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STUDY ON SPREAD OF DENGUE USING SIR EPIDEMIC MODEL IN FUZZY ENVIRONMENT

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Abstract

The purpose of this study is to investigate the SIR epidemic model in a fuzzy environment with a nonlinear incidence rate. The technique suggested to utilize the statistics to determine the spread of infection in the case of dengue epidemic in India. In addition, the researcher assessed the model's stability and concluded that the model for disease-free equilibrium is stable.

1. Introduction

In the dynamic equations, Epidemic models have attempted to consider the role of disease infection and treatment [10]. Using fuzzy parameters in differential equations defining the dynamical system, Barros et al., Bassanezi and Barros proposed a new technique to the analysis of an epidemic model.

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[4]. The recent fuzzy parameter analysis conducted in [6, 7, 10] identified the interval valued parameter set in a prey-predator model harvested and an epidemic model with fuzzy parameters studied for the computer network. The complicated behavior of an epidemic model with fuzzy transmission has investigated by Mondal et al. [7]. Verma et al. investigated the Fuzzy epidemic model for influenza virus propagation and potential control [10]. The following are the significant objectives of this paper:

- Define an SIR Epidemic model with uncertain parameters.
- Examine the proposed framework as well as provide evidence of the existence and stability. The model was constructed using secondary data collected from the Ministry of Health India official website.
- Finally, the results were used to analyze the spread of infection using fuzzy parameters, and the research concluded with conclusions and references.

2. SIR Epidemic Model

For many years, the emergence and spread of disease has been questioned and studied. Scientists are attempting to test inoculation with the ability to predict disease, and many of them will have a significant influence on the morality rate of a certain epidemic. Infectious disease modelling is a tool that has been used to study how diseases spread, predict the path of an outbreak, and evaluate infection management techniques.

These three compartments S (for susceptible), I (for infectious) and R are the normal conversion marks (for recovered). The model can be used to treat a wide variety of infectious diseases, including cancer, measles, mumps, rubella, and cholera. The letters also indicate how many individuals are in each compartment at any particular time. To imply that the number can change over time (each if the total population size remains constant). We make the precise numbers a function of t (time): $S(t)$, $I(t)$, $R(t)$. These functions can be developed for a specific disease in a specific population in order to predict and control possible breakdowns. The incidence rate is the number of new cases per population at risk in a certain period of time. Several academicians have proposed various incidence rates. An epidemic model with nonlinear incidence rate was studied by Nidhi et al. [9]. The rate and nature of infection play an important role in controlling the spread of infection.

3. Conditions for Stability

The suggested model needs to cover the following conditions in order to complete the stability analysis technique.

i. The model is stable if all real parts of the Eigenvalues are less than zero.

ii. The model is unstable if all real parts of the Eigenvalues are greater than zero.

There can be no conclusion if at least one of the Eigenvalues has a real part of zero. This is a borderline case between stability and instability.

4. Proposed Model

The spreading diseases between the susceptible and infected in the absence of vital dynamics have been modelled in the SIR model. An SIR Epidemic model with fuzzy parameters were proposed in this part. The suggested model is described by a set of differential equations.

$$\frac{dS}{dt} = A - \tilde{\beta}S - \tilde{\alpha}\lambda SI + \tilde{\mu}R$$

$$\frac{dI}{dt} = \tilde{\alpha}\lambda SI - (\tilde{\beta} + \tilde{b})I$$

$$\frac{dR}{dt}\tilde{b}I - (\tilde{\mu} + \tilde{\beta})R$$

Table 1. Parameters considered.

Variables	Description
$S(t)$	Susceptible rate at time t
$I(t)$	Infected rate at time t
$R(t)$	Recovered rate at time t

A	Recruitment rate
$\tilde{\beta}$	Uncertain Natural Death rate
$\tilde{b}$	Uncertain Death rate due to disease
$\tilde{\mu}$	Uncertain Recovery rate
$\tilde{a}$	Literacy rate
λ	Proportionality Constant

The above system has a disease-free equilibrium at the point $\left(\frac{A}{\tilde{\beta}} 0, 0\right)$.

The following values are obtained after resolving the system.

$$S = \frac{\tilde{\beta} + \tilde{b}}{a\lambda}$$

$$I = \frac{(\tilde{\beta} + \tilde{\mu})[\tilde{\beta}(\tilde{\beta} + \tilde{b}) - A\tilde{a}\lambda]}{\tilde{a}\lambda[\tilde{\mu}b - (\tilde{\beta} + \tilde{\mu})(\tilde{\beta} + \tilde{b})]}$$

$$R = \frac{\tilde{b} I}{\tilde{\mu} + \tilde{\beta}}$$

5. Stability Analysis for Disease free Equilibrium

Define the Jacobean matrix to check the stability of the disease's free equilibrium. Let us see

$$J(E_0) = \begin{bmatrix} -\tilde{\beta} & -a\lambda & \tilde{\mu} \\ 0 & \tilde{a}\lambda - (\tilde{\beta} + \tilde{b}) & 0 \\ 0 & \tilde{b} & (\tilde{\beta} + \tilde{\mu}) \end{bmatrix}$$

The characteristic equation of the system as follows.

$$x^3 + c_1 x^2 + c_2 x + c_3 = 0$$

where, the Eigenvalues written as

$$c_1 = -\tilde{\beta}$$

$$c_2 = \widetilde{b} - \widetilde{a}\lambda - \widetilde{\beta}$$

$$c_3 = -\widetilde{\beta} - \widetilde{\mu}$$

Using the collected crisp parameters in the Eigenvalues, we fix

$$c_1 = -0.37274,\ c_2 = -87.4,\ c_2 = -2.48387 - i1.47103.$$

Here, we can conclude that all Eigenvalues have a negative real root. As a result, for the parameters acquired, the suggested model is stable.

6. Case Study

The difficulty of calibrating the epidemiological parameters of an SIR model representing the progression of the dengue pandemic across time is addressed in this study. The secondary data that was gathered is listed below.

Table 2. Parameters collected from Ministry of Health India.

Variable\Year	2015	2016	2017	2018	2019	2020	2021
$\tilde{b}$ – Death Rate due to disease	0.002202	0.001897	0.001725	0.001700	0.001055	0.001256	0.000731
$\tilde{\mu}$ – Recovery rate	0.9978	0.9981	0.9983	0.9983	0.9989	0.9987	0.9993
˜ – Natural Death Rate	7.253	7.247	7.242	7.237	7.237	7.309	7.344
$\tilde{a}$ – Literacy Rate	64.08	69.03	69.03	69.1	69.1	74.04	74.04
A – Recruitment Rate	1.2503×10^9						

Table 3. Variable values obtained from Proposed Model.

Variable	Solution
S	(0.1132, 0.1051, 0.1049, 0.1049, 0.1048, 0.0987, 0.0991)
I	(0.0591, 0.0675, 0.0677, 0.0679, 0.068, 0.0723, 0.071)
R	$(1.576 \times 10^5,\ 1.553 \times 10^5,\ 1.417 \times 10^5,\ 1.401 \times 10^5,\ 0.871 \times 10^{-5},\ 1.093 \times 10^{-5},\ 0.622 \times 10^{-5})$

For more outcome analysis, we need to defuzzify the fuzzy numbers into crisp values. Using the proposed way of defuzzification, we obtain.

$$S = 0.2016,\ I = 0.1292,\ R = 0.1258 \times 10^{-4}$$

Thus, the rate of the susceptible population is 0.2016. The infected population rate is 0.1292. The rate of recovered population is 0.1258×10^{-4}.

7. Conclusion

In this study, the SIR epidemic model was discussed and analyzed for stability. The model concludes that it is stable. In addition, the described model was used to interpret Dengue infection in India, and the findings demonstrate that the model's infection spread is equivalent to the real-world results. One can implement the next phase in the given model in order to monitor the spread of disease.

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