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ON BIVARIATE COM-POISSON POLYA-AEPPLI DISTRIBUTION

S. THILAGARATHINAM, V. SAAVITHRI and R. SEETHALAKSHMI

Research Scholar
PG and Research Department of Mathematics
Nehru Memorial College
Affiliated to Bharathidasan University
Tiruchirappalli, Tamil Nadu, India
E-mail: rathinam.thilaga93@gmail.com

Associate Professor
PG and Research Department of Mathematics
Nehru Memorial College
Affiliated to Bharathidasan University
Tiruchirappalli, Tamil Nadu, India
E-mail: saavithriramani@gmail.com

Senior Assistant Professor
Department of Mathematics
SASTRA Deemed University
Thanjavur, Tamil Nadu, India
E-mail: seethalakshmi@maths.sastra.edu

Abstract

The present research paper introduces bivariate COM-Poisson Polya-Aeplli distribution. It is a generalization of bivariate Polya-Aeppli distribution. The properties of bivariate COM-Poisson Polya-Aeplli distribution are derived. The simulation study is executed for computing the probabilities, expectation, variance, covariance and correlation coefficients.

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1. Introduction

In 1930, Polya introduced, Polya-Aeppli distribution, he ascribed the derivation of the distribution to his student Aeppli in a Zurich thesis in 1924. It is a particular case of the compound Poisson distribution. The Polya-Aeppli distribution can be derived as a model for the number of objects where the objects occur in clusters, the clusters follow a Poisson distribution and the number of objects within a cluster follows a geometric distribution [4]. It is also called the geometric Poisson distribution.

In 2011a, Gamze Ozel introduced, bivariate versions of the Neyman type A, Neyman type B, geometric-Poisson, Thomas distributions and the usefulness of these distributions are illustrated in the analysis of earthquake data. In 2014, Minkova and Balakrishnan proposed bivariate Poisson distribution obtained by the trivariate reduction method and compound it with a geometric distribution to derive a bivariate Polya-Aeppli distribution.

In 2018, J. Priyadharshini and V. Saavithri, introduced compound COM-Poisson Polya-Aeppli distribution. It is compounding COM-Poisson with the geometric distribution. COM-Poisson Polya-Aeppli distribution is a generalization of Polya-Aeppli distribution.

In this research paper, bivariate COM-Poisson Polya-Aeppli distribution is introduced and its properties are derived. The simulation study is carried out for computing the probabilities, expectation, variance, covariance and correlation coefficient.

This paper is organised as follow, in section 2, the univariate COM-Poisson Polya-Aeppli distribution is given. Bivariate COM-Poisson Polya-Aeppli distribution is introduced and its properties are also given in section 3. In section 4, the simulation study is given. The conclusion is given in section 5.

2. Univariate COM-Poisson Polya-Aeppli Distribution

In this section, the univariate compound COM-Poisson Polya-Aeplli distribution [7] is derived by using the following assumptions,

(i) Y denotes the number of clusters and Y follows COM-Poisson distribution. (i.e.) $Y \sim CMP(\lambda, \nu)$.

(ii) X_i denotes the number of objects within i^{th} cluster and X follows a geometric distribution. (i.e.) $X \sim Geo(1 - \rho)$.

(iii) Assume that X_i's are independent and identically distributed random variables and are independent of Y.

(iv) Let $N = \sum_{i=1}^{Y} X_i$. This random variable N, formed by compounding in this fashion, gives rise to the univariate COM-Poisson Polya-Aeppli distribution.

The probability generating function of N is

$$G_N(s) = \frac{1}{Z(\lambda, \nu)} \sum_{j=0}^{\infty} \frac{1}{(j!)^{\nu}} [(\lambda s(1-\rho))^j (1-\rho s)^{-j}] \tag{1}$$

Now, expand the probability generating function of N in (1) and then, collecting the coefficient of s^n, we find an explicit expression for the probability mass function of N as

$$P(N = n) = \frac{1}{Z(\lambda, \nu)} \sum_{j=1}^{n} \frac{1}{(j!)^{\nu}} \binom{n-1}{j-1} [\lambda(1-\rho)]^j \rho^{n-j} \text{ for } n = 0, 1, 2, \ldots \tag{2}$$

This is the probability mass function of univariate COM-Poisson Polya-Aeppli distribution, its denoted by $N \sim CPPA(\lambda, \nu, \rho)$.

The mean, variance and the ration between variance and mean are given by

$$Mean(N) = \frac{\lambda Z_{\lambda}(\lambda, \nu)}{(1-\rho) Z(\lambda, \nu)}$$

$$Var(N) = \frac{1}{Z(\lambda, \nu)(1-\rho)^2} \left[\lambda^2 Z_{\lambda\lambda}(\lambda, \nu) + (1-\rho)\lambda Z_{\lambda}(\lambda, \nu) - \frac{\lambda^2 [Z_{\lambda}(\lambda, \nu)]^2}{Z(\lambda, \nu)} \right]$$

$$R(\lambda, \nu, \rho) = \frac{\lambda}{(1-\rho)} \left[\frac{Z_{\lambda\lambda}(\lambda, \nu)}{Z_{\lambda}(\lambda, \nu)} - \frac{Z_{\lambda}(\lambda, \nu)}{Z(\lambda, \nu)} \right] + \left(\frac{1+\rho}{1-\rho} \right)$$

3. Bivariate COM-Poisson Polya-Aeppli Distribution

Compound Poisson distribution plays a very important role in probability theory and has applications in biology, seismology, risk theory, health science, etc. In this section, the bivariate compound COM-Poisson Polya-Aeppli distribution is derived. It is a generalization of bivariate Polya-Aeppli distribution [5].

Definition. Let the pairs $X_i, Y_i, i = 0, 1, 2, \ldots$ are separately independent and identically distributed random variables, independent from each other and follow geometric distribution with the probability mass functions

$$P(X_i = x) = (1 - \rho_1)\rho_1^{x-1}, \ x = 0, 1, 2, \ldots$$

$$P(Y_i = y) = (1 - \rho_2)\rho_2^{y-1}, \ y = 0, 1, 2, \ldots$$

where $\rho_1 > 0$ and $\rho_2 > 0$.

The probability generating function of X_i and Y_i are

$$G_X(z_1) = \frac{(1 - \rho_1)z_1}{1 - z_1\rho_1}, \ | z_1 | \leq 1 \tag{3}$$

$$G_Y(z_2) = \frac{(1 - \rho_2)z_2}{1 - z_2\rho_2}, \ | z_2 | \leq 1 \tag{4}$$

Let N be another discrete random variable which is independent of X_i and $Y_i, i = 0, 1, 2, \ldots$ and N follows COM-Poisson distribution with parameters $\lambda > 0$ and $\nu \geq 0$. The probability mass function of N [1] is

$$P(N = n) = \frac{\lambda^n}{(n!)^\nu} \frac{1}{\psi(\lambda, \nu)}$$

where $\psi(\lambda, \nu) = \sum_{j=0}^{\infty} \frac{\lambda^j}{(j!)^\nu}, \ \lambda > 0$ and $\nu \geq 0$.

The probability generating function of N [1] is

$$G_N(z) = \frac{\psi(\lambda z, \nu)}{\psi(\lambda, \nu)}, \ | z_1 | \leq 1 \tag{5}$$

Then the bivariate compound COM-Poisson random variables are given by

$$S_1 = \sum_{i=1}^{N} X_i,\ S_2 = \sum_{i=1}^{N} Y_i \tag{6}$$

Joint probability generating function of (S_1, S_2).

The joint probability generating functions of S_1 and S_2 [3] is given by

$$G_{S_1, S_2}(z_1, z_2) = G_N(G_X(z_1)G_Y(z_2))$$

$$= \frac{\psi(\lambda G_X(z_1)G_Y(z_2), \nu)}{\psi(\lambda, \nu)}$$

$$= \frac{1}{\psi(\lambda, \nu)} \psi\left(\frac{\lambda(1-\rho_1)(1-\rho_2)z_1 z_2}{(1-\rho_1 z_1)(1-\rho_2 z_2)}, \nu\right)$$

$$= \frac{1}{\psi(\lambda, \nu)} \sum_{j=0}^{\infty} \frac{\left(\frac{\lambda z_1 z_2(1-\rho_1)(1-\rho_2)}{(1-\rho_1 z_1)(1-\rho_2 z_2)}\right)^j}{(j!)^{\nu}}$$

$$G_{S_1, S_2}(z_1, z_2) = \frac{1}{\psi(\lambda, \nu)} \sum_{j=0}^{\infty} \frac{1}{(j!)^{\nu}} [\lambda z_1 z_2 (1-\rho_1)(1-\rho_2)]^j [(1-\rho_1 z_1)(1-\rho_2 z_2)]^{-j}$$

Now, expanding the above equation (7) and collecting the coefficient of $z_1^1 z_2^1, \ldots, z_1^{n_1} z_2^{n_2}$, we find an explicit expression for the probabilities of bivariate COM-Poisson Polya-Aeppli distribution as.

$$P(N_1 = 0, N_2 = 0) = \frac{1}{\psi(\lambda, \nu)}$$

$$P(N_1 = 1, N_2 = 1) = \frac{\lambda(1-\rho_1)(1-\rho_2)}{\psi(\lambda, \nu)(1!)^{\nu}}$$

$$P(N_1 = 2, N_2 = 2) = \frac{1}{\psi(\lambda, \nu)} \sum_{j=1}^{2} \frac{(\lambda(1-\rho_1)(1-\rho_2))^j}{(j!)^{\nu}} \rho_1^{2-j} \rho_2^{2-j}$$

$$P(N_1 = 3, N_2 = 3) = \frac{1}{\psi(\lambda, \nu)} \sum_{j=1}^{3} \frac{(\lambda(1-\rho_1)(1-\rho_2))^j}{(j!)^\nu} \binom{2}{j-1}\binom{2}{j-1} \rho_1^{3-j}\rho_2^{3-j}$$

$$P(N_1 = 4, N_2 = 4) = \frac{1}{\psi(\lambda, \nu)} \sum_{j=1}^{4} \frac{(\lambda(1-\rho_1)(1-\rho_2))^j}{(j!)^\nu} \binom{3}{j-1}\binom{3}{j-1} \rho_1^{4-j}\rho_2^{4-j}$$

$$P(N_1 = 5, N_2 = 5) = \frac{1}{\psi(\lambda, \nu)} \sum_{j=1}^{5} \frac{(\lambda(1-\rho_1)(1-\rho_2))^j}{(j!)^\nu} \binom{4}{j-1}\binom{4}{j-1} \rho_1^{5-j}\rho_2^{5-j}$$

In general

$$P(N_1 = n_1, N_2 = n_2)$$

$$= \frac{1}{\psi(\lambda, \nu)} \sum_{j=1}^{n} \frac{(\lambda(1-\rho_1)(1-\rho_2))^j}{(j!)^\nu} \binom{n_1-1}{j-1}\binom{n_2-1}{j-1} \rho_1^{n_1-j}\rho_2^{n_2-j}$$

$$n_1 = n_2 = 0, 1, 2, \ldots \tag{8}$$

This is the joint probability mass function of the bivariate compound COM-Poisson Polya-Aeplli distribution.

3.1 Properties. *In this section, expressions for mean, variance, covariance and correlation coefficient are given below.*

$$E(S_1^r S_2^s) = \frac{\frac{\partial^{r+s} G_{S_1, S_2}(z_1, z_2)}{\partial z_1^r \partial z_2^s}\Big|_{z_1=z_2=0}}{r!\, s!}, \; r, s = 0, 1, 2, \ldots \tag{9}$$

Differentiating the joint probability generating function given by Equation (7), we have

$$E(N_1) = \frac{\lambda \psi_1(\lambda, \nu)}{(1-\rho_1)\psi(\lambda, \nu)}$$

$$E(N_2) = \frac{\lambda \psi_1(\lambda, \nu)}{(1-\rho_2)\psi(\lambda, \nu)}$$

$$E(N_1 N_2) = \frac{\lambda}{(1-\rho_1)(1-\rho_2)\psi(\lambda, \nu)} [\lambda \psi_2(\lambda, \nu) + \psi_1(\lambda, \nu)]$$

$$Var(N_1) = \frac{\lambda}{(1-\rho_1)^2 \psi(\lambda, \nu)} \left[(1-\rho_1)\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]$$

$$Var(N_2) = \frac{\lambda}{(1-\rho_2)^2 \psi(\lambda, \nu)} \left[(1-\rho_2)\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]$$

$$Cov(N_1, N_2) = \frac{\lambda}{(1-\rho_1)(1-\rho_2)\psi(\lambda, \nu)} \left[\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]$$

$$Cov(N_1, N_2) = \frac{\frac{\lambda}{(1-\rho_1)(1-\rho_2)\psi(\lambda, \nu)} \left[\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]}{\sqrt{\frac{\lambda}{(1-\rho_1)^2 \psi(\lambda, \nu)} \left[(1-\rho_1)\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]}}$$

$$\times \sqrt{\frac{\lambda}{(1-\rho_2)^2 \psi(\lambda, \nu)} \left[(1-\rho_2)\psi_1(\lambda, \nu) + \lambda \left(\psi_2(\lambda, \nu) - \frac{(\psi_1(\lambda, \nu))^2}{\psi(\lambda, \nu)} \right) \right]}$$

4. Simulation Study

In this section, a simulation study is carried out for computing the probabilities, expectation, variance, covariance and correlation coefficient. The simulation considered two samples of sizes 50 and 100 for various parameter settings.

Table 1. Parameter settings.

Sample size (n)	50, 100
ρ_1	0.5
ρ_2	0.4
λ	0.4, 0.2, 0.3, 0.25,0.5
ν	0.1, 0.3, 0.4, 0.2, 0.5

The expectation and variance, covariance and correlation coefficient for different parameter settings are given in the following table.

Table 2. Simulation results for samples of size 50, 100.

	$E(N_1)$	$E(N_2)$	$Var(N_1)$	$Var(N_2)$	$E(N_1, N_2)$	$Cov(N_1, N_2)$	$Corr(N_1, N_2)$
Parameters $\lambda = 0.4$, $\nu = 0.1$, $\rho_1 = 0.5$, $\rho_2 = 0.4$							
Theoretical value	3.1249	2.6040	6.4844	4.0691	10.9369	2.7996	0.5450
Sample size: 50	2.4531	1.9115	4.0224	5.3915	6.0337	1.3447	0.2888
Sample size: 100	2.8317	2.4132	4.4545	3.1957	8.7330	1.8997	0.5035
Parameters $\lambda = 0.2$, $\nu = 0.3$, $\rho_1 = 0.5$, $\rho_2 = 0.4$							
Theoretical value	2.3703	1.9753	3.2122	1.9014	5.3835	0.7015	0.2839
Sample size: 50	2.2137	1.7663	2.7524	4.3299	4.4266	0.5166	0.1496
Sample size: 100	2.3181	1.9511	2.8225	1.8093	5.1438	0.6212	0.2743
Parameters $\lambda = 0.3$, $\nu = 0.4$, $\rho_1 = 0.5$, $\rho_2 = 0.4$							
Theoretical value	2.5371	2.1142	3.7978	2.2850	6.4145	1.0506	0.3566
Sample size: 50	1.3444	1.8241	2.9262	1.3447	4.9145	2.4622	1.2412
Sample size: 100	2.4558	2.0726	3.2101	2.1219	5.9876	0.8978	0.3440
Parameters $\lambda = 0.25$, $\nu = 0.2$, $\rho_1 = 0.5$, $\rho_2 = 0.4$							
Theoretical value	2.5308	2.1090	3.8203	2.3015	6.4121	1.0745	0.3624
Sample size: 50	2.2904	1.8163	3.0672	1.3380	4.8702	0.7100	0.3505
Sample size: 100	2.4465	2.0648	3.2052	2.1216	5.9539	0.9024	0.3461
Parameters $\lambda = 0.5$, $\nu = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0.4$							
Theoretical value	2.8930	2.4108	5.1116	3.1479	8.8235	1.8488	0.4609
Sample size: 50	2.4455	1.9154	3.7253	1.6061	5.8054	1.1214	0.4584
Sample size: 100	2.7256	2.3132	3.9742	2.7610	7.7740	1.4691	0.4435

From the above table, it is clear that for the sample size 50 and 100, the values of expectation, variance, covariance and correlation coefficients obtained from simulation and theoretical values are approximately the same for different sets of parameters. The probability values of the bivariate COM-Poisson Polya-Aeppli distribution for a different set of parameters are given in the following tables.

Table 3. Parameter settings $\lambda = 0.4$, $\nu = 0.1$, $\rho_1 = 0.5$, $\rho_2 = 0.4$.

n_2 \ n_1	0	1	2	3	4	5	Total
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.1902	0.0761	0.0304	0.0122	0.0049	0.3138
2	0.0000	0.0951	0.0593	0.0323	0.0163	0.0079	0.2109
3	0.0000	0.0476	0.0403	0.0269	0.0160	0.0089	0.1397
4	0.0000	0.0238	0.0255	0.0200	0.0135	0.0084	0.0912
5	0.0000	0.0119	0.0154	0.0139	0.0105	0.0071	0.0587
6	0.0000	0.0059	0.0090	0.0091	0.0076	0.0056	0.0373
7	0.0000	0.0030	0.0052	0.0058	0.0053	0.0042	0.0235
8	0.0000	0.0015	0.0029	0.0036	0.0035	0.0030	0.0146
9	0.0000	0.0007	0.0016	0.0022	0.0023	0.0021	0.0090
10	0.0000	0.0004	0.0009	0.0013	0.0015	0.0014	0.0055
Total	0.0000	0.3800	0.2363	0.1455	0.0887	0.0535	0.9040

Table 4. Parameter settings $\lambda = 0.2$, $\nu = 0.3$, $\rho_1 = 0.5$, $\rho_2 = 0.4$.

n_1 \ n_2	0	1	2	3	4	5	Total
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.2522	0.1009	0.0404	0.0161	0.0065	0.4161
2	0.0000	0.1261	0.0627	0.0300	0.0140	0.0064	0.2392
3	0.0000	0.0631	0.0375	0.0205	0.0106	0.0053	0.1369
4	0.0000	0.0315	0.0218	0.0132	0.0074	0.0040	0.0780
5	0.0000	0.0158	0.0125	0.0082	0.0050	0.0028	0.0442
6	0.0000	0.0079	0.0070	0.0050	0.0032	0.0019	0.0250

7	0.0000	0.0039	0.0039	0.0030	0.0020	0.0013	0.0141
8	0.0000	0.0020	0.0021	0.0017	0.0012	0.0008	0.0079
9	0.0000	0.0010	0.0012	0.0010	0.0007	0.0005	0.0044
10	0.0000	0.0005	0.0006	0.0006	0.0004	0.0003	0.0025
Total	0.0000	0.5040	0.2502	0.1236	0.0607	0.0297	0.9682

Table 5. Parameter settings $\lambda = 0.3$, $\nu = 0.4$, $\rho_1 = 0.5$, $\rho_2 = 0.4$.

n_1 \ n_2	0	1	2	3	4	5	Total
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3865
1	0.0000	0.2343	0.0937	0.0375	0.0150	0.0060	0.2338
2	0.0000	0.1172	0.0628	0.0315	0.0152	0.0071	0.1401
3	0.0000	0.0586	0.0394	0.0231	0.0125	0.0065	0.0832
4	0.0000	0.0293	0.0237	0.0157	0.0093	0.0052	0.0491
5	0.0000	0.0146	0.0138	0.0101	0.0065	0.0039	0.0287
6	0.0000	0.0073	0.0079	0.0063	0.0044	0.0028	0.0167
7	0.0000	0.0037	0.0045	0.0039	0.0028	0.0019	0.0097
8	0.0000	0.0018	0.0025	0.0023	0.0018	0.0013	0.0056
9	0.0000	0.0009	0.0014	0.0014	0.0011	0.0008	0.0032
10	0.0000	0.0005	0.0007	0.0008	0.0007	0.0005	0.0000
Total	0.0000	0.4682	0.2505	0.1325	0.0694	0.0360	0.9566

Table 6. Parameter settings $\lambda = 0.25$, $\nu = 0.2$, $\rho_1 = 0.5$, $\rho_2 = 0.4$.

n_1 \ n_2	0	1	2	3	4	5	Total
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.00000
1	0.0000	0.2360	0.0944	0.0378	0.0151	0.0060	0.38924
2	0.0000	0.1180	0.0626	0.0312	0.0149	0.0070	0.23369
3	0.0000	0.0590	0.0390	0.0227	0.0123	0.0063	0.13931
4	0.0000	0.0295	0.0234	0.0154	0.0092	0.0051	0.08249

5	0.0000	0.0147	0.0136	0.0099	0.0064	0.0039	0.04854
6	0.0000	0.0074	0.0078	0.0062	0.0043	0.0028	0.02839
7	0.0000	0.0037	0.0044	0.0038	0.0028	0.0019	0.01651
8	0.0000	0.0018	0.0024	0.0023	0.0018	0.0013	0.00955
9	0.0000	0.0009	0.0013	0.0013	0.0011	0.0008	0.00550
10	0.0000	0.0005	0.0007	0.0008	0.0007	0.0005	0.00315
Total	0.0000	0.4715	0.2495	0.1312	0.0686	0.0356	0.9564

Table 7. Parameter settings $\lambda = 0.5$, $\nu = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0.4$.

n_1 \ n_2	0	1	2	3	4	5	Total
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.2016	0.0806	0.0323	0.0129	0.0052	0.3325
2	0.0000	0.1008	0.0617	0.0332	0.0167	0.0081	0.2205
3	0.0000	0.0504	0.0415	0.0270	0.0157	0.0085	0.1432
4	0.0000	0.0252	0.0261	0.0196	0.0128	0.0076	0.0914
5	0.0000	0.0126	0.0157	0.0133	0.0095	0.0062	0.0574
6	0.0000	0.0063	0.0092	0.0087	0.0067	0.0047	0.0356
7	0.0000	0.0031	0.0053	0.0054	0.0046	0.0034	0.0218
8	0.0000	0.0016	0.0030	0.0033	0.0030	0.0023	0.0132
9	0.0000	0.0008	0.0017	0.0020	0.0019	0.0016	0.0079
10	0.0000	0.0004	0.0009	0.0012	0.0012	0.0010	0.0047
Total	0.0000	0.4028	0.2457	0.1461	0.0850	0.0485	0.9282

5. Conclusion

In this study, a bivariate compound COM-Poisson Polya-Aeppli distribution is defined and its properties are derived. The simulation study is executed for computing the probabilities, expectation, variance, covariance and correlation coefficient for various sets of parameters and it is observed that the values obtained coincide with the theoretical values for the sample sizes of 50 and 100.

References

[1] R. W. Conway and W. L. Maxwell, A queuing model with state dependent service rates, Journal of Industrial Engineering 12 (1962), 132-136.

[2] Gamze Ozel, The probability function of the compound poisson distribution using integer partions and ferre's graph, Bulletin of Statistics and Economics 2(1) (2008), 75-87.

[3] G. Suresh and T. Gayathri, Graphoidal covering degree sequence number of product graphs, Malaya Journal of Matematik S(1) (2021), 441-442.

[4] G. Suresh and T. Gayathri, E-graphs, Advances and Applications in Mathematical Sciences 21(2) (2021), 1015-1019.

[5] G. Suresh et al., Prediction of absenteeism at work with multinomial logistic regression model, Advances and Applications in Mathematical Sciences 21(3) (2022), 1479-1489.

[6] Gamze Ozel, A bivariate compound Poisson model for the occurrence of foreshock and aftershock sequence in Turkey, Environmetrics 22 (2011a), 847-856.

[7] N. L. Johnson, S. Kotz and A. W. Kemp, Univariate discrete distributions (3rd ed.), New York Wiley, (2005).

[8] A. Ramanathan and G. Suresh, Effect of magnetic field dependent viscosity and anisotropy of porous medium on ferro convection, International Journal of Engineering Science 42(3-4) (2004), 411-425.

[9] G. Suresh and T. Gayathri, Numerical analysis of magnetic field dependent viscosity and anisotropy of porous medium, Advances and Applications in Mathematical Sciences 21(2) (2021), 937-949.

[10] G. Suresh and R. Vasanthakumari, Numerical analysis of thermal convection of a fluid with inclined axis of rotation using galerkin method, International Research Journal of Engineering and Technology 3(1) (2016), 466-473.

[11] G. Suresh et al., Comparison With Theoretical Approach To Soret-Driven Ferro Thermohaline Convection by Computational Method, Dogo Rangsang Research Journal 10(7) 12 (2020), 256-271.

[12] L. D. Minkova and N. Balakrishnan, A bivariate Polya-Aeppli distribution, Communication in Statistics-Theory and Methods 43 (2014), 5026-5038.

[13] G. Polya, Sur quelques points de la theorie des probabilities, Annales de l'Institut H. Poincre 1 (1930), 117-161.

[14] J. Priyadharshini, V. Saavithri and R. Seethalakshmi, COM- Poisson Polya-Aeppli Distribution, International Journal of Mathematics Trends and Technology (IJMTT), Special Issue, ISSN NO: 2231-5373, (2018), 125-132.

[15] G. Shmeli, T. P. Minka, J. B. Kadane, S. Borle and P. Boatwright, A useful distribution for fitting discrete data: Revival of the COM-Poisson distribution, J. R. Stat. Soc. Ser. C (Appl. Stat) 54 (2005), 127-142.