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A STUDY ON GRAPH COLOURING IN SUDOKU

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Abstract

Graph colouring enjoys many practical applications as well as theoretical challenges. Beside the classical types of problems, different limitations can also be set on the graph or on the way a colour is assigned or even on the colour itself. It has even reached popularity with the general public in the form of the popular number puzzle Sudoku. Graph colouring is still a very active field of research.

Introduction

The game of Sudoku has become, in the last few years, the rage among puzzle and game enthusiasts looking for a more intellectual (and cheaper) challenge than the one provided by an X-Box. Sudoku is addictive and even ordinary people that are not drawn to video games are hooked on it. These days practically every major newspaper carries a daily Sudoku puzzle.

It seems that these days everyone is enjoying the game of Sudoku wherever they are. The Sudoku puzzle is ideal for whenever you have a few spare minutes and want to indulge in a little bit of thinking power. Sudoku, sometimes spelled “Sudoku”, is a puzzle that originated in Japan. The puzzle is known as a “placement” puzzle. In the United States Sudoku is sometimes called the “Number Place” puzzle.

People of all ages and from all backgrounds are finding that Sudoku is a great way to keep their mind active and thinking. Puzzles range all the way

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from easy for the beginner to extremely difficult for the more advanced puzzler. Sudoku is easy to take with you wherever you go so that you can indulge in a little bit of number guessing whenever you have a few spare minutes.

If you haven't played Sudoku yet, the rules are quite simple: You start with a grid of 81 squares called cells. The grid is also subdivided into nine sub grids called boxes. Some of the cells are already filled with the numbers 1 through the labels 1 through 9 on the columns and a through i on the rows are not part of the puzzle, but they provide a convenient way to refer to the cells. Just for fun, you may want to try this one out before you read on. (Hint: Try to figure out what number should go in cell. Once you have that one figured, go to cell $d3$. That's enough help for now.

These are called the givens. The challenge of the game is to complete the grid by filling the remaining cells with the numbers 1 through 9.

To complete the Sudoku puzzle requires a lot of patience as well as the ability to think logically. The basic layout of the Sudoku grid is much like a chess game or crossword puzzles. Sudoku is not just a mathematical or arithmetic type of puzzle. It works just as well if the numbers are substituted with letters or other symbols. However, numbers work best.

The requirements are.

(1) Every row and every column of the grid must have the numbers 1 through 9.

(2) Appear once; each of the nine boxes must have the numbers 1 through 9 appears once.

A typical Sudoku puzzle may have somewhere between 25 and 40 givens, depending on the level of difficulty. Figure 1 is an example of a moderately easy Sudoku puzzle.

The labels 1 through 9 on the columns and a through i on the rows are not part of the puzzle, but they provide a convenient way to refer to the cells. Just for fun, you may want to try this one out before you read on. (Hint: Try to figure out what number should go in cell. Once you have that one figured, go to cell d3.That's enough help for now.

Sudoku is usually played as a 9×9 grid which contains 3×3 regions. Although this is the most common grid layout there are many variations which can be found. The following grid layouts are not uncommon and can add an even more challenging level to the puzzle:

(a) 4×4 grid with 2×2 regions

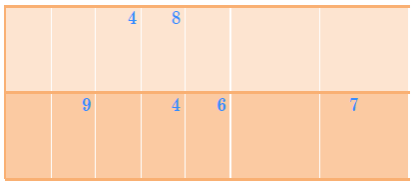
(b) 5×5 grid with pentomino regions (these puzzles are known as "Logi-5")

(c) 6×6 grid with 2×3 regions (grid used in the World Puzzle Championship)

(d) 7×7 grid with six heptomino regions as well as a "disjoint" region.

(e) 9×9 grid that generally has nonomino regions.

Larger grid puzzles are possible, such as the 16×16 grid layout published by Dell that is known as "Number Place Challenger". As well, Nikoli in Japan as published a 25×25 grid. Yet another variant for the Sudoku puzzle is the for the numbers in the main diagonal areas to be completely unique.



Terminology and Rules

The Sudoku puzzle is quite easy to solve, at least in the general concept. Your goal will be to fill in each of the empty cells with one number. Every row, column, and region will contain the numbers from one to nine exactly one time. This means that every number in the solution of the puzzle will occur only one time in three directions.

The reason that so many people are attracted to a Sudoku puzzle is that, even though the solving rules are simple, the reasoning behind the path to the correct solution can be very difficult. Most puzzles will be ranked according to how difficult they are. Still other puzzles will give you an estimated time of how long it should take you to solve the puzzle. In most

cases, the more "givens" there are, the easier the puzzle will be to solve. The bottom line on how easy it is to solve a Sudoku puzzle will depend on how easy it is to determine the logical order of all of the numbers.

Answer for Figure 1.

6	2	4	8	7	1	9	5	3
1	9	3	4	6	5	8	7	2
7	5	8	3	9	2	6	1	4
2	1	9	6	4	3	5	8	7
5	8	6	7	2	9	3	4	1
4	3	7	1	5	8	2	6	9
3	4	5	2	1	6	7	9	8
8	6	1	9	3	7	4	2	5
9	7	2	5	8	4	1	3	6

The Sudoku Graph

To see the connection between Sudoku and graph colouring, we will first describe the Sudoku graph, which for convenience we will refer to as S. The graph S has 81 vertices, with each vertex representing a cell. When two cells cannot have the same number (either because they are in the same row, in the same column, or in the same box) we put an edge connecting the corresponding vertices of the Sudoku graph S. For example, since cells $a3$ and $a7$ are in the same row, there is an edge joining their corresponding vertices; there is also an edge connecting $a1$ and $b3$ (They are in the same box), and so on.

When everything is said and done, each vertex of the Sudoku graph has degree 20 and the graph have a total of 810 edges. S is too large to draw, but we can get a sense of the structure of S by looking at a partial drawing such as the one in Figure 2. The drawing shows all 81 vertices of S, but only two ($a1$ and $e5$) have their full set of incident edges showing.

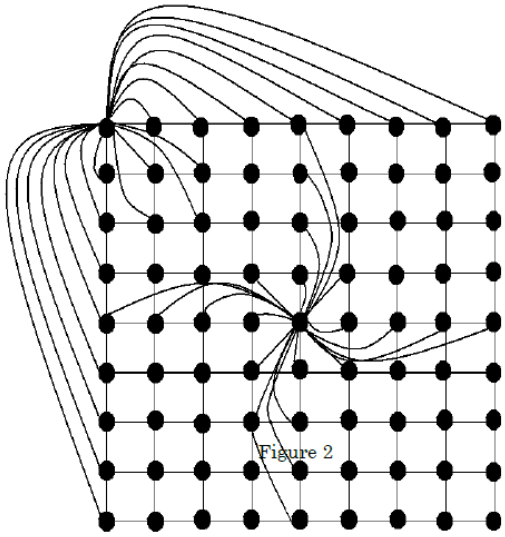


Figure 2

The second step in converting a Sudoku puzzle into a graph colouring problem is to assign colors to the numbers 1 through 9. This assignment is arbitrary, and is not a priority ordering of the colours as in the greedy algorithm it's just a simple correspondence between numbers and colours. Figure 3 shows one such assignment.

Conclusion

In this Project, the basic definitions of Graph, Graph colouring and types of colouring are discussed. The main aim of this study is to know where the Graph colouring used in various field. This project is a surprising application of the graph colouring techniques we developed in this project, and involves an important real-life problem. This paper gives an overview of the applications of graph colouring in heterogeneous fields to some extent applications that uses graph colouring concepts. Various papers based on graph colouring have been studied related to scheduling concepts, computer science applications and an overview has been presented here.

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