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BIPOLAR SINGLE VALUED NEUTROSOPHIC APPROACH TO MULTI CRITERIA DECISION MAKING PROBLEM

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Abstract

In this paper we have applied the concept of bipolar single valued neutrosophic set. Bipolar neutrosophic set theory is a strong tool for dealing with uncertainty, indeterminate and imprecise information. We used score, certainty and accuracy functions to compare the bipolar neutrosophic sets. We develop a bipolar single valued neutrosophic multicriteria decision making problem in which the values of alternatives take the form of bipolar single valued neutrosophic numbers. Based on bipolar neutrosophic weighted average operator A_w and bipolar neutrosophic weighted geometric operator G_w we can aggregate the bipolar neutrosophic information. Using score, certainty and accuracy functions we can select the most desirable one. Finally we proposed an approach which is applied to multicriteria decision making problem and demonstrated the advantage of the approach by a numerical example.

1. Introduction

The notion of fuzzy set was introduced by Zadeh in 1965 [10] to deal with vagueness. In 1986, Atanassov [1] extended the concept of fuzzy set to

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intuitionistic fuzzy set by offering the degree of non membership as an independent component. Smarandache [7] grounded the concept of degree of indeterminacy as independent component and defined neutrosophic set. Kharel [4] used neutrosophic numbers in multicriteria decision making problems. Hagar et al. [2] introduced TOPSIS method to solve neutrosophic multicriteria decision making problems. Wang et al. [8] introduced the concept of single valued neutrosophic set which is an instance of a neutrosophic set to deal real scientific and engineering applications. Ye [9] introduced subtraction and division for neutrosophic sets. Lee [5] instituted bipolar fuzzy sets, as an extension of fuzzy sets. In bipolar fuzzy sets, the degree of membership is extended from $[0, 1]$ to $[-1, 1]$. If the degree of membership of an element is zero, then we say that the element is unrelated to the corresponding property, the membership $(0, 1]$ of an element specifies that the element somewhat satisfies the property and the membership degree $[-1, 0)$ of an element specifies that the element somewhat satisfies the implicit counter property.

Deli et al. [3] dened bipolar neutrosophic set and showed numerical example for multi criteria decision making problem. Shiny Jose [6] proposed a decision making model in interval valued intuitionistic fuzzy environment. In this paper we applied the concept of bipolar single valued neutrosophic sets which is an extension of fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. In section 2, we study the concept of bipolar neutrosophic set and its operations including the score, uncertainty and accuracy functions to compare the bipolar neutrosophic sets. Also we used the bipolar neutrosophic weighted average operator (A_w), bipolar neutrosophic weighted geometric operator (G_w) to aggregate the bipolar neutrosophic information. In section 3, based on A_w, G_w operators and the score, certainty and accuracy fn, we develop a bipolar neutrosophic MCDM approach in which the values of alternatives on the attributes take the form of bipolar neutrosophic numbers to select the most desirable one and give a numerical example to demonstrate the application of the developed method. Section 4 concludes the paper.

2. Preliminaries

Definition 2.1. Neutrosophic set. Let X be a universe of discourse. Then a neutrosophic set is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$$

which is characterized by a truth membership function $T_A : X \to]0^-, 1^+[$ and indeterminacy membership function $I_A : X \to]0^-, 1^+[$ and a falsity membership function $F_A : X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$.

So $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$.

Definition 2.2. Single valued neutrosophic set. Let X be a universe of discourse. A single valued neutrosophic set is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$$

which is characterized by a truth membership function $T_A : X \to [0, 1]$, an indeterminacy membership function $I_A : X \to [0, 1]$ and a falsity membership function $F_A : X \to [0, 1]$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$.

So $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$.

For any two single valued neutrosophic sets

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\} \text{ and}$$

$$B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle / x \in X\}$$

(1) The subset $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$ and $F_A(x) \geq F_B(x)$.

(2) $A = B$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$.

(3) The complement of A is denoted by A^c and it is defined as

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in X\}$$

(4) The intersection

$$A \cap B = \{\langle x, \min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \max\{F_A(x), F_B(x)\}\rangle / x \in X\}$$

(5) The union

$$A \cup B = \{\langle x, \max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\}\rangle / x \in X\}$$

A single valued neutrosophic number is denoted by $\widetilde{A} = \langle T, I, F\rangle$ for our convenience.

Definition 2.3. Let $\widetilde{A}_1 = \langle T_1, I_1, F_1\rangle$ and $\widetilde{A}_2 = \langle T_2, I_2, F_2\rangle$ be Two single valued neutrosophic numbers, then the operation of neutrosophic numbers are defined as

(1) $\lambda\widetilde{A}_1 = \langle 1 - (1 - T_1)^{\lambda}, I_1^{\lambda}, F_1^{\lambda}\rangle$

(2) $\widetilde{A}_1^{\lambda} = \langle T_1^{\lambda}, 1 - (1 - I_1)^{\lambda}, 1 - (1 - F_1)^{\lambda}\rangle$

(3) $\widetilde{A}_1 + \widetilde{A}_2 = \langle T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2\rangle$

(4) $\widetilde{A}_1 \cdot \widetilde{A}_2 = \langle T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2\rangle$ where $\lambda > 0$.

Definition 2.4. Score function. Let $\widetilde{A} = \langle T, I, F\rangle$ be a single valued neutrosophic number. Then the score function $S(\widetilde{A})$ is defined as follows

$$S(\widetilde{A}) = \frac{T + (1 - I) + (1 - F)}{3}$$

Accuracy function

$$A(\widetilde{A}) = T - F$$

Certainty function

$$C(\widetilde{A}) = T$$

Definition 2.5. Comparison of two neutrosophic numbers. Let $\widetilde{A}_1 = \langle T_1, I_1, F_1 \rangle$ and $\widetilde{A}_2 = \langle T_2, I_2, F_2 \rangle$ be two single valued neutrosophic numbers

(1) If $S(\widetilde{A}_1) > S(\widetilde{A}_2)$, then $\widetilde{A}_1 > \widetilde{A}_2$. That is $\widetilde{A}_1$ is higher than $\widetilde{A}_2$.

(2) If $S(\widetilde{A}_1) = S(\widetilde{A}_2)$ and $A(\widetilde{A}_1) = A(\widetilde{A}_2)$ then $\widetilde{A}_1 > \widetilde{A}_2$. That is $\widetilde{A}_1$ is higher than $\widetilde{A}_2$.

(3) If $S(\widetilde{A}_1) = S(\widetilde{A}_2)$, $A(\widetilde{A}_1) = A(\widetilde{A}_2)$ and $C(\widetilde{A}_1) > C(\widetilde{A}_2)$ then $\widetilde{A}_1 > \widetilde{A}_2$. That is $\widetilde{A}_1$ is higher than $\widetilde{A}_2$.

(4) If $S(\widetilde{A}_1) = S(\widetilde{A}_2)$, $A(\widetilde{A}_1) = A(\widetilde{A}_2)$ and $C(\widetilde{A}_1) = C(\widetilde{A}_2)$ then $\widetilde{A}_1 = \widetilde{A}_2$. That is $\widetilde{A}_1$ is same as $\widetilde{A}_2$.

Definition 2.6. Bipolar valued fuzzy set. Let X be a non empty set. Then a bipolar valued fuzzy set denoted by A_{BF} is defined as

$$A_{BF} = \{\langle x, \mu_B^+(x), \mu_B^-(x)\rangle / x \in X\} \text{ where}$$

$$\mu_B^+ : X \to [0, 1] \text{ and } \mu_B^- : X \to [-1, 0]$$

The positive membership degree $\mu_B^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to A_{BF} and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of x to some implicit counter property of A_{BF}.

Definition 2.7. Bipolar neutrosophic set. A bipolar neutrosophic set A in X is defined as an object of the form

$$A = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\rangle / x \in X\}$$

where $T^+, I^+, F^+ : X \to [0, 1]$ and $T^-, I^-, F^- : X \to [-1, 0]$.

The positive membership degree T^+, I^+, F^+ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree T^{-}, I^{-}, F^{-} denotes the truth membership, indeterminate membership and false membership function of an element $x \in X$ some implicit counter property corresponding to a bipolar neutrosophic set A.

Example.

Let $X = \{x_1, x_2, x_3\}$. Then

$$A = \begin{Bmatrix} \langle x_1, 0.5, 0.8, 0.6, -0.6, -0.4, -0.01 \rangle \\ \langle x_2, 0.8, 0.7, 0.5, -0.2, -0.3, -0.4 \rangle \\ \langle x_3, 0.4, 0.3, 0.6, -0.8, -0.7, -0.6 \rangle \end{Bmatrix}$$

is a bipolar neutrosophic subset of X.

Definition 2.8.

Let $A_1 = \{\langle x, T_1^{+}(x), I_1^{+}(x), F_1^{+}(x), T_1^{-}(x), I_1^{-}(x), F_1^{-}(x)\rangle / x \in X\}$ and $A_2 = \{\langle x, T_2^{+}(x), I_2^{+}(x), F_2^{+}(x), T_2^{-}(x), I_2^{-}(x), F_2^{-}(x)\rangle / x \in X\}$ be two bipolar neutrosophic sets. Then $A_1 \subseteq A_2$ if and only if

$T_1^{+}(x) \leq T_2^{+}(x), I_1^{+}(x) \leq I_2^{+}(x), F_1^{+}(x) \geq F_2^{+}(x),$

$T_1^{-}(x) \geq T_2^{-}(x), I_1^{-}(x) \geq I_2^{-}(x), F_1^{-}(x) \leq F_2^{-}(x)$ for every $x \in X$.

Definition 2.9.

Let $A_1 = \{\langle x, T_1^{+}(x), I_1^{+}(x), F_1^{+}(x), T_1^{-}(x), I_1^{-}(x), F_1^{-}(x)\rangle / x \in X\}$ and $A_2 = \{\langle x, T_2^{+}(x), I_2^{+}(x), F_2^{+}(x), T_2^{-}(x), I_2^{-}(x), F_2^{-}(x)\rangle / x \in X\}$ be two bipolar neutrosophic sets. Then $A_1 = A_2$ if and only if

$T_1^{+}(x) = T_2^{+}(x), I_1^{+}(x) = I_2^{+}(x), F_1^{+}(x) = F_2^{+}(x),$

$T_1^{-}(x) = T_2^{-}(x), I_1^{-}(x) = I_2^{-}(x), F_1^{-}(x) = F_2^{-}(x)$ for every $x \in X$.

Definition 2.10.

$$A_1 \cup A_2(x) = \left(\max(T_1^+(x), T_2^+(x)), \frac{I_1^+(x) + I_2^+(x)}{2}, \min(F_1^+(x), F_2^+(x)), \right.$$

$$\left. \min(T_1^-(x), T_2^-(x)), \frac{I_1^-(x) + I_2^-(x)}{2}, \max(F_1^-(x), F_2^-(x)) \right)$$

for every $x \in X$.

Definition 2.11.

$$A_1 \cap A_2(x) = \left(\min(T_1^+(x), T_2^+(x)), \frac{I_1^+(x) + I_2^+(x)}{2}, \max(F_1^+(x), F_2^+(x)), \right.$$

$$\left. \max(T_1^-(x), T_2^-(x)), \frac{I_1^-(x) + I_2^-(x)}{2}, \min(F_1^-(x), F_2^-(x)) \right)$$

for every $x \in X$.

Definition 2.12.

Let $A = \{\langle x, T_1^+(x), I_1^+(x), F_1^+(x), T_1^-(x), I_1^-(x), F_1^-(x)\rangle / x \in X\}$ be a bipolar neutrosophic set in X. Then the complement of A is denoted by A^c and it is defined by

$$T_{A^c}^+(x) = \{1^+\} - T_A^+(x)$$

$$I_{A^c}^+(x) = \{1^+\} - I_A^+(x)$$

$$F_{A^c}^+(x) = \{1^+\} - F_A^+(x)$$

$$T_{A^c}^-(x) = \{1^-\} - T_A^-(x)$$

$$I_{A^c}^-(x) = \{1^-\} - I_A^-(x)$$

$$F_{A^c}^-(x) = \{1^-\} - F_A^-(x)$$

for every $x \in X$.

Example. Let $X = \{x_1, x_2, x_3\}$. Then

$$A = \begin{Bmatrix} \langle x_1, 0.3, 0.4, 0.5, -0.6, -0.9, -0.4 \rangle \\ \langle x_2, 0.6, 0.7, 0.8, -0.3, -0.4, -0.6 \rangle \\ \langle x_3, 0.7, 0.2, 0.1, -0.2, -0.8, -0.7 \rangle \end{Bmatrix}$$

is a bipolar neutrosophic set in X. Then the complement of A is given as follows:

$$A^c = \begin{Bmatrix} \langle x_1, 0.7, 0.6, 0.5, -0.4, -0.1, -0.6 \rangle \\ \langle x_2, 0.4, 0.3, 0.2, -0.7, -0.6, -0.4 \rangle \\ \langle x_3, 0.3, 0.8, 0.9, -0.8, -0.2, -0.3 \rangle \end{Bmatrix}$$

Definition 2.13.

Let $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ and $\tilde{a}_2 = \langle T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^- \rangle$ be two bipolar neutrosophic number. Then the operations for neutrosophic numbers are defined as below.

(1) $\lambda\tilde{a}_1 = (\langle 1 - (1 - T_1^+)^\lambda), (I_1^+)^\lambda, (F_1^+)^\lambda, -(-T_1^-)^\lambda, -(-I_1^-)^\lambda,$

$-(1 - (1 - (-F_1^-))^\lambda)\rangle$

(2) $\tilde{a}_1^\lambda = \langle (T_1^+)^\lambda, 1 - (1 - I_1^+)^\lambda, 1 - (1 - F_1^+)^\lambda, -(1 - (1 - (-T_1^-)^\lambda))$

$-(-I_1^-)^\lambda, -(-F_1^-))^\lambda)\rangle$

(3) $\tilde{a}_1 + \tilde{a}_2 = \langle T_1^+ + T_2^+ - T_1^+T_2^+, I_1^+I_2^+, F_1^+F_2^+, -T_1^-T_2^-,$

$-(-I_1^- - I_2^- - I_1^-I_2^-), -(-F_1^- - F_2^- - F_1^-F_2^-)\rangle$

(4) $\tilde{a}_1 \cdot \tilde{a}_2 = \langle T_1^+T_2^+, I_1^+ + I_2^+ - I_1^+I_2^+, F_1^+ + F_2^+ - F_1^+F_2^+,$

$-(-T_1^- - T_2^- - T_1^-T_2^-), -(I_1^-I_2^-) - (F_1^-F_2^-)\rangle$

where $\lambda > 0$.

Definition 2.14. Score function. If $\tilde{a} = \langle T^+, I^+, F^+, T^-, I^-, F^- \rangle$ be a bipolar neutrosophic number. Then the score function

$$S(\tilde{a}) = \frac{(T^+ + (1 - I^+) + (1 - F^+) + (1 - T^-) - I^- - F^-)}{6}$$

Accuracy function

$$A(\tilde{a}) = T^{+} - F^{+} + T^{-} - F^{-}$$

Certainty function

$$C(\tilde{a}) = T^{+} - F^{-}$$

Definition 2.15.

(1) If $S(\tilde{a}_1) > S(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$.

(2) If $S(\tilde{a}_1) = S(\tilde{a}_2)$ and $A(\tilde{a}_1) > A(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$.

(3) If $S(\tilde{a}_1) = S(\tilde{a}_2)$, $A(\tilde{a}_1) = A(\tilde{a}_2)$ and $C(\tilde{a}_1) > C(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$.

4) If $S(\tilde{a}_1) = S(\tilde{a}_2)$, $A(\tilde{a}_1) = A(\tilde{a}_2)$ and $C(\tilde{a}_1) = C(\tilde{a}_2)$ then $\tilde{a}_1 = \tilde{a}_2$.

3. Some Weighted Aggregation Operators Related to Bipolar Neutrosophic Set

Bipolar neutrosophic weighted average operator 3.1.

Let $\tilde{a}_j = \langle T_j^{+}, I_j^{+}, F_j^{+}, T_j^{-}, I_j^{-}, F_j^{-} \rangle_{j=1, 2, \ldots, n}$ be a family of bipolar neutrosophic numbers. A mapping $A_w : Q_n \to Q$ is called bipolar neutrosophic weighted average operator if it satisfies

$$A_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_j$$

$$= \left\langle 1 - \prod_{j=1}^{n}(1 - T_j^{+})^{w_j}, \prod_{j=1}^{n}(I_j^{+})^{w_j}, \prod_{j=1}^{n}(F_j^{+})^{w_j}, -\prod_{j=1}^{n}(-T_j^{-})^{w_j}, \right.$$

$$\left. -(1 - \prod_{j=1}^{n}(1 - (-I_j^{-}))^{w_j}), -(1 - \prod_{j=1}^{n}(1 - (-F_j^{-}))^{w_j}) \right\rangle$$

where w_j is the weight of $\tilde{a}_j$ $(j = 1, 2, \ldots, n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3.2. Bipolar neutrosophic weighted geometric operator

Let $\tilde{a}_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle_{j=1, 2, \ldots, n}$ be a family of bipolar neutrosophic numbers. A mapping $G_w : Q_n \to Q$ is called bipolar neutrosophic weighted geometric operator if it satisfies

$$G_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{w_j}$$

$$= \left\langle \prod_{j=1}^{n} (T_j^+)^{w_j}, \prod_{j=1}^{n} (1 - I_j^+)^{w_j}, 1 - \prod_{j=1}^{n} (1 - F_j^+)^{w_j}, \right.$$

$$\left. -(1 - \prod_{j=1}^{n} (1 - (-T_j^-)^{w_j})), -\prod_{j=1}^{n} (-I_j^-)^{w_j}, -\prod_{j=1}^{n} (-F_j^-)^{w_j} \right\rangle$$

where w_j is the weight of $\tilde{a}_j$ $(j = 1, 2, \ldots, n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Multi-criteria decision making problem

We establish an approach based on A_w (or G_w) operator to deal with muticriteria decision making problems with bipolar neutrosophic information.

Suppose that $A = \{A_1, A_2, \ldots, A_m\}$ and $C = \{C_1, C_2, \ldots, C_n\}$ is the set of alternatives and criterions or attributes respectively. Let $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of attributes, such that $\sum_{j=1}^{n} w_j = 1, w_j \geq 0$ $(j = 1, 2, \ldots, n)$ and w_j refers to the weight of attribute C_j.

An alternative on criterions is evaluated by the decision maker and the evaluation values are represented by the form of bipolar neutrosophic numbers.

Assume that $(a_{ij})_{m \times n} = (\langle T_{ij}^+, I_{ij}^+, F_{ij}^+, T_{ij}^-, I_{ij}^-, F_{ij}^- \rangle)$ is the decision matrix provided by the decision matrix, $\widetilde{a}_{ij}$ is a bipolar neutrosophic number for alternative A_i associated with the criterion C_j. We have the condition $T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}$ and $F_{ij}^{-} \in [0, 1]$ such that $0 \leq T_{ij}^{+} + I_{ij}^{+} + F_{ij}^{+} - T_{ij}^{-} - I_{ij}^{-} - F_{ij}^{-} \leq 6$, for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ we can develop an algorithm as follows:

Step (1) Construct the decision matrix provided by the decision maker as

$$(a_{ij})_{m\times n} = (\langle T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} \rangle)_{m\times n}$$

Step (2) Compute $\widetilde{a}_i = A_w(\widetilde{a}_{i1}, \widetilde{a}_{i2}, \ldots, \widetilde{a}_{in})$ or $G_w(\widetilde{a}_{i1}, \widetilde{a}_{i2}, \ldots, \widetilde{a}_{in})$ for each $i = 1, 2, \ldots, m$.

Step (3) Calculate the score values $\widetilde{S}(\widetilde{a}_i)$ for the collective overall bipolar neutrosophic number of $\widetilde{a}_i$ $(i = 1, 2, \ldots, m)$.

Step (4) Rank all the $\widetilde{a}_i$ $i = 1, 2, \ldots, m$ according to the score values.

Numerical example

Let us consider the following decision making problem.

Tamilnadu government wants to give a "BEST TEACHER AWARD". Four government school teachers were elected from votes of the government school students. These 4 teachers (Alternatives) A_i $(i = 1, 2, 3, 4)$ are available in schools. Government has to select one teacher from these 4 teachers by taking into four attributes to evaluate these 4 teachers. This 4 attributes are C_1 = Teaching skill, C_2 = sincerity, C_3 = Punctuality, C_4 = Academic records.

Use the bipolar neutrosophic values to evaluate the 4 teachers A_i $(i = 1, 2, 3, 4)$ under the above 4 attributes.

Also the weight vector of the attributes C_j $(j = 1, 2, 3, 4)$ is $w = (0.50, 0.25, 0.125, 0.125)^T$.

Algorithm

Step 1. Construct the decision matrix provided the students as follows:

$$
\begin{array}{c} \\ A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{array}
\begin{array}{cc} C_1 & C_2 \\ C_3 & C_4 \end{array}
\begin{pmatrix}
(0.7, 0.5, 0.2, -0.6, -0.5, -0.4) & (0.6, 0.5, 0.4, -0.8, -0.7, -0.6) \\
(0.8, 0.7, 0.6, -0.6, -0.5, -0.4) & (0.7, 0.6, 0.5, -0.5, -0.4, -0.6) \\
(0.8, 0.6, 0.5, -0.8, -0.5, -0.4) & (0.9, 0.5, 0.4, -0.7, -0.4, -0.6) \\
(0.5, 0.7, 0.6, -0.6, -0.8, -0.7) & (0.7, 0.8, 0.6, -0.7, -0.4, -0.3) \\
(0.5, 0.4, 0.3, -0.6, -0.5, -0.4) & (0.3, 0.4, 0.6, -0.3, -0.4, -0.5) \\
(0.8, 0.7, 0.5, -0.3, -0.2, -0.4) & (0.7, 0.6, 0.5, -0.7, -0.5, -0.4) \\
(0.6, 0.7, 0.8, -0.4, -0.3, -0.2) & (0.6, 0.7, 0.5, -0.6, -0.5, -0.4) \\
(0.5, 0.6, 0.7, -0.5, -0.6, -0.7) & (0.6, 0.5, 0.3, -0.5, -0.6, -0.7)
\end{pmatrix}
$$

Step 2. Compute $\tilde{a}_i = G_w(\tilde{a}_{j1}, \tilde{a}_{j2}, \tilde{a}_{j3}, \tilde{a}_{j4})$ for each $i = 1, 2, 3, 4$

$$\tilde{a}_1 = G_w(\tilde{a}_{11}, \tilde{a}_{12}, \tilde{a}_{13}, \tilde{a}_{14}) = \prod_{j=1}^{n} \tilde{a}_{ij}^{w_j}$$

$$= \left\langle \prod_{j=1}^{4} (T_{1j}^{+})^{w_j}, 1 - \prod_{j=1}^{4} (1 - I_{1j}^{+})^{w_j}, 1 - \prod_{j=1}^{4} (1 - F_{1j}^{+})^{w_j}, \right.$$

$$\left. -(1 - \prod_{j=1}^{4} (1 - (-T_{1j}^{-})^{w_j})), - \prod_{j=1}^{4} (-I_{1j}^{-})^{w_j}, - \prod_{j=1}^{4} (-F_{1j}^{-})^{w_j} \right\rangle$$

where w_j is the weight $\tilde{a}_j$, $j = 1, 2, 3, 4$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

$$\tilde{a}_1 = \langle 0.68, 0.54, 0.36, -0.65, -0.54, -0.46 \rangle$$

$$\tilde{a}_2 = G_w(\tilde{a}_{21}, \tilde{a}_{22}, \tilde{a}_{23}, \tilde{a}_{24}) = \prod_{j=1}^{n} \tilde{a}_{2j}^{w_j}$$

$$= \left\langle \prod_{j=1}^{4} (T_{2j}^{+})^{w_j}, 1 - \prod_{j=1}^{4} (1 - I_{2j}^{+})^{w_j}, 1 - \prod_{j=1}^{4} (1 - F_{2j}^{+})^{w_j}, \right.$$

$$- (1 - \prod_{j=1}^{4}(1 - (-T_{2j}^{-})^{w_j})), - \prod_{j=1}^{4}(-I_{2j}^{-})^{w_j}, - \prod_{j=1}^{4}(-F_{2j}^{-})^{w_j}\Big\rangle$$

$$= \langle 0.76, 0.62, 0.51, -0.74, -0.48, -0.45\rangle$$

$$\tilde{a}_3 = G_w(\tilde{a}_{31}, \tilde{a}_{32}, \tilde{a}_{33}, \tilde{a}_{34}) = \prod_{j=1}^{n}\tilde{a}_{3j}^{w_j}$$

$$= \Big\langle \prod_{j=1}^{4}(T_{3j}^{+})^{w_j}, 1 - \prod_{j=1}^{4}(1 - I_{3j}^{+})^{w_j}, 1 - \prod_{j=1}^{4}(1 - F_{3j}^{+})^{w_j},$$

$$- (1 - \prod_{j=1}^{4}(1 - (-T_{3j}^{-})^{w_j})), - \prod_{j=1}^{4}(-I_{3j}^{-})^{w_j}, - \prod_{j=1}^{4}(-F_{3j}^{-})^{w_j}\Big\rangle$$

$$= \langle 0.48, 0.478, 0.44, -0.52, -0.42, -0.42\rangle$$

$$\tilde{a}_4 = G_w(\tilde{a}_{41}, \tilde{a}_{42}, \tilde{a}_{43}, \tilde{a}_{44}) = \prod_{j=1}^{n}\tilde{a}_{4j}^{w_j}$$

$$= \Big\langle \prod_{j=1}^{4}(T_{4j}^{+})^{w_j}, 1 - \prod_{j=1}^{4}(1 - I_{4j}^{+})^{w_j}, 1 - \prod_{j=1}^{4}(1 - F_{4j}^{+})^{w_j},$$

$$- (1 - \prod_{j=1}^{4}(1 - (-T_{4j}^{-})^{w_j})), - \prod_{j=1}^{4}(-I_{4j}^{-})^{w_j}, - \prod_{j=1}^{4}(-F_{4j}^{-})^{w_j}\Big\rangle$$

$$= \langle 0.58, 0.67, 0.69, -0.48, -0.40, -0.32\rangle$$

Step 3. Calculate the score values of $S(\tilde{a}_i)$ $i = 1, 2, 3, 4$ for the collective overall bipolar neutrosophic number $\tilde{a}_1$ $(i = 1, 2, 3, 4)$ as follows:

$$S(\tilde{a}_1) = \frac{(T_1^{+} + (1 - I_1^{+}) + (1 - F_1^{+}) + (1 - T_1^{-}) - I_1^{-} - F_1^{-})}{6}$$

$S(\tilde{a}_1) = 0.52$

$S(\tilde{a}_2) = 0.47$

$S(\tilde{a}_3) = 0.48$

$S(\tilde{a}_4) = 0.41$

$A(\tilde{a}_1) = T_1^+ - F_1^+ + T_1^- - F_1^- = 0.13$

$A(\tilde{a}_2) = -0.04$

$A(\tilde{a}_3) = -0.06$

$A(\tilde{a}_4) = -0.27$

$C(\tilde{a}_1) = T_1^+ - F_1^- = 0.68 + 0.46 = 1.14$

$C(\tilde{a}_2) = 1.21$

$C(\tilde{a}_3) = 0.9$

$C(\tilde{a}_4) = 0.9$

Step 4. Based on the score values $A_1 > A_3 > A_2 > A_4$ so A_1 is the best alternative.

4. Conclusion

In this paper, we have discussed a bipolar neutrosophic set and its score, certainty and accuracy functions. Also we have developed a bipolar neutrosophic multicriteria decision making approach in which the values of alternatives taken from bipolar neutrosophic numbers. Based on the values of bipolar neutrosophic weighted average operator A_w, bipolar neutrosophic weighted geometric operator G_w, score, certainty and accuracy functions, we can aggregate the bipolar neutrosophic information and rank the alternatives to choose the best one. A numerical example is given to demonstrate the application and effectiveness of the developed method. For future work, we can divide any criterion to sub criterion then we can find the best alternative using corresponding formula.

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