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SOME REMARKABLE OBSERVATIONS OF FAREY SEQUENCE

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Abstract

Both Farey sequence and continued fractions are recently developing fields of number theory. Their inter relationships are studied in this paper. Farey sequence is analyzed through continued fractions as Farey sequence consist of fractals from 0 to 1. It is considered as sum of rational numbers. An attempt has been made to write the mediant inserted in Farey sequence of order as a matrix whose diagonal elements are unity. For the matrix representation only the new inserted elements have been considered.

1. Introduction

The terms of Farey sequence are fractals in [0, 1]. The representations of rationals as continued fractions is really an interesting one. Continued fractions on its own has no particular properties on addition or multiplications. In developing the Farey sequence for successive iterations the mediants are inserted between corresponding ratios. In this paper the terms of the sequence are represented as continued fractions and some properties are analyzed. Also without finding the mediant the development of Farey sequence in terms of continued fractions have been given as an algorithm. It is established that the mediant elements of F_N derived from

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F_{N-1} has been expressed as rational sum of $F_m, F_n,$ where m and n are relatively prime factors of N. Also the new inserted elements of $F_{n\times(n-1)}$ from the previous sequence has been written as direct sum $F_n \oplus F_n$. Treating this as a matrix it is a matrix whose diagonal elements are unity and can be written as sum of a symmetric upper triangular, lower triangular matrices.

1.1. Continued Fraction

An expression of the form

$$\frac{a}{b} = p_0 + \cfrac{q_0}{p_1 + \cfrac{q_1}{p_2 + \cfrac{q_2}{p_3 + \cfrac{q_3}{\ddots}}}}$$

Where p_i, q_i are real or complex numbers and is called a continued fraction.

Table 1. Farey sequence continued fractions.

F_1 :	$\frac{0}{1}$ $\langle 0, 0\rangle$	$\frac{1}{1}$ $\langle 0, 1\rangle$					
F_2 :	$\frac{0}{1}$ $\langle 0, 0\rangle$	$\frac{1}{2}$ $\langle 0, 2\rangle$	$\frac{1}{1}$ $\langle 0, 1\rangle$				
F_3 :	$\frac{0}{1}$ $\langle 0, 0\rangle$	$\frac{1}{3}$ $\langle 0, 3\rangle$	$\frac{1}{2}$ $\langle 0, 2\rangle$	$\frac{2}{3}$ $\langle 0, 1, 2\rangle$	$\frac{1}{1}$ $\langle 0, 1\rangle$		
F_4 :	$\frac{0}{1}$ $\langle 0, 0\rangle$	$\frac{1}{4}$ $\langle 0, 4\rangle$	$\frac{1}{3}$ $\langle 0, 3\rangle$	$\frac{1}{2}$ $\langle 0, 2\rangle$	$\frac{2}{3}$ $\langle 0, 1, 2\rangle$	$\frac{3}{4}$ $\langle 0, 1, 3\rangle$	$\frac{1}{1}$ $\langle 0, 1\rangle$
F_5 :	$\frac{0}{1}$ $\langle 0, 0\rangle$	$\frac{1}{5}$ $\langle 0, 5\rangle$	$\frac{1}{4}$ $\langle 0, 4\rangle$	$\frac{1}{3}$ $\langle 0, 3\rangle$	$\frac{2}{5}$ $\langle 0, 2, 2\rangle$	$\frac{1}{2}$ $\langle 0, 2\rangle$	$\frac{3}{5}$ $\langle 0, 1, 1, 2\rangle$

$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{1}{1}$
$\langle 0, 1, 2\rangle$	$\langle 0, 1, 3\rangle$	$\langle 0, 1, 4\rangle$	$\langle 0, 1\rangle$

2. Algorithm to Develop Farey Sequence

1. In each sequence first and last terms are $\langle 0, 0\rangle$ and $\langle 0, 1\rangle$ respectively.

2. In every sequence F_n, the second and last but one terms are $\langle 0, n\rangle$ and $\langle 0, 1, n-1\rangle$ respectively.

3. The number of terms in each sequence are $\sum_{k=1}^{n} \varphi(k) + 1$.

4. In each sequence the number of terms added is $\varphi(n)$, the Euler function.

5. If 'd' is the number such that the $\gcd(n, d) = 1$ then the continued fraction of $\frac{d}{n}$ occupies the r^{th} position from left with continued fraction $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor, 2\rangle$ lies between $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor + 1\rangle$ and $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor\rangle$.

6. The complement $\frac{n-d}{n}$ occupies r^{th} position from right with continued fraction $\langle 0, 1, \left\lfloor \frac{n}{d} \right\rfloor - 1, 2\rangle$.

7. If the continued fraction $\frac{d_i}{n}$ consist of three quotients, one of the following cases may arise $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor, k\rangle$ must be inserted between $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor, k-1\rangle$ and $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor\rangle$, where $k = 2, 3, 4$.

8. If the continued fraction $\frac{d_i}{n}$ consist of four quotients, one of the following cases may arise $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor, 1, k\rangle$ must be inserted between $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor + 1\rangle$ and $\langle 0, \left\lfloor \frac{n}{d} \right\rfloor, 1, k-1\rangle$, where $k = 2, 3, 4$.

9. The centre term is always taken by $\langle 0, 2\rangle$.

Theorem 2.1. *The terms in* F_{mn} *which are mediant of terms in* F_{mn-1} *can be written as the sum of rational numbers* $\frac{k}{m} \in F_m$ *and* $\frac{l}{n} \in F_n$, *where m and n are relatively primes*, $k = 1, 2, \ldots, (m-1)$ *and* $l = 1, 2, \ldots, (n-1)$.

Proof. Let $\frac{k}{m}, \frac{l}{n}$ be any two rational numbers belonging to F_m and F_n respectively, where m, n are relatively primes

For $k < m, l < n$

$$\frac{k}{m} + \frac{l}{n} = \frac{kn + lm}{mn}$$

If $kn + lm < mn$ then $\frac{k}{m} + \frac{l}{n} = \frac{kn + lm}{mn}$

If $kn + lm > mn$ then $\frac{k}{m} + \frac{l}{n} \equiv \frac{kn + lm}{mn} (\operatorname{mod} mn)$

$$= 1 + \frac{kn + lm - mn}{mn}$$

$$\equiv \frac{kn + lm - mn}{mn}$$

$$Claim : kn + lm - mn < mn$$

$$kn + lm < 2mn$$

If $k < m$ then $kn < mn.$

And if $l < n$ then $lm < mn$

$$\therefore kn + lm - mn < mn.$$

Consider the members $\frac{1}{m}, \frac{2}{m}, \ldots, \frac{m-1}{m}$ and $\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}$ and their sum taking all pairs, clearly the resulting $\varphi_{(mn)}$ numbers are in F_{mn}.

$\varphi_{(mn)}$ numbers formed above are mediants of elements in F_{mn-1}. The element $\frac{1}{mn}$ is the median of $\frac{0}{1}$ and $\frac{1}{mn-1}$.

The number $\frac{2}{15}$ is the median of $\frac{1}{\left\lceil \frac{mn}{2} \right\rceil}$ and $\frac{1}{\left\lceil \frac{mn}{2} \right\rceil + 1}$ and so on.

$\frac{mn-1}{mn}$ is the median of $\frac{mn-2}{mn-1}$ and $\frac{1}{1}$.

Illustration 2.2.

Mediant of elements of F_{14} in F_{15} as $F_5 \oplus F_3$.

Table 2.

$F_5 \oplus F_3$	$\frac{1}{3}$	Components in F_{14}	$\frac{2}{3}$	Components in F_{14}
$\frac{2}{5}$	$\frac{11}{15}$	$\left\langle \frac{8}{11}, \frac{3}{4} \right\rangle$	$\frac{1}{15}$	$\left\langle \frac{0}{1}, \frac{1}{14} \right\rangle$
$\frac{3}{5}$	$\frac{14}{15}$	$\left\langle \frac{13}{14}, \frac{1}{1} \right\rangle$	$\frac{4}{15}$	$\left\langle \frac{1}{4}, \frac{3}{11} \right\rangle$
$\frac{4}{5}$	$\frac{2}{15}$	$\left\langle \frac{1}{8}, \frac{1}{7} \right\rangle$	$\frac{7}{15}$	$\left\langle \frac{6}{13}, \frac{1}{2} \right\rangle$

F_{14} : $\frac{0}{1}$ $\langle 0, 0\rangle$ $\frac{1}{14}$ $\langle 0, 14\rangle$ $\frac{1}{13}$ $\langle 0, 13\rangle$ $\frac{1}{12}$ $\langle 0, 12\rangle$ $\frac{1}{11}$ $\langle 0, 11\rangle$ $\frac{1}{10}$ $\langle 0, 10\rangle$ $\frac{1}{9}$ $\langle 0, 9\rangle$ $\frac{1}{8}$ $\langle 0, 8\rangle$

$\frac{1}{7}$ $\langle 0, 7\rangle$ $\frac{2}{13}$ $\langle 0, 6, 2\rangle$ $\frac{1}{6}$ $\langle 0, 6\rangle$ $\frac{2}{11}$ $\langle 0, 5, 2\rangle$

$\frac{1}{5}$ $\langle 0, 5\rangle$ $\frac{3}{14}$ $\langle 0, 4, 1, 2\rangle$ $\frac{2}{9}$ $\langle 0, 4, 2\rangle$ $\frac{3}{13}$ $\langle 0, 4, 3\rangle$ $\frac{1}{4}$ $\langle 0, 4\rangle$ $\frac{3}{11}$ $\langle 0, 3, 1, 2\rangle$ $\frac{2}{7}$ $\langle 0, 3, 2\rangle$

$\frac{3}{10}$ $\langle 0, 3, 3\rangle$ $\frac{4}{13}$ $\langle 0, 3, 4\rangle$ $\frac{1}{3}$ $\langle 0, 3\rangle$

$\frac{5}{14}$ $\langle 0, 2, 1, 4\rangle$ $\frac{4}{11}$ $\langle 0, 2, 1, 3\rangle$ $\frac{3}{8}$ $\langle 0, 2, 1, 2\rangle$ $\frac{5}{13}$ $\langle 0, 2, 1, 1\rangle$ $\frac{2}{5}$ $\langle 0, 2, 2\rangle$ $\frac{5}{12}$ $\langle 0, 2, 2, 2\rangle$

$$\underset{\langle 0,\,2,\,3\rangle}{\frac{3}{7}} \quad \underset{\langle 0,\,2,\,4\rangle}{\frac{4}{9}} \quad \underset{\langle 0,\,2,\,5\rangle}{\frac{5}{11}} \quad \underset{\langle 0,\,2,\,6\rangle}{\frac{6}{13}}$$

$$\underset{\langle 0,\,2\rangle}{\frac{1}{2}} \quad \underset{\langle 0,\,1,\,1,\,6\rangle}{\frac{7}{13}} \quad \underset{\langle 0,\,1,\,1,\,5\rangle}{\frac{6}{11}} \quad \underset{\langle 0,\,1,\,1,\,4\rangle}{\frac{5}{9}} \quad \underset{\langle 0,\,1,\,1,\,3\rangle}{\frac{2}{7}} \quad \underset{\langle 0,\,1,\,1,\,2,\,2\rangle}{\frac{7}{12}}$$

$$\underset{\langle 0,\,1,\,1,\,2\rangle}{\frac{3}{5}} \quad \underset{\langle 0,\,1,\,1,\,1,\,1,\,2\rangle}{\frac{8}{13}} \quad \underset{\langle 0,\,1,\,1,\,1,\,2\rangle}{\frac{5}{8}} \quad \underset{\langle 0,\,1,\,1,\,1,\,3\rangle}{\frac{6}{11}}$$

$$\underset{\langle 0,\,1,\,1,\,1,\,4\rangle}{\frac{9}{14}} \quad \underset{\langle 0,\,1,\,2\rangle}{\frac{2}{3}} \quad \underset{\langle 0,\,1,\,2,\,4\rangle}{\frac{9}{13}} \quad \underset{\langle 0,\,1,\,2,\,3\rangle}{\frac{7}{10}} \quad \underset{\langle 0,\,1,\,2,\,2\rangle}{\frac{5}{7}} \quad \underset{\langle 0,\,1,\,2,\,1\rangle}{\frac{8}{11}}$$

$$\underset{\langle 0,\,1,\,3\rangle}{\frac{3}{4}} \quad \underset{\langle 0,\,1,\,3,\,3\rangle}{\frac{10}{13}} \quad \underset{\langle 0,\,1,\,3,\,2\rangle}{\frac{7}{9}} \quad \underset{\langle 0,\,1,\,3,\,1\rangle}{\frac{11}{14}}$$

$$\underset{\langle 0,\,1,\,4\rangle}{\frac{4}{5}} \quad \underset{\langle 0,\,1,\,4,\,2\rangle}{\frac{9}{11}} \quad \underset{\langle 0,\,5\rangle}{\frac{5}{6}} \quad \underset{\langle 0,\,1,\,5,\,2\rangle}{\frac{11}{13}} \quad \underset{\langle 0,\,1,\,6\rangle}{\frac{6}{7}} \quad \underset{\langle 0,\,1,\,7\rangle}{\frac{7}{8}}$$

$$\underset{\langle 0,\,1,\,8\rangle}{\frac{8}{9}} \quad \underset{\langle 0,\,1,\,9\rangle}{\frac{9}{10}} \quad \underset{\langle 0,\,1,\,10\rangle}{\frac{10}{11}} \quad \underset{\langle 0,\,1,\,11\rangle}{\frac{11}{12}}$$

$$\underset{\langle 0,\,1,\,12\rangle}{\frac{12}{13}} \quad \underset{\langle 0,\,1,\,13\rangle}{\frac{13}{14}} \quad \underset{\langle 0,\,1\rangle}{\frac{1}{1}}$$

Table 3. Farey sequences F_{14} and F_{15}.

$$F_{15}: \quad \underset{\langle 0,\,0\rangle}{\frac{0}{1}} \quad \underset{\langle 0,\,15\rangle}{\frac{1}{15}} \quad \underset{\langle 0,\,14\rangle}{\frac{1}{14}} \quad \underset{\langle 0,\,13\rangle}{\frac{1}{13}} \quad \underset{\langle 0,\,12\rangle}{\frac{1}{12}} \quad \underset{\langle 0,\,11\rangle}{\frac{1}{11}} \quad \underset{\langle 0,\,10\rangle}{\frac{1}{10}}$$

$$\underset{\langle 0,\,9\rangle}{\frac{1}{9}} \quad \underset{\langle 0,\,8\rangle}{\frac{1}{8}} \quad \underset{\langle 0,\,7,\,2\rangle}{\frac{2}{15}} \quad \underset{\langle 0,\,7\rangle}{\frac{1}{7}}$$

$$\underset{\langle 0,\,6,\,2\rangle}{\frac{2}{13}} \quad \underset{\langle 0,\,6\rangle}{\frac{1}{6}} \quad \underset{\langle 0,\,5,\,2\rangle}{\frac{2}{11}} \quad \underset{\langle 0,\,5\rangle}{\frac{1}{5}} \quad \underset{\langle 0,\,4,\,1,\,2\rangle}{\frac{3}{14}} \quad \underset{\langle 0,\,4,\,2\rangle}{\frac{2}{9}} \quad \underset{\langle 0,\,4,\,3\rangle}{\frac{3}{13}}$$

$$\frac{1}{4} \quad \frac{4}{15} \quad \frac{3}{11}$$
$$\langle 0,\, 4\rangle \quad \langle 0,\, 3,\, 1,\, 4\rangle \quad \langle 0,\, 3,\, 1,\, 2\rangle$$

$$\frac{2}{7} \quad \frac{3}{10} \quad \frac{4}{13} \quad \frac{1}{3} \quad \frac{5}{14} \quad \frac{4}{11}$$
$$\langle 0,\, 3,\, 2\rangle \quad \langle 0,\, 3,\, 3\rangle \quad \langle 0,\, 3,\, 4\rangle \quad \langle 0,\, 3\rangle \quad \langle 0,\, 2,\, 1,\, 4\rangle \quad \langle 0,\, 2,\, 1,\, 3\rangle$$

$$\frac{3}{8} \quad \frac{5}{13} \quad \frac{2}{5} \quad \frac{5}{12}$$
$$\langle 0,\, 2,\, 1,\, 2\rangle \quad \langle 0,\, 2,\, 1,\, 1,\, 2\rangle \quad \langle 0,\, 2,\, 2\rangle \quad \langle 0,\, 2,\, 2,\, 2\rangle$$

$$\frac{3}{7} \quad \frac{4}{9} \quad \frac{5}{11} \quad \frac{6}{13} \quad \frac{7}{15} \quad \frac{1}{2}$$
$$\langle 0,\, 2,\, 3\rangle \quad \langle 0,\, 2,\, 4\rangle \quad \langle 0,\, 2,\, 5\rangle \quad \langle 0,\, 2,\, 6\rangle \quad \langle 0,\, 2,\, 7\rangle \quad \langle 0,\, 2\rangle$$

$$\frac{8}{15} \quad \frac{7}{13} \quad \frac{6}{11}$$
$$\langle 0,\, 1,\, 1,\, 7\rangle \quad \langle 0,\, 1,\, 1,\, 6\rangle \quad \langle 0,\, 1,\, 1,\, 5\rangle$$

$$\frac{5}{9} \quad \frac{4}{7} \quad \frac{7}{12} \quad \frac{3}{5} \quad \frac{8}{13}$$
$$\langle 0,\, 1,\, 1,\, 4\rangle \quad \langle 0,\, 1,\, 1,\, 3\rangle \quad \langle 0,\, 1,\, 1,\, 2,\, 2\rangle \quad \langle 0,\, 1,\, 1,\, 2\rangle \quad \langle 0,\, 1,\, 1,\, 1,\, 1,\, 2\rangle$$

$$\frac{5}{8} \quad \frac{7}{11}$$
$$\langle 0,\, 1,\, 1,\, 1,\, 2\rangle \quad \langle 0,\, 1,\, 1,\, 1,\, 3\rangle$$

$$\frac{9}{14} \quad \frac{2}{3} \quad \frac{9}{13} \quad \frac{7}{10} \quad \frac{5}{7}$$
$$\langle 0,\, 1,\, 1,\, 1,\, 4\rangle \quad \langle 0,\, 1,\, 2\rangle \quad \langle 0,\, 1,\, 2,\, 4\rangle \quad \langle 0,\, 1,\, 2,\, 3\rangle \quad \langle 0,\, 1,\, 2,\, 2\rangle$$

$$\frac{8}{11} \quad \frac{11}{15}$$
$$\langle 0,\, 1,\, 2,\, 1,\, 2\rangle \quad \langle 0,\, 1,\, 2,\, 1,\, 3\rangle$$

$$\frac{7}{9} \quad \frac{11}{14} \quad \frac{4}{5} \quad \frac{9}{11} \quad \frac{5}{6}$$
$$\langle 0,\, 1,\, 3,\, 2\rangle \quad \langle 0,\, 1,\, 3,\, 1,\, 2\rangle \quad \langle 0,\, 1,\, 4\rangle \quad \langle 0,\, 1,\, 4,\, 2\rangle \quad \langle 0,\, 5\rangle$$

$$\frac{11}{13} \quad \frac{6}{7} \quad \frac{13}{15}$$
$$\langle 0,\, 1,\, 5,\, 2\rangle \quad \langle 0,\, 1,\, 6\rangle \quad \langle 0,\, 1,\, 6,\, 2\rangle$$

$\frac{7}{8}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{10}{11}$	$\frac{11}{12}$	$\frac{12}{13}$
$\langle 0, 1, 7\rangle$	$\langle 0, 1, 8\rangle$	$\langle 0, 1, 9\rangle$	$\langle 0, 1, 10\rangle$	$\langle 0, 1, 11\rangle$	$\langle 0, 1, 12\rangle$
$\frac{13}{14}$	$\frac{14}{15}$	$\frac{1}{1}$			
$\langle 0, 1, 13\rangle$	$\langle 0, 1, 14\rangle$	$\langle 0, 1\rangle$			

3. Proposition

$F_n \oplus F_n$ is $n \times n$ an matrix whose elements are the mediant elements of inserted in $F_{n-1\times n}$ from the previous Farey sequence. This matrix is a sum of symmetric upper diagonal matrix and symmetric lower diagonal matrix whose diagonal consist of unity.

$$F_4 \oplus F_4$$

$$\begin{array}{c|c}
F_4 \oplus F_4 & \begin{matrix} \frac{3}{4} & \frac{2}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \end{matrix} \\
\begin{matrix} \frac{1}{4} \\ \frac{1}{3} \\ \frac{1}{2} \\ \frac{2}{3} \\ \frac{3}{4} \end{matrix} & \begin{bmatrix} \frac{1}{1} & \frac{11}{12} & \frac{3}{4} & \frac{7}{12} & \frac{1}{2} \\ \frac{1}{12} & \frac{1}{1} & \frac{5}{6} & \frac{2}{3} & \frac{7}{12} \\ \frac{1}{4} & \frac{1}{6} & \frac{1}{1} & \frac{5}{6} & \frac{3}{4} \\ \frac{5}{12} & \frac{1}{3} & \frac{1}{6} & \frac{1}{1} & \frac{11}{12} \\ \frac{1}{2} & \frac{5}{12} & \frac{1}{4} & \frac{1}{12} & \frac{1}{1} \end{bmatrix}
\end{array}$$

$$= \begin{bmatrix} \frac{0}{1} & \frac{0}{12} & \frac{0}{4} & \frac{0}{12} & \frac{0}{2} \\ \frac{1}{12} & \frac{0}{1} & \frac{0}{6} & \frac{0}{3} & \frac{0}{12} \\ \frac{1}{4} & \frac{1}{6} & \frac{0}{1} & \frac{0}{6} & \frac{0}{4} \\ \frac{5}{12} & \frac{1}{3} & \frac{1}{6} & \frac{0}{1} & \frac{0}{12} \\ \frac{1}{2} & \frac{5}{12} & \frac{1}{4} & \frac{1}{12} & \frac{0}{1} \end{bmatrix} + \begin{bmatrix} \frac{0}{1} & \frac{11}{12} & \frac{3}{4} & \frac{7}{12} & \frac{1}{2} \\ \frac{0}{12} & \frac{0}{1} & \frac{5}{6} & \frac{2}{3} & \frac{7}{12} \\ \frac{0}{4} & \frac{0}{6} & \frac{0}{1} & \frac{5}{6} & \frac{3}{4} \\ \frac{0}{12} & \frac{0}{3} & \frac{0}{6} & \frac{0}{1} & \frac{11}{12} \\ \frac{0}{2} & \frac{0}{12} & \frac{0}{4} & \frac{0}{12} & \frac{0}{1} \end{bmatrix}$$

$$+\begin{bmatrix} \frac{1}{1} & \frac{0}{15} & \frac{0}{15} & \frac{0}{15} & \frac{0}{15} \\ \frac{0}{15} & \frac{1}{1} & \frac{0}{15} & \frac{0}{15} & \frac{0}{15} \\ \frac{0}{15} & \frac{0}{15} & \frac{1}{1} & \frac{0}{15} & \frac{0}{15} \\ \frac{0}{15} & \frac{0}{15} & \frac{0}{15} & \frac{1}{1} & \frac{0}{15} \\ \frac{0}{15} & \frac{0}{15} & \frac{0}{15} & \frac{0}{15} & \frac{1}{1} \end{bmatrix}.$$

4. Conclusion

In this paper Farey sequence whose elements are fractal has been analyzed in various fashions. Here Farey sequence is first expressed as continued fraction and then a sum of rational numbers. Also it is observed that the Farey sequence is represented as a matrix. Various approaches of the successive inserted terms has been discussed here. Likewise in future Farey sequence in terms of equations may be studied.

References

[1] A. Gnanam and S. Krithika, Ratios of polygonal numbers as continued fractions, International Journal of Engineering Science, Advanced Computing and Bio Technology 8(3) (2017), 143-155.

[2] P. Balamurugan and A. Gnanam, Pattern classification of continued fractions with triangular number as base, International Journal of Pure and Applied Mathematics 119(15) (2018), 3413-3418.

[3] P. Balamurugan and A. Gnanam, Pattern classification of continued fractions with centered polygonal number as base, International Journal of Management, IT and Engineering 9(1) (2019), 156-162.

[4] P. Balamurugan, A. Gnanam and B. Anitha, Expression of ratios of polygonal numbers as continued fractions, Advances and Applications in Mathematical Sciences 18(10) (2019), 997-1006.

[5] A. Gnanam, P. Balamurugan and C. Dinesh, Farey Grids, IOSR Journal of Mathematics (IOSR - JM) 12(6) (2016), 70-73. DOI: 10.9790/5728-1206027073

[6] Ivan Niven, Herbert S. Zukerman and Hugh L. Montgomery, An introduction to theory of Numbers, 5th edition, John Wiley and Sons 1991.

[7] Long, Calvin T. Elementary Introduction to Number Theory, Lexington, VA: D.C. Heath and Company, 2nd Ed, 1972.

[8] A. Gnanam and C. Dinesh, Extraction of Cantor Middle from Non-Reducible Farey Subsequence, International Journal of Scientific Engineering and Research (IJSER) 4(2) (2016), 18-20.

[9] A. Gnanam and C. Dinesh, Farey to Cantor, International Journal of Science and Research (IJSR) 4(11) (2015), 1219-1220.