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MATCHING IN FUZZY LABELING TREE

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Abstract

A graph is said to be a complete fuzzy labeling graph if it has every pair of adjacent vertices of the fuzzy graph. A matching is a set of non-adjacent edges. If every vertex of fuzzy graph is *M*-saturated then the matching is said to be complete or perfect. In this paper, we introduce the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labeling tree using matching and perfect matching.

1. Introduction

Graph theory is rapidly moving into mainstream of mathematics mainly because of its applications in diverse fields with include biochemistry (DNA double helix and SNP assembly Problem), chemistry (model chemical compounds) electrical engineering (communication networks and coding

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theory) computer science (algorithms and computations) and Operations Research (scheduling).

Many Problems of practical interest that can be modeled as graph theoretic problems may be uncertain. To deal with this uncertainty the concept of fuzzy theory was applied to graph theory.

A fuzzy set was defined by L. A. Zadeh in 1965. Every element in the universal set is assigned a grade of membership, a value in $[0, 1]$. The elements in the universal set along with their grades of membership form a fuzzy set. In 1965 Fuzzy relations on a set was first defined by Zadeh [13]. Among many branches of modern mathematics, the theory of sets (which was founded by G. Cantor occupies a unique place. The mathematical concept of a set can be used as foundation for many branches of modern mathematics.

Rosenfeld first introduced the concept of fuzzy graphs. After that fuzzy relation on a set was first defined by Zadeh in 1965. Based on Zadeh fuzzy relation the first definition of a fuzzy graph was introduced by Kaufmann in 1973.

Azriel Rosenfeld in 1975 developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts like bridges and tree [6]. A. Nagoorgani, D. Rajalaxmi [3] introduced the concept of fuzzy labelling tree and S. Yahya Mohamad, S. Suganthi [8] introduced matching in fuzzy labelling graph.

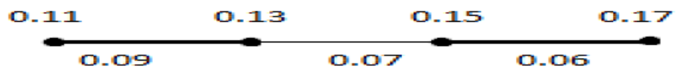
In this paper, we introduce the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labelling tree using matching and perfect matching. Here we consider the simple complete fuzzy graph with even number of vertices.

2. Preliminaries

Definition 2.1. Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$. A fuzzy graph $G = (\alpha, \beta)$ is a pair of functions $\alpha : V \to [0, 1]$ and $\beta : V \times V \to [0, 1]$ where for all $u, v \in V$, we have $\beta(u, v) \leq \min \{\alpha(u), \alpha(v)\}$.

Definition 2.2. If every vertex of fuzzy graph is M-saturated then the matching is said to be complete or perfect. It is denoted by C_M.

Example



Definition 2.3. Let $G : (\alpha, \beta)$ be a fuzzy graph and F is a subset of G. If nodes of F is contained (or) equal to the nodes of G then F is said to be a fuzzy subgraph.

Definition 2.4. A fuzzy sub graph F of the fuzzy labeling graph G is said to be fuzzy spanning sub graph [FSS] of G if nodes of fuzzy sub graph is equal to the nodes of fuzzy graph.

Definition 2.5. A fuzzy graph G is said to be fuzzy simple labelling graph [FSG] if G does not contain a line with same ends and multiple lines.

Definition 2.6. A fuzzy simple graph G is said to be fuzzy complete labelling graph [FCLG] if every pair of nodes of the graph are joined by line. *A* FCLG with n nodes are denoted by k_n.

Definition 2.7. A fuzzy labelling graph G is said to be fuzzy connected labelling graph [FCG] if there exists a path between all pair of nodes of G.

Definition 2.8. A cyclic graph G is said to be fuzzy cyclic graph if it has fuzzy labeling.

3. Main Results

Definition 3.1. A subset M of $\beta(v_i, v_{i+1}), 1 \leq i \leq n$ is called a matching in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be saturated under M.

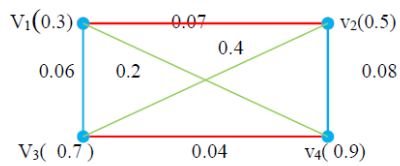
Definition 3.2. Let M be a matching in fuzzy labeling graph. An M-alternating path in G is a path whose edges alternatively in $\beta - M$ and M.

Definition 3.3. An M-Augmenting path is an M-alternating path whose origin and terminal vertices are M-unsaturated.

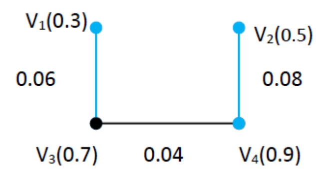
Definition 3.4. A graph $G = (\alpha, \beta)$ is said to be fuzzy labeling tree (FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path.

Example:

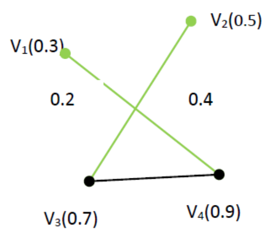
The fuzzy labelling trees of G are given below.



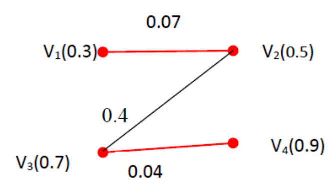
(FLG)



FLT$_1$



FLT$_2$



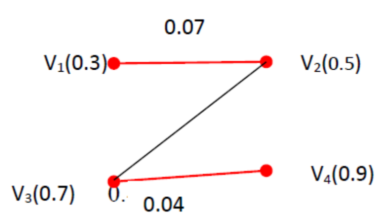
FLT$_3$

There are three distinct perfect matchings exists. Here the matchings are $M_1 = \{0.06, 0.08\}$, $M_2 = \{0.07, 0.04\}$ and $M_3 = \{0.2, 0.4\}$.

Similarly we can find the remaining nine fuzzy labelling trees.

Definition 3.6. The weight of the fuzzy labelling tree is the sum of the membership value of the lines in the spanning subgraph.

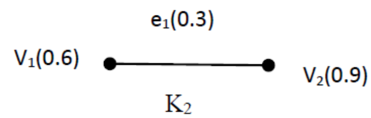
Example:



Here $W(FLT) = 0.07 + 0.4 + 0.04 = 0.51$.

Definition 3.7. An edge in a fuzzy labelling tree is said to be matching bridge if it belongs to any one of the perfect matching.

Example:



Here the line {0.3) is a matching bridge.

Theorem 3.8. *Every complete fuzzy labelling graph with even number of vertices* $(n \geq 2)$ *has a fuzzy labelling tree.*

Proof. Let G be a complete fuzzy labelling graph with even number of vertices and M be a perfect matching in G.

To prove G has a fuzzy labelling tree.

Since every fuzzy labelling graph has proper or improper subgraph, G always has the subgraph with fuzzy labeling.

The Matching is a set of non-adjacent edges. So every pair of nodes in spanning subgraph has an alternating path.

Therefore by the definition of Fuzzy labelling tree, G has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path.

Hence always G has a fuzzy labelling tree.

Theorem 3.9. *Every fuzzy labeling tree of a given fuzzy labelling graph has the same number of edges.*

Proof. Let T_1 and T_2 are two fuzzy labelling trees of a given fuzzy labelling graph G.

To prove T_1 and T_2 have the same number of edges.

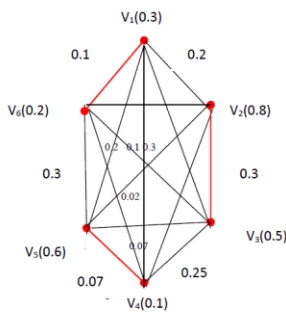
We know that every complete fuzzy labelling graph with even number of vertices $(n \geq 2)$ has a fuzzy labelling tree.

So G has fuzzy labelling trees. And also a tree is a connected acyclic graph.

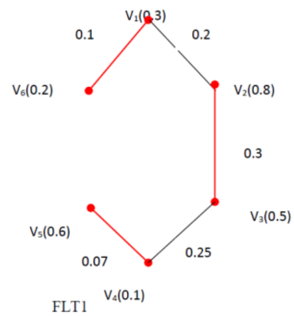
By the properties of a tree, "every tree with n vertices has $n - 1$ edges" we have all fuzzy labelling trees have same number of edges.

Hence every fuzzy labeling tree of a given fuzzy labelling graph has the same number of edges.

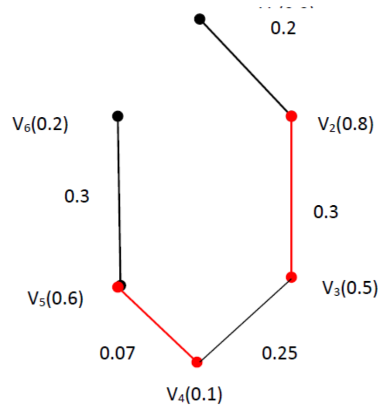
Example:



FLG



FLT1



Here FLT_1 and FLT_2 have five edges. Similarly we can find remaining fuzzy labelling trees with five edges.

Theorem 3.10. *Every fuzzy labelling tree contains at least one matching bridge.*

Proof. Let G be a fuzzy labelling graph and T be a fuzzy labelling tree of G.

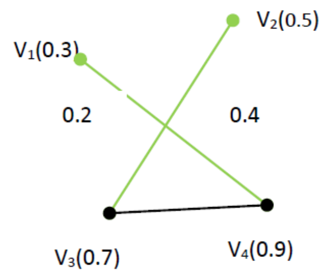
To prove T contains at least one matching bridge.

By the definition of fuzzy labelling tree, "A graph $G = (\alpha, \beta)$ is said to be fuzzy labeling tree(FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path", we have T has an alternating path between every pair of nodes in it.

In alternating path, the edges are alternatively in M and $\beta - M$. So T contains at least one edge from perfect matching M.

Therefore every fuzzy labelling tree contains at least one matching bridge.

Example:



Here the matching bridges are 0.2 and 0.4.

Theorem 3.11. *Let G be a fuzzy labeling graph and T be a fuzzy labelling tree of G. Then $G - G(T)$ is again a spanning sub graph which contains a matching.*

Proof. Let G be a fuzzy labeling graph and T be a fuzzy labelling tree of G. To prove $G - G(T)$ is again a spanning sub graph which contains a perfect matching.

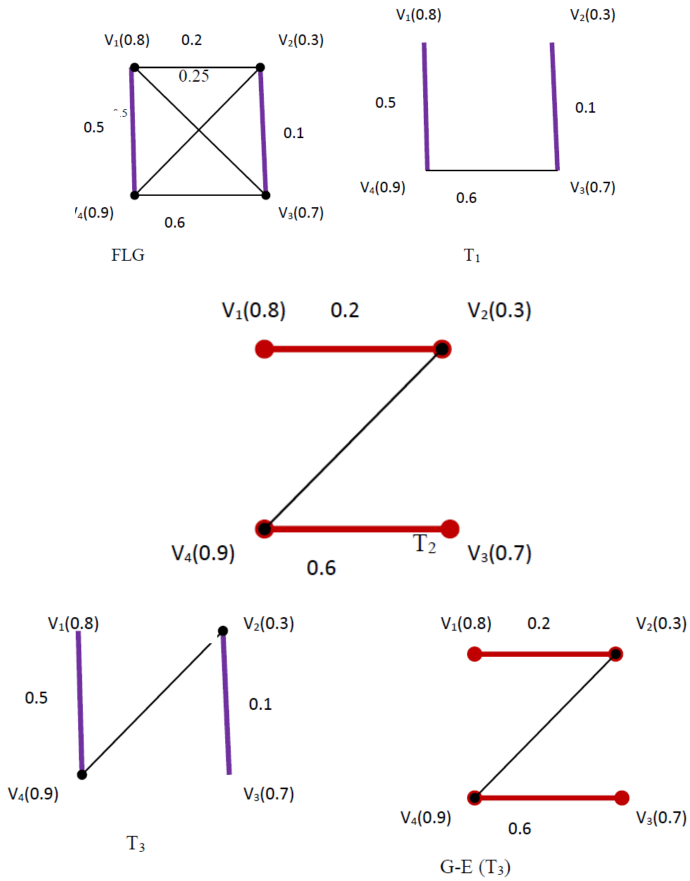
Since T be a fuzzy labelling tree. T contains a spanning subgraph S_1 in which every pair of vertices contains an alternating path.

Now we remove the edges of S_1 from G we obtain another spanning subgraph.

This spanning subgraph also contains an alternating path. It is also contains the edges in the matching.

Hence $G - G(T)$ is again a spanning sub graph which contains a matching.

Example:



4. Conclusion

In this paper, we introduced the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labelling tree using matching and perfect matching. In Future, we will find centre and eccentricity of fuzzy labelling tree using matching and perfect matching.

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