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k-SUPER ROOT SQUARE MEAN LABELING OF ROOTED PRODUCT OF PATH GRAPH AND DIAMOND GRAPH

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Abstract

The graph labeling was established by Alexander Rosa [4] in 1967. In 2003 Somasundaram and Ponraj [6] introduced a labeling method called Mean labeling. Later various methodologies evolved and one such technique is k-Super root square mean labeling Akilandeswari [1] introduced the concept of k-Super root square mean labeling. We examine k-Super root square mean labeling of some path related graphs i.e. rooted product of path graph and diamond graph.

1. Motivation and Main Results

In [2, Theorem 1], it was established k-super root square mean labeling of rooted product of path graph and diamond graph.

Our main results can be stated as the following theorem.

Theorem 1. *The graph* $P_n \circ k_{1,2,3}$ *admits a k-Super root square mean graph for* $n \leq 2$.

2 Proof of Theorem 1. Consider the graph G be $P_n \circ k_{1,2,3}$. Denote the vertices of the diamond graph as u_i, v_i, w_i, s_i where $1 \leq i \leq n$ and edges $\{e_i, e_i', e_i', \ldots\}$ where $1 \leq i \leq n$. The edge between the two vertices u_i and v_i

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are denoted as e_i follows.

The total number of vertex in diamond graph $(K_{1,1,n}) = 4n$ and the total number of edges in diamond graph $(K_{1,1,n}) = 5n,$ without loss of generality we initiate the labeling from the vertex u_i.

Define the vertex label as follows,

Let $(P_n \circ K_{1,2,3}) = \{u_i, v_i, w_i, s_i; 1 \leq i \leq n\}$

Define the edge label as follows,

$$E(P_n \circ K_{1,2,3}) = \{e_i = (u_i, v_i); 1 \leq i \leq n\} \cup$$

$$\{e_i' = (u_i, w_i); 1 \leq i \leq n\} \cup$$

$$\{e_i' = (u_i, s_i); 1 \leq i \leq n\} \cup$$

$$\{e_i'' = (v_i, s_i); 1 \leq i \leq n\} \cup$$

$$\{e_i'^{v} = (w_i, s_i); 1 \leq i \leq n\}.$$

Be the vertices and edges $(P_n \circ K_{1,1,2})$ respectively.

Define the vertex labels of diamond graph $(K_{1,1,2})$ as follows,

Let $f : V(P_n \circ K_{1,1,2}) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$

Here, $p = 4n$ and $q = 5n + (n-1)$

$f : V(P_n \circ K_{1,1,2}) \to \{k, k+1, k+2, \ldots, k+10n-2\}$

$$f(u_i) = k + 10i - 1, 1 \leq i \leq n$$

$$f(v_i) = k + 10i - 2, 1 \leq i \leq n$$

$$f(w_i) = k + 10i - 4, 1 \leq i \leq n$$

$$f(s_i) = k + 10i - 8, 1 \leq i \leq n$$

Clearly, the vertex labels are distinct.

Define the edge labels of diamond graph $(K_{1,1,2})$ as follows,

$$\{e_i = (u_i, v_i); 1 \le i \le n\} \cup$$

$$\{e_i' = (u_i, w_i); 1 \le i \le n\} \cup$$

$$\{e_i' = (u_i, s_i); 1 \le i \le n\} \cup$$

$$\{e_i'' = (v_i, s_i); 1 \le i \le n\} \cup$$

$$\{e_i'^{v} = (w_i, s_i); 1 \le i \le n\}.$$

Be the vertices and edges $(P_n \circ k_{1,2,3})$ respectively.

Then f induced a function $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$, for each edge $e = uv$

Compute the edge labels of diamond graph $K_{1,1,2}$ as follows,

$$f^*(e_i) = \left\lceil \sqrt{\frac{f(u_i)^2 + f(v_i)^2}{2}} \right\rceil,$$

$$= \sqrt{\frac{(k + 10i - 1)^2 + (k + 10i - 2)^2}{2}}$$

$$f^*(e_i) = k + 10i - 1; 1 \le i \le n$$

$$f^*(e_i') = \left\lceil \sqrt{\frac{f(u_i)^2 + f(w_i)^2}{2}} \right\rceil,$$

$$= \sqrt{\frac{(k + 10i - 1)^2 + (k + 10i - 4)^2}{2}}$$

$$f^*(e_i') = k + 10i - 3; 1 \le i \le n$$

$$f^*(e_i'') = \left\lceil \sqrt{\frac{f(u_i)^2 + f(s_i)^2}{2}} \right\rceil,$$

$$= \sqrt{\frac{(k+10i-1)^2 + (k+10i-8)^2}{2}}$$

$$f^*(e_i') = k + 10i - 5; 1 \le i \le n$$

$$f^*(e_i'') = \left\lceil \sqrt{\frac{f(v_i)^2 + f(s_i)^2}{2}} \right\rceil,$$

$$= \sqrt{\frac{(k+10i-2)^2 + (k+10i-8)^2}{2}}$$

$$f^*(e_i'') = k + 10i - 6; 1 \le i \le n$$

$$f^*(e_i^{\prime v}) = \left\lceil \sqrt{\frac{f(w_i)^2 + f(s_i)^2}{2}} \right\rceil,$$

$$= \sqrt{\frac{(k+10i-4)^2 + (k+10i-8)^2}{2}}$$

$$f^*(e_i^{\prime v}) = k + 10i - 6; 1 \le i \le n$$

Clearly, $f(V) \cup \{f^*(e) : e \in E(P_n \circ K_{1,1,3}) = \{k, k+1, \ldots, k+10n-2\}\}$

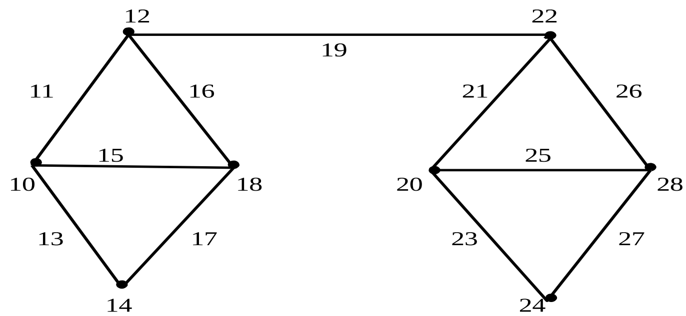
The edge labels of diamond graph $K_{1,1,2}$ are distinct.

Hence f admits k-super root square mean labeling.

Hence $P_n \circ k_{1,1,2}$ admits a k-Super root square mean graph.

Illustration

10-SRSML of $P_2 \circ K_{1,1,2}$ is given in Figure



10-SRSML of $P_2 \circ K_{1,1,2}$.

The proof of theorem 1 is complete.

3. Remarks

Finally, we list several remarks on our main results and closely related things.

Remark 1.

Definition 1. Let G be a (p, q) graph and $f : V(G) \rightarrow \{k, k+1, k+2, \ldots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ then f is called k-Super root square mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$. A graph that admits a k-Super root square mean labeling is called k-super root square mean graph.

Definition 2. Let G and H be graph then rooted product of G and H is denoted by $G \circ H$ and obtained by taking $|V(G)|$ copies of H, and for every vertex v_i of G, identify v_i with the root node of the i^{th} copy of H.

Definition 3. The diamond graph is the simple graph on 4 nodes and 5 edges. It is isomorphic to the complete tripartite graph $k_{1,1,2}$. The diamond graph is also known as double triangle graph.

Remark 2. The motivations in this papers are [1,3, 5, 7] are same as the one in the paper.

Remark 3. This paper is a slightly modified version of the preprint [3].

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