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ANALYSIS ON HIRING TIME IN WORKFORCE STRUCTURE WITH VOLUNTARY AND INVOLUNTARY CLUMP OF EGRESSES AND CONSTANT THRESHOLD

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Abstract

Consider a marketing organization (workforce structure) admitting clump of voluntary and involuntary egresses of employees. Two stochastic exemplars are composed by utilizing CUM and MAX hiring strategies for respective exemplars. Analytical result for mean square deviation for hiring time is derived while (i) total number of egresses is a compound Poisson process (ii) deprivation of workforce due to voluntary egress in any clump are independent and exponentially distributed (iii) deprivation of workforce due to involuntary egress in any clump are independent and exponentially distributed with a parameter different from that for voluntary egress and (iv) threshold is a non-negative constant. Expected total deprivation of workforce in the hiring interval is also obtained. The impact of the leading parameters on the mean square deviation of hiring time is quantitatively illustrated with relevant conclusions. A remark on the suitability of the exemplars is given with justification.

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1. Introduction

As an explorer, the author of [3] and [4] has analyzed workforce design by employing different statistical processes and methods. A comprehensive research utilizing CUM hiring strategy on the analysis of hiring problem in structuring workforce is reported in the literature. For details one may refer to ([8], [10], [15], [12], [16], [13], [14]). In [5], the author has studied the problem when the structure has different epochs for egress and decisions which cause egress. In [2], mean square deviation of hiring time is determined for a workforce structure with deprivation occurring due to two types of non-clumped egresses of employees. Recently in [7], authors have analyzed the structure by considering clump of non-classified egresses and random threshold for cumulative number of egresses. Some research work has been done on this problem using MAX hiring strategy also. One may refer ([17], [11], [1], [9], [5], [8]) to know the development. Research work in [7] is studied in [6] utilizing MAX hiring strategy. Present paper is an extension of [6] and [7] when (i) each clump of egresses in the workforce structure consists of independent and exponentially distributed deprivation of workforce due to voluntary egress and has another independent and exponentially distributed deprivation of workforce due to involuntary egress with a parameter different from that for voluntary egress and (ii) threshold is a non-negative constant.

2. Description of Exemplar – 1

Consider a workforce structure having clump of egresses of employees in $(0, \infty]$. Egress is classified as voluntary and involuntary in each clump. Let $C(u)$, the count for clumps of egresses in $(0, u]$, be a Poisson process with degree $a, (a > 0)$. For each i, clump i consists of Y_i voluntary and Z_i involuntary egresses respectively. It is presumed that Y_i is independent of $Z_i, i = 1, 2, 3, \ldots, \{Y_i\}_{i=1}^{\infty}$ is a sequence of independent and identically distributed exponential random variables with parameter $\theta_1, (\theta_1 > 0)$ and $\{Z_i\}_{i=1}^{\infty}$ is a sequence of independent and identically distributed exponential random variables with parameter $\theta_2, (\theta_2 > 0)$. The total deprivation of workforce $Y_i + Z_i$ for i^{th} clump is denoted by $X_i, i = 1, 2, 3\ldots.$ Note that

$\sum_{i=1}^{j} Y_i \sim$Gamma (θ_1, j), $\sum_{i=1}^{j} Z_i \sim$Gamma (θ_2, j), and $\sum_{i=1}^{j} X_i \sim$Gamma $(\theta_1 + \theta_2, j)$. The cumulative deprivation in $(0, u]$ is denoted by $S(u)$. Note that $S(u) = \sum_{i=1}^{C(u)} X_i$ is a compound Poisson process. Let c be a positive real number representing the threshold for the cumulative deprivation of workforce. It is presumed that $\{C(u)\}, \{Y_i\}_{i=1}^{\infty}$ and $\{Z_i\}_{i=1}^{\infty}$ are statistically independent. **CUM hiring strategy is to hire when cumulative deprivation of workforce exceeds the threshold.** Let T_{CUM} be the time to hiring with mean $E(T_{CUM})$ and mean square deviation $V(T_{CUM})$. Let $E[S(T_{CUM})]$ be the expected cumulative deprivation of workforce in the interval of hiring.

3. Main Result for Exemplar – 1

From the hiring strategy we note that

$$P(T_{CUM} > u) = P(S(u) \leq c) = P\left[\sum_{i=1}^{C(u)} X_i \leq c\right] \tag{1}$$

Conditioning upon $C(u)$ and noting that $\{C(u); u \geq 0\}$ is independent of X_i, $i = 1, 2, 3 \ldots$ we get

$$P(T_{CUM} > u) = \sum_{j=0}^{\infty} P\left[\sum_{i=1}^{j} X_i \leq c\right] P[C(u) = j]$$

Since $\sum_{i=1}^{j} X_i \leq c$ is an impossible event when $j = 0$, we can write

$$P(T_{CUM} > u) = \sum_{j=1}^{\infty} P\left[\sum_{i=1}^{j} X_i \leq c\right] P[C(u) = j] \tag{2}$$

Since $E[T_{CUM}] = \int_0^{\infty} P(T_{CUM} > u)du$, using the hypothesis on $\{C(u)\}$ we get

$$E[T_{CUM}] = \sum_{j=1}^{\infty} P\left[\sum_{i=1}^{j} X_i \le c\right] \int_0^{\infty} \frac{e^{-au}(au)^j}{j!} du$$

i.e., $$E[T_{CUM}] = \frac{1}{a} \sum_{j=1}^{\infty} P\left[\sum_{i=1}^{j} X_i \le c\right] \tag{3}$$

Since $\sum_{i=1}^{j} X_i \sim \text{Gamma } (\theta_1 + \theta_2,\ j),$ from (3) and on simplification it can be shown that

$$E[T_{CUM}] = \frac{(\theta_1 + \theta_2)}{a} \int_0^{c} e^{(\theta_1 + \theta_2)x} e^{-(\theta_1 + \theta_2)x} dx$$

i.e., $$E[T_{CUM}] = \frac{c(\theta_1 + \theta_2)}{a} \tag{4}$$

We next determine $E[T_{CUM}^2]$.

we know that $E[T_{CUM}^2] = 2\int_0^{\infty} uP(T_{CUM} > u)du.$

Therefore, from (2) we get

$$E[T_{CUM}^2] = 2\sum_{j=1}^{\infty} P\left[\sum_{i=1}^{j} X_i \le c\right] \int_0^{\infty} u \frac{e^{-au}(au)^j}{j!} du$$

i.e., $$E[T_{CUM}^2] = \frac{2}{a^2} \sum_{j=1}^{\infty} (j+1) \int_0^{c} \frac{e^{-(\theta_1 + \theta_2)x}(\theta_1 + \theta_2)^j x^{j-1}}{(j-1)!} dx \tag{5}$$

Writing j as $(j-1)+1$ and using the result $\sum_{j=1}^{\infty} \frac{y^{j-1}}{(j-1)!} = e^y$ and $\sum_{j=1}^{\infty} \frac{y^{j-1}}{(j-2)!} = ye^y,$ on simplification it can be shown that

$$E[T_{CUM}^2] = \frac{2[\theta_1 + \theta_2]}{a^2}\int_0^c [(\theta_1 + \theta_2)xe^{(\theta_1+\theta_2)x} + 2e^{(\theta_1+\theta_2)x}]e^{-(\theta_1+\theta_2)x}dx$$

i.e., $E[T_{CUM}^2] = \dfrac{c(\theta_1 + \theta_2)[4 + c(\theta_1 + \theta_2)]}{a^2}$ (6)

From (4) and (6) we find that

$$V[T_{CUM}] = \frac{4c(\theta_1 + \theta_2)}{a^2} \tag{7}$$

(4) and (7) give the performance measures for the present exemplar.

Next we compute $E[S(T_{CUM})]$.

Since $E[C(T_{CUM})] = aE(T_{CUM})$ and $E[X_i] = \dfrac{(\theta_1 + \theta_2)}{\theta_1\theta_2}$ by hypothesis, invoking to Wald's lemma we find that

$$E[S(T_{CUM})] = E[C(T_{CUM})]E[X_i] = \frac{c(\theta_1 + \theta_2)^2}{\theta_1\theta_2} \tag{8}$$

4. Description of Exemplar – 2

In this section we study the work in Exemplar – 1 utilizing MAX hiring strategy. Define $F(c) = P[X_i \leq c]$, $i = 1, 2, 3, \ldots$. Here c, a positive real number, is the threshold for the maximum deprivation of workforce in the structure. **The MAX strategy for hiring states that hiring is done when the maximum deprivation of workforce due to clump of egresses exceeds the threshold.** The other assumptions and notations are as in Exemplar – 1.

5. Main Result for Exemplar – 2

In this section, we determine the mean $E(T_{MAX})$ and mean square deviation $V(T_{MAX})$ of the time to hiring for exemplar – 2.

By the hiring strategy,

$$P(T_{MAX} > u) = P[\max_{1 \le i \le C(u)} X_i \le c] \tag{9}$$

Conditioning upon $C(u)$ in (9) and noting that $\{C(u)\}$ is independent of X_i, $i = 1, 2, 3, \ldots$ we get

$$P(T_{MAX} > u) = \sum_{j=0}^{\infty} P[\max_{1 \le i \le C(u)} X_i \le c] P[C(u) = j]$$

i.e., $$P(T_{MAX} > u) = \sum_{j=0}^{\infty} P[C(u) = j][F(c)]^j \tag{10}$$

Since $C(u)$~Poises (a) (10) can be written as

$$P(T_{MAX} > u) = e^{-au[1 - F(c)]} \tag{11}$$

By hypothesis, X_i being the sum of two independent exponentially random variables with respective parameters θ_1 and θ_2, we know that X_i follows hypo-exponential distribution for $i = 1, 2, 3, \ldots$ and its distributions $F(x)$ is given by

$$F(x) = 1 - \left(\frac{\theta_2}{\theta_2 - \theta_1}\right)e^{-\theta_1 x} + \left(\frac{\theta_1}{\theta_2 - \theta_1}\right)e^{-\theta_2 x} \tag{12}$$

From (11) and (12) we get

$$P(T_{MAX} > u) = e^{-\lambda u} \tag{13}$$

where $$\lambda = a\left[\left(\frac{\theta_2}{\theta_2 - \theta_1}\right)e^{-\theta_1 c} - \left(\frac{\theta_1}{\theta_2 - \theta_1}\right)e^{-\theta_2 c}\right] \tag{14}$$

where λ is given in (14). As λ has to be positive, we should choose the values of θ_1, θ_2, and c, in (14) such that $\left(\frac{\theta_2}{\theta_2 - \theta_1}\right)e^{-\theta_1 c} > \left(\frac{\theta_1}{\theta_2 - \theta_1}\right)e^{-\theta_2 c}$,

Note that $E(T_{MAX}) > 0$ if and only if $\theta_2 > \frac{\theta_1}{e^{(\theta_2 - \theta_1)c}}$.

From (13) we get

$$E(T_{MAX}) = \frac{1}{\lambda} \text{ if } \theta_2 > \frac{\theta_1}{e^{(\theta_2 - \theta_1)c}} \tag{15}$$

$$V(T_{MAX}) = \frac{1}{\lambda^2} \tag{16}$$

Equations (15) and (16) give mean and mean square deviation of time to hiring for exemplar – 2.

6. Quantitative Illustration and Finding for Exemplars 1 and 2

For exemplar – 1, from equations (4), (7) and (8) we note the following:

(i) As 'a' alone increases, both $E(T_{CUM})$ and $V(T_{CUM})$ decrease.

(ii) $E(T_{CUM})$ and $V(T_{CUM})$ increase when 'a' is fixed and only one of the parameters θ_1, θ_2 and c is increases and (iii) $E[C(T_{CUM})]$ increases when only one of the parameters θ_1, θ_2 and c increase. The observations on $E(T_{CUM})$ in (i) and (ii) are also true logically.

For exemplar – 2, it is important to note from (15) that we should choose the values of θ_1, θ_2 and c such that the condition in (15) is satisfied for $E(T_{MAX})$ to be positive. There is no such restriction for $V(T_{MAX})$.

The effect of the leading parameters a, θ_1, θ_2 and c for exemplar – 2 in the context of the monotonicity of $E(T_{MAX})$ and $V(T_{MAX})$ can be seen from the following table.

Table 1.

a	c	θ_1	θ_2	$E(T_{MAX})$	$V(T_{MAX})$
10	30	0.4	0.2	20.20	407.90
15	30	0.4	0.2	13.46	181.29
20	30	0.4	0.2	10.10	101.97
10	31	0.4	0.2	24.66	608.24

10	32	0.4	0.2	30.12	907.05
10	33	0.4	0.2	36.78	1352.75
10	30	0.3	0.1	1.34	1.80
10	30	0.5	0.1	1.61	2.58
10	30	0.7	0.1	1.72	2.96
10	30	0.2	0.7	28.82	830.38
10	30	0.2	0.8	30.26	915.50
10	30	0.2	0.9	31.38	984.57

Remark. Since the partial sum of deprivation of workforce in each clump is always greater than the maximum of this deprivation, time to hiring is advanced for CUM strategy of hiring, but is postponed for MAX strategy of hiring. This justifies the reason for the suitability of the choice of one exemplar over the other in the present study. Thus exemplar – 1 is suitable if the plan is to prepone the hiring time. Exemplar – 2 is suitable if the plan is to postpone the hiring time.

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