + R_\mu(\tilde{B})$$

Theorem 2. *The Li-Samuel ranking associated with non-membership function satisfies* $R_\gamma(\tilde{A}+\tilde{B}) = R_\gamma(\tilde{A}) + R_\gamma(\tilde{B})$ *only if* $u_{\tilde{A}} = u_{\tilde{B}}$ *and* $v_{\tilde{A}} = v_{\tilde{B}}$ *in* $\tilde{A} = (a_1, a_2, a_3; u_{\tilde{A}}, v_{\tilde{A}})$ *and* $\tilde{A} = (b_1, b_2, b_3; u_{\tilde{B}}, v_{\tilde{B}})$.

Proof. We know that $\tilde{A}+\tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; u_{\tilde{A}}, v_{\tilde{A}})$. Using equation (11) we obtain,

$$R_\mu(\tilde{A}+\tilde{B}) = \left(\frac{\sqrt{3}}{3}\right) \times [(2(a_2 + b_2) - (a_1 + b_1 + a_3 + b_3)v_{\tilde{A}})$$

$$\frac{(a_1 + b_1 - 2(a_2 + b_2) + a_3 + b_3)(1 + v_{\tilde{A}})}{2}] = R_\gamma(\tilde{A}) + R_\gamma(\tilde{B})$$

4. Algorithm for MAGDM problem under TIFS with Proposed Li-Samuel Ranking Method

Let the set of alternatives be $A_i(i = 1, 2, \ldots, m)$ and the set of attributes be $G_i(j = 1, 2, \ldots, n)$ and ω_j be the weighting vector of G_j derived from solving triangular intuitionistic fuzzy matrix games, where $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. Let the set of decision makers be $D_k(k = 1, 2, \ldots, t)$ whose weighting vectors are w_k, where $\omega_j \in [0, 1]$, $\sum_{k=1}^{t} w_k = 1$. Then $\widetilde{R}_k = (\widetilde{r}_{ij}^k)_{m\times n} = ((a_1)_{ij}^k, (a_2)_{ij}^k, (a_3)_{ij}^k; u_{\widetilde{r}_{ij}^k}, v_{\widetilde{r}_{ij}^k})_{m\times n}$ is the TIFN decision matrix, where $u_{\widetilde{r}_{ij}^k}$ represents the value of A_i satisfying G_j in $\widetilde{R}_k$ and $v_{\widetilde{r}_{ij}^k}$ represents the value of A_i not satisfying G_j in $\widetilde{R}_k$.

Step 1. The TIFWAA operator [18] is utilized in $\widetilde{R}_k$ to get the individual overall TIFNs.

Step 2. The TIFOWA operator [18] is utilized to get the collective overall .preference of TIFNs.

Step 3. Using the correlation coefficient for TIFNs [14], the correlation between $\widetilde{r}^i$ and the positive ideal solution for TIFN $\widetilde{r}^+ = (1, 1, 1; 1, 0)$ is calculated.

Step 4. Order the alternative according to the correlation coefficient $K_{TIFN}(\widetilde{r}^i, \widetilde{r}^+)$ and select the best one with the highest correlation coefficient.

5. Deriving Weights for MAGDM Problem from Triangular Intuitionistic Fuzzy Matrix Games (TIFMG) with Proposed Li-Samuel Ranking Method

Consider a type of matrix games with TIFN payoff, in which pure strategy sets S_1 and S_2 and mixed strategy sets Y and Z are used by players I and II, respectively. Which is given by the decision makers as a weighting vector information for the MAGDM problem. The Table 1 contains the triangular intuitionistic fuzzy payoff matrix.

Table 1. TIFN Payoff Matrix.

	1	2	3
1	(40,60,90;0.5,0.4)	(50,60,80;0.6,0.2)	(30,50,90;0.6,0.1)
2	(30,40,60;0.5,0.2)	(30,40,90;0.7,0.1)	(50,60,80;0.7,0.2)
3	(40,50,70;0.6,0.3)	(40,60,80;0.7,0.2)	(30,40,60;0.5,0.3)

The TIFN in the payoff matrix is converted into crisp values using the proposed Li-Samuel ranking associated with membership function which is calculated as,

$$R_{\mu}(40,\ 60,\ 90;0.5,\ 0.4)=\left(\frac{\sqrt{3}}{3}\right)\left[(40+90)0.5-\frac{(40-2(60)+90)0.5}{2}\right] \quad (12)$$

$$= 36.0844$$

Table 2 contains the defuzzified payoff matrix and the computations for all other elements in Table 1.

Table 2. Defuzzified Payoff Matrix using proposed Li-Samuel ranking associated with membership.

	1	2	3
1	36.0844	43.3013	38.1051
2	24.5374	40.4145	50.5181
3	36.3731	48.4974	24.5374

The payoff matrix is checked for existence of saddle point using maxmin and minmax principle. Because there is no such saddle point present in the payoff, the mixed strategy method given in [3] is used to solve the matrix games. Construct the linear programming problems using payoff matrix in Table 2.

Minimize $p_1 + p_2 + p_3$

$$\text{Subject to } \begin{cases} 36.0844 p_1 + 24.5374 p_2 + 36.3731 p_3 \geq 1, \\ 43.3013 p_1 + 40.4145 p_2 + 48.4974 p_3 \geq 1, \\ 38.1051 p_1 + 50.5181 p_2 + 24.5374 p_3 \geq 1, \\ p_1,\ p_2,\ p_3 \geq 0. \end{cases} \tag{13}$$

And Maximize $q_1 + q_2 + q_3$

$$\text{Subject to } \begin{cases} 36.0844 q_1 + 43.3013 q_2 + 38.1051 q_3 \leq 1, \\ 24.5374 q_1 + 40.4145 q_2 + 50.5181 q_3 \leq 1, \\ 36.3731 p_3 + 48.4974 p_3 + 24.5374 p_3 \leq 1, \\ q_1,\ q_2,\ q_3 \geq 0. \end{cases} \tag{14}$$

Solving equations (13) and (14) using simplex method and the basic solutions are $p = (0.236,\ 0.0040,\ 0.2196)$ and $q = (0.0271,\ 0.0006,\ 3058)$ respectively. The solutions are normalized to get the weighting vectors which are $Y = (0.0955,\ 0.0162,\ 0.8883)$ and $Z = (0.0813,\ 0.0018,\ 9169)$.

Now the TIFNs in the payoff matrix shown in Table 1 is converted into crisp values using the proposed Li-Samuel ranking method based on non-membership function which is calculated as,

$R_\gamma(40,\ 60,\ 90;\, 0.5,\ 0.4)$

$$= \left(\frac{\sqrt{3}}{3}\right) \times \left[(2(60) - (40 + 90)0.4) + \frac{(40 - 2(60) + 90)(1 + 0.4)}{2}\right] = 43.3013 \tag{15}$$

Table 3 contains the defuzzified payoff matrix and the computations for all other elements in Table 1.

Table 3. Defuzzified Payoff Matrix using proposed Li-Samuel ranking associated with non-membership.

	1	2	3
1	43.3013	57.7350	57.1577
2	39.2598	51.9615	57.7350
3	42.4352	55.4256	34.3523

The payoff matrix is checked for existence of saddle point using maxmin and minmax principle. Because there is no such saddle point present in the payoff, the mixed strategy method given in [3] is used to solve the matrix games. Construct the linear programming problems using payoff matrix in Table 3. Solve the problems using simplex method and the basic solutions are $p = (0.0231, 0.3333, 0.3200)$ and $q = (0.0231, 0.0933, 0.0200)$ respectively. The solutions are normalized to get the weighting vectors which are $Y = (0.0342, 0.4928, 0.4731)$ and $Z = (0.1694, 0.6840, 0.1466)$.

The solutions of the linear programming problems and the decision maker weights obtained by using robust ranking [3], Proposed Li-Samuel ranking with membership and Proposed Li-Samuel ranking with non-membership values are listed in Table 4.

Table 4. Comparison of expert weight vectors.

	Basic solutions	Weights
Robust ranking	$p = (0.0110, 0.0041, 0.0359)$	$Y = (0.2157, 0.0804, 0.739)$
	$q = (0.0041, 0.0110, 0.2072)$	$Z = (0.0184, 0.0495, 0.9321)$
Proposed Li-Samuel ranking associated with membership	$p = (0.0236, 0.0040, 0.2196)$	$Y = (0.0955, 0.0162, 0.8883)$
	$q = (0.0271, 0.0006, 0.3058)$	$Z = (0.0813, 0.0018, 0.9169)$
Proposed Li-Samuel ranking associated with non-membership	$p = (0.0231, 0.3333, 0.3200)$	$Y = (0.0342, 0.4928, 0.4731)$
	$q = (0.0231, 0.0933, 0.0200)$	$Z = (0.1694, 0.6840, 0.1466)$

6. Numerical Illustration: MAGDM problem under TIFS with Proposed Li-Samuel Ranking Method

There is an organization interested in investing in the best of five possible alternatives $A_i(1, 2, 3, 5)$. The Company has to take a decision according to three attributes, G_1 is the risk study, G_2 is the growth study, G_3 is the environmental influence study. The alternatives A_i are evaluated by three decision makers using TIFN and its weighting vectors which are derived from the triangular intuitionistic fuzzy matrix games which are $\omega = (0.0955, 0.0162, 0.8883)$ and $w = (0.0813, 0.0018, 9169)$. $\widetilde{R}_k = (\tilde{r}_{ij}^k)$ the decision matrix, where $k = 1, 2, 3$ are listed:

$$\widetilde{R}_1 = \begin{bmatrix} (0.1, 0.3, 0.5; 0.4, 0.2) & (0.3, 0.5, 0.6; 0.6, 0.3) & (0.2, 0.6, 0.8; 0.5, 0.2) \\ (0.2, 0.4, 0.8; 0.5, 0.4) & (0.2, 0.4, 0.7; 0.3, 0.2) & (0.2, 0.8, 0.9; 0.2, 0.5) \\ (0.4, 0.6, 0.9; 0.7, 0.2) & (0.4, 0.5, 0.8; 0.5, 0.1) & (0.2, 0.5, 0.8; 0.5, 0.3) \\ (0.2, 0.4, 0.6; 0.3, 0.6) & (0.1, 0.2, 0.4; 0.4, 0.3) & (0.2, 0.4, 0.5; 0.5, 0.1) \\ 0.3, 0.4, 0.6; 0.3, 0.6 & (0.2, 0.5, 0.7; 0.3, 0.6) & (0.2, 0.7, 0.9; 0.3, 0.2) \end{bmatrix}$$

$$\widetilde{R}_1 = \begin{bmatrix} (0.5, 0.8, 0.9; 0.6, 0.3) & (0.2, 0.4, 0.7; 0.7, 0.2) & (0.4, 0.4, 0.6; 0.2, 0.5) \\ (0.4, 0.7, 0.8; 0.5, 0.3) & (0.3, 0.4, 0.6; 0.5, 0.4) & (0.1, 0.3, 0.5; 0.3, 0.4) \\ (0.2, 0.6, 0.8; 0.4, 0.5) & (0.3, 0.5, 0.8; 0.6, 0.3) & (0.2, 0.5, 0.8; 0.5, 0.4) \\ (0.5, 0.6, 0.7; 0.2, 0.5) & (0.3, 0.4, 0.8; 0.7, 0.2) & (0.5, 0.7, 0.8; 0.4, 0.2) \\ (0.2, 0.5, 0.8; 0.6, 0.2) & (0.5, 0.6, 0.7; 0.6, 0.3) & (0.2, 0.3, 0.4; 0.7, 0.1) \end{bmatrix}$$

$$\widetilde{R}_1 = \begin{bmatrix} (0.2, 0.4, 0.5; 0.6, 0.1) & (0.3, 0.4, 0.5; 0.5, 0.3) & (0.5, 0.6, 0.8; 0.6, 0.2) \\ (0.2, 0.3, 0.5; 0.5, 0.4) & (0.1, 0.5, 0.6; 0.3, 0.5) & (0.5, 0.6, 0.8; 0.4, 0.4) \\ (0.4, 0.6, 0.8; 0.5, 0.1) & (0.3, 0.5, 0.7; 0.8, 0.1) & (0.6, 0.8, 0.9; 0.7, 0.2) \\ (0.4, 0.6, 0.8; 0.5, 0.1) & (0.3, 0.4, 0.6; 0.4, 0.5) & (0.4, 0.5, 0.7; 0.5, 0.4) \\ (0.2, 0.4, 0.7; 0.5, 0.3) & (0.2, 0.5, 0.7; 0.3, 0.2) & (0.2, 0.3, 0.5; 0.3, 0.4) \end{bmatrix}$$

Solve the given decision matrices using algorithm for MAGDM problem under TIFS with Li-Samuel ranking method.

6.1 Computation using Weights Derived from TIFMG using Li-Samuel Ranking Associated with Membership Function

Step 1 and **Step 2.** Applying the TIFWAA operator in $\widetilde{R}_k$ and $\omega = (0.0955, 0.0162, 0.8883)$ the weighting vector, we get the individual overall decision matrices and applying the TIFOWA operator in the individual overall decision matrix $\widetilde{R}_k$ and $w(0.0813, 0.0018, 0.9169)$, we get the collective overall decision matrix as follows:

$$\tilde{r}^1 = (0.3890, 0.5769, 0.7554; 0.4776, 0.2005);$$

$$\tilde{r}^2 = (0.4381, 0.5514, 0.7490; 0.3191, 0.40006)$$

$$\tilde{r} = (0.2213, 0.4956, 0.8089; 0.5217, 0.2740)$$

$$\tilde{r}^4 = (0.3823, 0.4905, 0.6743; 0.4690, 0.2082);$$

$\tilde{r}^5 = (0.2095,\ 0.3237,\ 0.5165; 0.3213,\ 0.2133)$.

Step 3. Calculating the correlation coefficient for $\tilde{r}^i$ and $\tilde{r}^+ = (1, 1, 1; 1, 0)$, we get, $K(\tilde{r}^1, \tilde{r}^+) = 08586$, $K(\tilde{r}^2, \tilde{r}^+) = 0.4904$, $K(\tilde{r}^3, \tilde{r}^+) = 0.8029$, $K(\tilde{r}^4, \tilde{r}^+) = 0.8470$, $K(\tilde{r}^5, \tilde{r}^+) = 0.7296$.

Step 4. The alternatives are ordered according to $K(\tilde{r}^i, \tilde{r}^+)$ which is $A_1 > A_4 > A_3 > A_5 > A_2$. Since A_1 has the highest correlation with $\tilde{r}^+$, A_1 is the best alternative.

6.2 Computation using Weights Derived from TIFMG using Li-Samuel Ranking Associated with Non-membership Function

Similarly, computing the MADGM problem using weights derived from TIFMG with proposed Li-Samuel ranking method associated with non-membership degree which are $\omega = (0.0342,\ 0.4928,\ 0.4731)$ and $w = (0.1694,\ 0.6840,\ 0.1466)$ and proceeding with the same computational procedure as above, we get:

$\tilde{r}^1 = (0.3540,\ 0.4794,\ 0.6787; 0.5471,\ 0.2881)$;

$\tilde{r}^2 = (0.3769,\ 0.5438,\ 0.7420; 0.3506,\ 0.3453)$;

$\tilde{r}^3 = (0.3916,\ 0.5845,\ 0.7992; 0.6950,\ 0.2695)$;

$\tilde{r}^4 = (0.3516,\ 0.4925,\ 0.7168; 0.4694,\ 0.3372)$;

$\tilde{r}^5 = (0.3022,\ 0.5392,\ 0.7251; 0.56580,\ 0.2691)$.

Calculating the correlation coefficient for $\tilde{r}^i$ and $\tilde{r}^+(1, 1, 1; 1, 0)$ we get, $K(\tilde{r}^1, \tilde{r}^+) = 0.8550$, $K(\tilde{r}^2, \tilde{r}^+) = 0.6061$, $K(\tilde{r}^3, \tilde{r}^+) = 0.9313$, $K(\tilde{r}^4, \tilde{r}^+) = 0.7702$, $K(\tilde{r}^5, \tilde{r}^+) = 0.8733$. By ordering all the alternatives we get, $A_3 > A_5 > A_1 > A_4 > A_2$. Since A_3 has the highest correlation with $\tilde{r}^+$, A_3 is the best alternative.

Table 5. Comparison of Ranking.

Expert Weights Derived from Matrix Games using Different Methods	Final Ranking of the Alternatives
Robust ranking method [3]	$A_5 > A_3 > A_1 > A_4 > A_2$
Proposed Li-Samuel ranking associated with membership	$A_1 > A_4 > A_3 > A_5 > A_2$
Proposed Li-Samuel ranking associated with non-membership	$A_3 > A_5 > A_1 > A_4 > A_2$

The weights derived from TIFMG are utilized in the MAGDM and the TIFNs are ranked with the three methods namely Robust ranking method [3], proposed Li-Samuel ranking associated with membership and proposed Li-Samuel ranking associated with non-membership degree where the final alternative rankings are presented in Table 5. It can be observed that the final ordering of the three methods are different.

7. Discussion

The robust ranking method gives the same ranking for TIFNs with same triangular numbers but with different membership and nonmembership degrees. The advantages and merits of the proposed Li-Samuel ranking associated with membership and proposed Li-Samuel ranking associated with non-membership degree are that the TIFNs with same triangular numbers but with different membership degrees and nonmembership degrees are given different ranking respectively. The TIFN with higher membership degree is given higher order of ranking while using and the TIFN with higher non-membership degree is given least order of ranking while using.

8. Conclusion

A new method for ranking TIFNs called Li-Samuel ranking method associated with membership and non-membership function for the TIFNs are proposed in this paper, which is an improved form of the robust ranking method [3]. The proposed Li-Samuel ranking methods are used to defuzzify the TIFNs in the payoff matrix of the TIFMG. The defuzzified matrix game is solved using linear programing method to derive the decision maker's weights. Then the weights are used with the TIFOWAA operator and TIFOWA operator to aggregate the decision matrices in the MAGDM problem. The best alternative is selected according the highest correlation coefficient. In future, intuitionistic fuzzy linear programming method may be used to solve the TIFN matrix games without defuzzifying the payoff matrix.

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