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SOME PROPERTIES ON STRONG FULLY COMPLETE DOMINATION IN PICTURE FUZZY GRAPH

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Abstract

Picture fuzzy graph is a useful mathematical tool for dealing with uncertain real-world problems when fuzzy graphs and intuitionistic fuzzy graphs are ineffective. It is especially helpful in cases where there are numerous options of the same type, such as yes, no, abstain, and refusal. The main objective of this study is to define the strong fully complete domination in a picture fuzzy graph with strong edges. The strong fully complete picture fuzzy dominating set is introduced based on the importance of the notion of domination and its applications in several instances. Furthermore, some important properties relating to this parameter are determined. The relationship between the strong fully complete picture fuzzy domination number and the picture fuzzy domination number is established. Some theorems have been proved with examples.

1. Introduction

L. A. Zadeh [16] first proposed the concept of fuzzy sets in 1965, and it has been successfully applied to a range of uncertain real-life scenarios. A fuzzy set is an extended version of a crisp set-in which members have varying degrees of membership. This crisp set can't handle uncertain real-world

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problems because it just has two values: 0 and 1 (no or yes). Instead of considering 0 or 1, a fuzzy gives its elements with membership values between 0 and 1 for a better outcome. In other situations, however, such single membership degree values are unable to cope with the uncertainty. To deal with this type of unknown scenario, Atanassov [1] introduced the intuitionistic fuzzy set, which includes an extended membership degree known as the hesitation margin. The intuitionistic fuzzy set is an advanced version of Zadeh's fuzzy set. It is more accessible and effective to work with uncertainty than a standard fuzzy set because of the presence of hesitation margin. When human perception and knowledge are completely unexpected and unclear, the intuitionistic fuzzy set is implemented in real-world scenarios. In recent years, scientists and analysts have successfully applied the concept of intuitionistic fuzzy set to image processing, social networks, machine learning, decision making, and medical diagnosis among other fields. The concept of neutrality degree, however, is rejected in intuitionistic fuzzy set theory. However, the degree of neutrality must be addressed in many common scenarios, such as democratic election stations, medical diagnosis recognition, social networks, decision making and so on.

Cuong [3] introduced the picture fuzzy set as an improved kind of intuitionistic fuzzy set to satisfy the neutrality degree. The degree of positive membership value $\mu : X \to [0, 1]$, neutral membership value $\eta : X \to [0, 1]$, and negative membership value $\gamma : X \to [0, 1]$ build up the picture fuzzy set under the condition $0 \leq \mu(x) + \eta(x) + \nu(x) \leq 1$, where $\Pi(x) = 1 - (\mu(x) + \eta(x) + \gamma(x))$ is the degree of refusal membership values of a vertex. The notion of picture fuzzy graph was suggested by Cen Zuo et al. [2], which is based on picture fuzzy relations for the effective way of expressing ambiguity. Phong et al. [12] have proposed a variety of picture fuzzy relation compositions. Xiao Wei [15] investigated the regular picture fuzzy graphs and its properties. This motivated us to introduce the concept of fully complete picture fuzzy dominating set.

The paper is constructed as follows. Section 3 provides the primary definitions of picture fuzzy graphs, whereas Section 4 provides the strong fully complete picture fuzzy dominating set and strong fully complete picture fuzzy domination number. Some propositions and theorems related to this domination parameter are discussed.

2. Preliminaries

Definition 1. A fuzzy graph G is said to be Picture Fuzzy Graph (PFG) if (i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0, 1]\, \eta_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$, positive membership value, neutral membership value and negative membership value of the vertex $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \eta_1(v_i) + \gamma_1(v_i) \leq 1, \forall v_i \in V, i = 1, 2, \ldots, n.$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1], \eta_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ such that

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$$

$$\eta_2(v_i, v_j) \leq \eta_1(v_i) \wedge \eta_1(v_j)$$

$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$$

where $0 \leq \mu_2(v_i, v_j) + \eta_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1, (v_i, v_j) \in E, \forall i, j = 1, 2, \ldots, n$

Example 2.

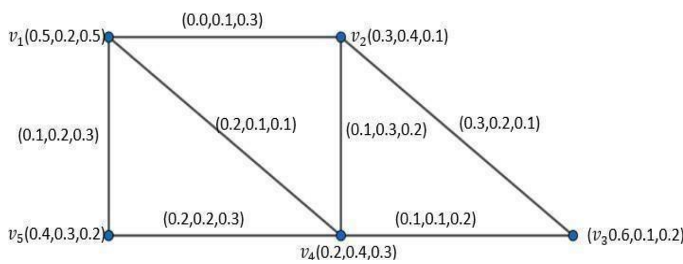

Figure 1.

Definition 3. In a picture fuzzy graph $G = (V, E)$, if $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j), \eta_2(v_i, v_j) = \eta_1(v_i) \wedge \eta_1(v_j)$ and $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$, $\forall v_i, v_j \in V$, then it is called complete picture fuzzy graph.

Definition 4. In a picture fuzzy graph $G = (V, E)$, if $\mu_2(v_i, v_j) \geq \mu_2'^{\infty}(v_i, v_j), \eta_2(v_i, v_j) \geq \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) \leq \gamma_2'^{\infty}(v_i, v_j)\ \forall v_i, v_j \in V$, then the edge is called the strong edge.

Definition 5. The sum of weights of the strong edges occurs at v_i in a picture fuzzy graph $G = (V, E)$ is called the strong degree and is denoted by $d_s(v_i)$.

Definition 6. The maximum and minimum strong degree of a picture fuzzy graph are defined as $\Delta_s(G) = \max\{d_s(v_i)/v_i \in V\}$ and $\delta_s(G) = \min\{d_s(v_i)/v_i \in V\}$.

Definition 7. If two vertices v_i and v_j are called neighbors in the picture fuzzy graph $G = (V, E)$, then one of the below conditions holds,

(i) $\mu_2(v_i, v_j) > 0, \eta_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$

(ii) $\mu_2(v_i, v_j) = 0, \eta_2(v_i, v_j) \geq 0, \gamma_2(v_i, v_j) > 0$

(iii) $\mu_2(v_i, v_j) > 0, \eta_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) \geq 0$

(iv) $\mu_2(v_i, v_j) \geq 0, \eta_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) = 0, \forall v_i, v_j \in V$.

Definition 8. Let v_i be a vertex in a PFG $G = (V, E)$. Then $N_S(v_i) = \{v_j \in V : (v_i, v_j)$ is a strong edge$\}$ is called open strong neighborhood of v_i. $N_S(v_i) = N_S(v_i) \cup \{v_i\}$ is called the closed strong neighborhood of u.

Definition 9. Let $G = (V, E)$ be a PFG. Let $v_i, v_j \in V$. Then v_i dominates v_j in G if there exists a strong edge between them.

Definition 10. A subset D of V is called a picture fuzzy dominating set in a PFG G, if for every vertex $v_j \in V - D$, there exists some vertices $v_i \in D$ which dominates v_j.

Definition 11. A dominating set D of the PFG G is said to be minimal picture fuzzy dominating set if there is no proper subset of D is a picture fuzzy dominating set.

Definition 12. The minimum fuzzy cardinality among all picture fuzzy dominating set is called domination number or lower domination number of G and it is denoted by $\gamma_{pf}(G)$.

Definition 13. Let v_i and v_j be any two vertices in a picture fuzzy graph $G = (V, E)$. Then v_i strongly dominates v_j (i) (v_i, v_j) is a strong edge, (ii) $d_s(v_i) \geq d_s(v_j)$. Otherwise v_i weakly dominates v_j.

Definition 14. Let $G = (V, E)$ be a PFG. Then $D \subset V$ is said to be strong picture fuzzy dominating set of G if every vertex $v_j \in V - D$ is strongly dominated by some vertex $v_i \in D$.

3. Strong Fully Complete Domination in Picture Fuzzy Graph

This section covers the strong fully complete picture fuzzy dominating set and the strong fully complete picture fuzzy domination number. Some related theorems and propositions related are discussed.

Definition 1. Let $G = (V, E)$ be a PFG. Let $v_i, v_j \in V$ and $D_f \subset V$, then D_f is a fully complete picture fuzzy dominating set, if every $v_i \in D_f$ dominates to some vertices $v_j \in V - D_f$ such that the edge (v_i, v_j) is a strong edge.

Definition 2. The fully complete picture fuzzy domination number of G is the maximum fuzzy cardinality between all fully complete picture fuzzy dominating sets, and it is represented by $\gamma_{fpf}(G)$.

Example 3.

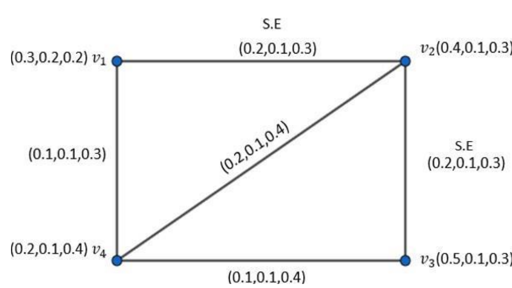

Figure 2.

In Figure 2, (v_1, v_2) and (v_2, v_3) are strong edges. Therefore, the fully complete picture fuzzy dominating set is $D_f = \{v_1, v_3\}$. The picture fuzzy domination number is $\gamma_{pf} = 0.85$ and the fully complete picture fuzzy domination number is $\gamma_{fpf} = 1.00$.

Definition 4. The maximal fully complete picture fuzzy dominating set is a fully complete picture fuzzy dominating set D_f of PFG G, if $\forall v_i \in V - D_f$, then the set $D_f \cup \{v_i\}$ is not a fully complete picture fuzzy dominating set.

Definition 5. The set D_{sf} is said to be strong fully complete picture fuzzy dominating set, if every vertex v_i in D_{sf} strongly dominates at least one vertex v_j in $V - D_{sf}$.

Example 6.

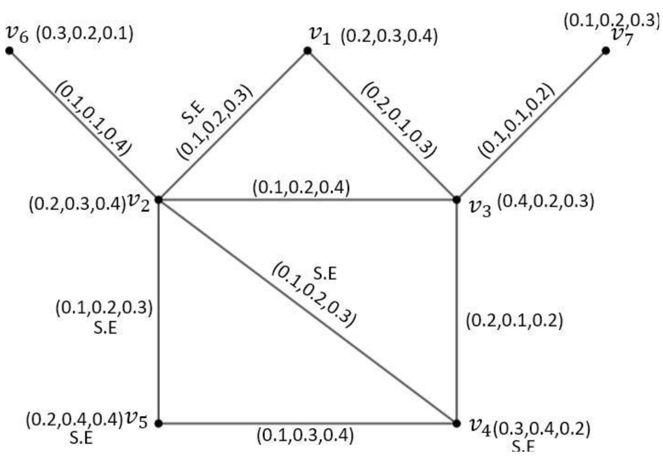

Figure 3.

In Figure 3, the strong fully complete picture fuzzy dominating set $D_{sf} = \{v_2, v_5, v_7\}$.

Definition 7. The strong fully complete picture fuzzy domination number of G is the maximum fuzzy cardinality between all strong fully complete picture fuzzy dominating sets and it is represented by $\gamma_{sfpf}(G)$.

Proposition 8. *Let* $G = (V, E)$ *be a picture fuzzy graph. The set* D_{sf} *is a strong fully complete picture fuzzy dominating set iff* $V - D_{sf}$ *is a weak picture fuzzy dominating set.*

Proof. Let D_{sf} be a strong fully complete picture fuzzy dominating set. By the definition of strong domination, every vertex $v_i \in D_{sf}$ strongly dominates some vertices $v_j \in V - D_{sf}$ which satisfies the condition of weak picture fuzzy dominating set $V - D_{sf}$. Hence $V - D_{sf}$ is a weak picture fuzzy dominating set.

Conversely, suppose that $V - D_{sf}$ is a weak picture fuzzy dominating set. To prove, D_{sf} is a strong fully complete picture fuzzy dominating set. Suppose D_f is not a strong fully complete picture fuzzy dominating set, there exists at least one vertex $v_i \in D_{sf}$ does not strongly dominates any vertices $V - D_{sf}$. Since $V - D_{sf}$ is a weak picture fuzzy dominating set, every vertex in D_{sf} must be weakly dominated by some vertices in $V - D_{sf}$ which contradicts our assumption. Hence D_{sf} is a strong fully complete picture fuzzy dominating set.

Example 9.

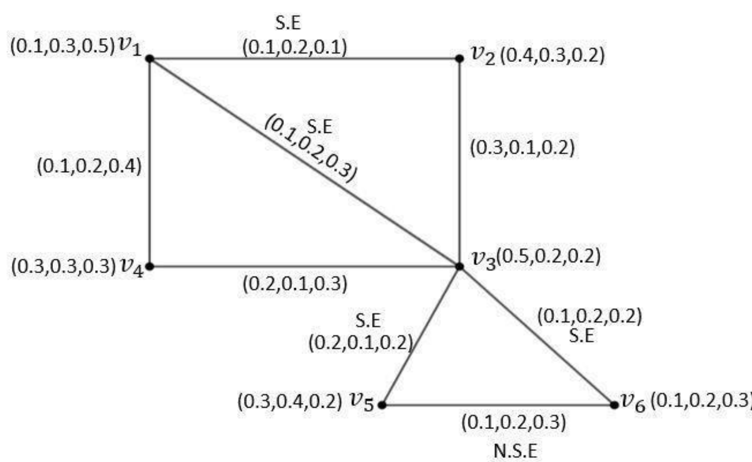

Figure 4.

In Figure 4, the strong fully complete picture fuzzy dominating set $D_{sf} = \{v_1, v_3\}$. Then $V - D_{sf} = \{v_2, v_4, v_5, v_6\}$. Here v_2 weakly dominates v_1 and v_5 and v_6 weakly dominates v_3 and v_4 is the independent vertex.

Proposition 3.10. *For any picture fuzzy graph $G = (V, E)$, if D_f and D_{sf} are fully and strong fully complete picture fuzzy dominating set, then their corresponding domination numbers satisfies the following inequalities.*

(i) $\gamma_{sfpf}(G) \leq \gamma_{fpf}(G)$

(ii) $\gamma_{wfpf}(G) \leq \gamma_{fpf}(G)$.

Proof. Let D_f and D_{sf} be fully complete and strong fully complete picture fuzzy dominating sets. By the definition, every strong fully complete picture fuzzy dominating set is a fully complete picture fuzzy dominating set of a picture fuzzy graph G. Therefore $\gamma_{sfpf}(G) \leq \gamma_{fpf}(G)$.

Similarly, every weak fully complete picture fuzzy dominating set is a fully complete picture fuzzy dominating set. Hence $\gamma_{wfpf}(G) \leq \gamma_{fpf}(G)$.

Example 11.

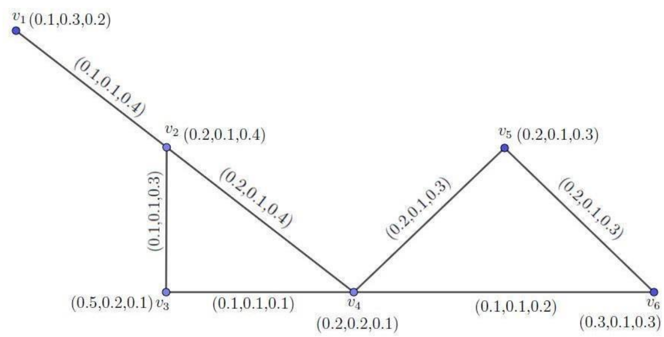

Figure 5.

In Figure 5, $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_6)$ and (v_5, v_6) are strong edges. The fully complete picture fuzzy dominating set $D_f = \{v_2, v_3, v_5\}$

$$\gamma_{fpf} = 0.5 + 0.8 + 0.55 = 1.95.$$

$D_{sf} = \{v_3, v_5\}$ is a strong fully complete picture fuzzy dominating set.

$$\gamma_{sfpf} = 0.8 + 0.55 = 1.85.$$

Therefore, the inequality holds for strong fully complete dominating set.

Theorem 12. *In picture fuzzy graph* $G = (V, E)$, *the strong fully complete picture dominating set exists if every vertex in* D_{sf} *are strongly dominates at least one vertex in* $V - D_{sf}$.

Proof. Let D_{sf} be a strong fully complete picture fuzzy dominating set. Let (v_i, v_j) be a strong edge. Every vertex in D_{sf} is strongly dominates to some vertices in $V - D_{sf}$. In particular, the vertices in the set D_{sf} are strongly dominates at least one vertex in $V - D_{sf}$. Suppose let us assume that the vertex $v_i \in D_{sf}$ does not strongly dominate $v_j \in V - D_{sf}$ such that (v_i, v_j) is not a strong edge which contradicts our assumption. Hence every vertex in D_{sf} strongly dominates at least one vertex in $V - D_{sf}$.

Example 13.

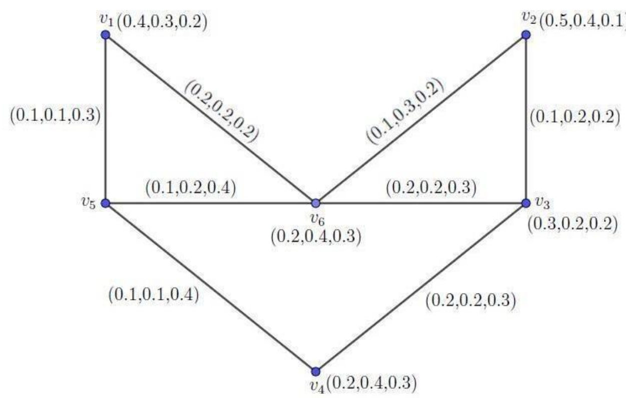

Figure 6.

In Figure 6, (v_2, v_3), (v_3, v_4), (v_3, v_6) and (v_5, v_6) are strong edges.

Let $D_{sf} = \{v_3, v_5\}$ be a strong fully complete picture fuzzy dominating set. Then $V - D_{sf} = \{v_1, v_2, v_3, v_4\}$ hence the vertices in D_{sf} strongly dominates the elements v_2, v_4 and v_6 in $V - D_{sf}$.

Theorem 14. *Every fully complete picture fuzzy dominating set* D_f *of picture fuzzy graph* $G = (V, E)$ *consists of at least one strong fully complete picture fuzzy dominating set* D_{sf} *in G.*

Proof. Let D_f be a fully complete picture fuzzy dominating set. Suppose let us assume that the fully complete picture fuzzy dominating set D_f doesn't contain strong fully complete picture fuzzy dominating set D_{sf} in G. This gives that any two vertices of D_{sf} are independent and there exist vertices which are not strong in D_f. Therefore, for every vertex $v_i \in D_{sf}$, there exists no $v_j \in V - D_{sf}$ such that v_i does not strongly dominates v_j which contradicts our assumption. Hence, D_f must contain at least one strong fully complete picture fuzzy dominating set D_{sf} in G.

Example 15.

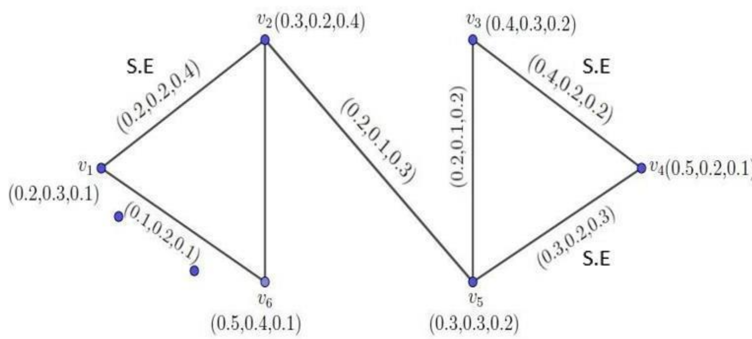

Figure 7.

In Figure 7, the fully complete picture fuzzy dominating set

$$D_f = \{v_2, v_3, v_4\}. \text{ Then } V - D_f = \{v_1, v_5, v_6\}$$

Therefore, D_f is a fully complete picture fuzzy dominating set.

$D_{sf} = \{v_2, v_4\}$. This gives that D_{sf} is a strong fully complete picture fuzzy dominating set which is contained in D_f.

Proposition 16. *Let $G = (V, E)$ be a picture fuzzy graph of order P. Let γ_{sfpf} and γ_{wfpf} be strong and weak fully complete picture fuzzy domination numbers of the strong and weak fully complete picture fuzzy dominating sets D_{sf} and D_{wf} respectively. Then*

(i) $\gamma_{sfpf} + \gamma_{wpf} = p$

(ii) $\gamma_{wfpf} + \gamma_{spf} = p$

Where γ_{sfpf} *and* γ_{wpf} *are strong and weak picture fuzzy domination numbers.*

Proof. Let P be the order of the PFG $G = (V, E)$. If D_w be a weak picture fuzzy dominating set, then by Proposition 8, $V - D_w$ will be a strong fully complete picture fuzzy dominating set.

Therefore, $| D_{sf} | \leq | V - D_w |$

This implies that $\gamma_{sfpf} \leq p - \gamma_{wpf}$ $\gamma_{sfpf} + \gamma_{wpf} \leq p$ (1)

Suppose let D_{sf} be the maximum strong fully complete picture fuzzy dominating set, then $V - D_{sf}$ is a weak picture fuzzy dominating set and hence, $| D_w | \leq | V - D_{sf} |$ $\gamma_{wpf} \leq p - \gamma_{sfpf}$ $\gamma_{sfpf} + \gamma_{wpf} \leq p$ (2)

From (1) and (2), we get $\gamma_{sfpf} + \gamma_{wpf} = p$. If D_s be a strong picture fuzzy dominating set, then by $V - D_s$ will be a weak fully complete picture fuzzy dominating set.

Therefore, $| D_{wf} | \geq | V - D_s |$ $\gamma_{wfpf} \geq p - \gamma_{spf}$ $\gamma_{wfpf} + \gamma_{spf} \geq p$ (3)

Suppose let D_{wf} be the maximum weak fully complete picture fuzzy dominating set, then $V - D_{wf}$ is a strong picture fuzzy dominating set and hence, $| D_s | \leq | V - D_{wf} |$ $\gamma_{spf} \leq p - \gamma_{wfpf}$ $\gamma_{spf} + \gamma_{wfpf} \leq p$ (4)

From (3) and (4), we get $\gamma_{spf} + \gamma_{wfpf} = p$.

Example 17.

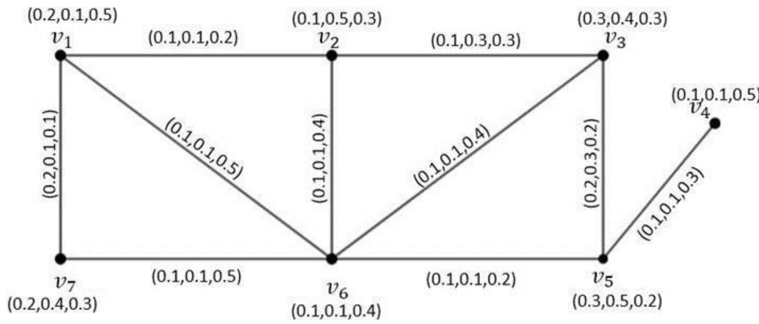

Figure 8.

In Figure 8, the strong fully complete picture fuzzy dominating set is $D_{sf} = \{v_1, v_3, v_5, v_6\}$ $V - D_{sf} = \{v_2, v_4, v_7\}$.

Then $V - D_{sf}$ is a weak picture fuzzy dominating set.

$\gamma_{sfpf} = 2.3$, $\gamma_{wpf} = 1.65$. The order $P = 3.95$. Hence $\gamma_{sfpf} + \gamma_{wpf} = p$.

Similarly, $\gamma_{wfpf} + \gamma_{sfpf} = p$.

Theorem 18. *If $G = (V, E)$ be a picture fuzzy graph with end vertices, then at least one strong fully complete picture fuzzy dominating set D_{sf} exists.*

Proof. Let $G = (V, E)$ be a picture fuzzy graph with end vertices. Let D_f be a fully complete picture fuzzy dominating set of V. Since $G = (V, E)$ has strong edges for some $v_j \in V - D_{sf}$, there exists $v_i \in D_{sf}$ such that D_{sf} is a fully complete picture fuzzy dominating set. Also, few vertices in $V - D_{sf}$ are strongly dominated by all vertices in D_{sf} which gives that at least one strong fully complete dominating set D_{sf} exists.

Example 19.

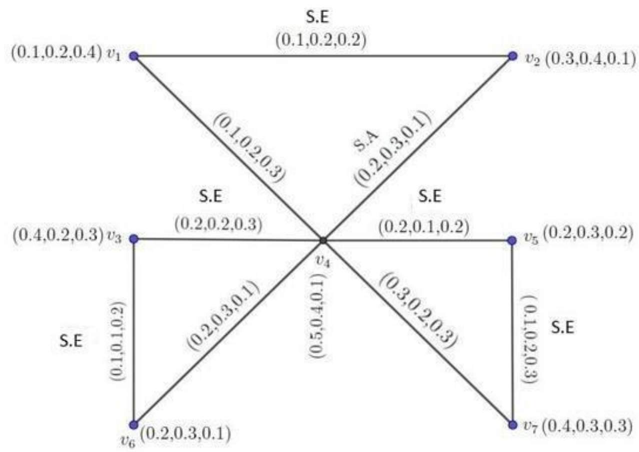

Figure 9.

In Figure 9, v_2, v_6 and v_7 are end vertices. Let $D_{sf} = \{v_1, v_4, v_7\}$.

Hence at least one strong fully complete picture fuzzy dominating set exists.

Theorem 16. *Let $G = (V, E)$ be a picture fuzzy graph with end vertices, then almost all end vertices exist in the complement of the strong fully complete picture fuzzy dominating set.*

Proof. Let $G = (V, E)$ be a picture fuzzy graph with end vertices. Let D_{sf} be a strong fully complete picture fuzzy dominating set. Then for every vertex $v_i \in D_{sf}$, there exists some vertices $v_j \in V - D_{sf}$, such that (v_i, v_j) is a strong edge. Since the strong degree of end vertices is less than or equal to their adjacent vertices, it must belong to the set $V - D_{sf}$. Hence almost all end vertices exist in the complement of the strong fully complete picture fuzzy dominating set.

Example 17.

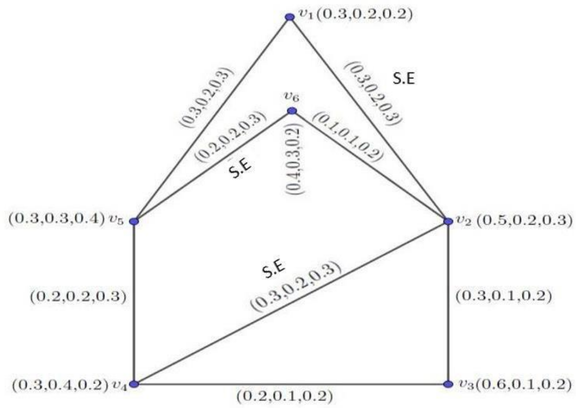

Figure 10.

In Figure 10, v_3, v_4 and v_6 are end vertices.

Let $D_{sf} = \{v_2, v_5\}$ be a strong fully complete picture fuzzy dominating set. Then almost all end vertices v_3, v_4 and v_6 exist in $V - D_{sf} = \{v_1, v_3, v_4, v_6\}$.

Theorem 18. *Let $G = (V, E)$ be a PFG and $\overline{G}$ be the complement of G with the vertices and strong edges which is same as G. If D_{sf} is a strong fully complete picture fuzzy dominating set of G, then it is also strong fully complete picture fuzzy dominating set in $\overline{G}$.*

Proof. Let G and $\overline{G}$ be the picture fuzzy graph and its complement respectively. Suppose let us assume that $\overline{G}$ contains minimum number of strong edges than G. The vertices v_i and v_j which are any two adjacent edges in G, then they are adjacent or independent in $\overline{G}$. This gives that there exists different strong fully complete picture fuzzy dominating set D_{sf} in $\overline{G}$ which is not equal strong fully complete picture fuzzy dominating set D_{sf} in G which contradicts to assumption that G consists of minimum number of vertices and strong edges than $\overline{G}$. Hence the strong fully complete fuzzy dominating set D_{sf} exists in G and $\overline{G}$.

Example 20.

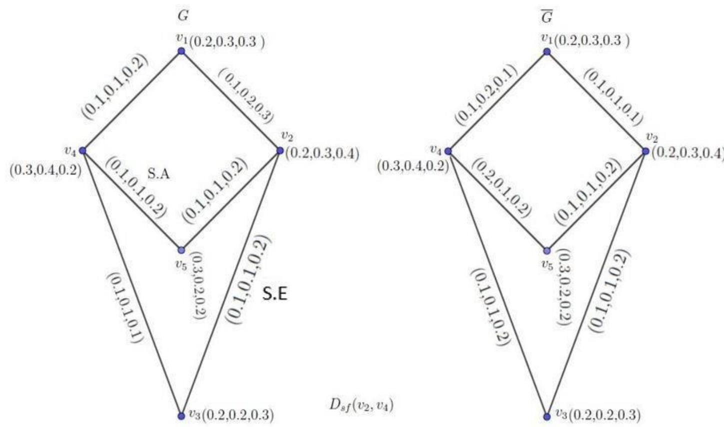

Figure 11.

In Figure 11, the strong fully complete picture fuzzy dominating set is $D_{sf} = \{v_2, v_4\}$ in G and $D_{sf} = \{v_2, v_4\}$ in $\overline{G}$.

Hence G and $\overline{G}$ have the same strong fully complete picture fuzzy dominating set.

4. Conclusion

In this paper, the strong fully complete dominating set and strong fully complete domination number is defined in picture fuzzy graph using strong edges. The relationship between the domination number and the strong fully complete domination number in picture fuzzy graph has been discussed. Bounds of some picture fuzzy graphs have been determined. Theorems and properties related to the strong fully complete domination number have been proved with examples.

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