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# MOLECULAR GRAPH OF LINEAR BENZENOID GRAPH AS FRACTALS USING DOMINATION SEQUENCE

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## Abstract

This paper indicates that dominating sequence in the molecular graph of Linear Benzenoid Chain. It obtains the greedy dominating sequence of the molecular graph of Anthracene and discusses about its greedy domination number. Then a proposition about the relation between domination number, fractional dominating number and greedy domination number for the molecular graph of Linear Benzenoid Chain is obtained.

## 1. Introduction

Graph Theory is one among the developing branches in Mathematics. It has a lot of applications in real time field such as Engineering, Basic sciences, Life sciences and Computer science. Nowadays Chemical graph theory is the fast growing research field. Chemical compounds are a graphical version in graph theory during which the each vertex of the graph expresses an atom of molecule and bonds between atoms are represented by the lines. The molecular graph of Linear Benzenoid Chain is $4h + 2$ vertices where $h$ is the number of hexagons [1, 2]. Throughout this paper, Graphs are considered as finite, simple, and undirected. The order of a graph is its number of vertices. Then $\delta(G)$ and $\Delta(G)$ are taken as the minimum and maximum degree of the

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graph $G$, respectively. $N[v]$ will denote the closed neighborhood of a vertex $v$. Consider any vertex of the graph dominates other vertex in the graph means that dominating vertex lies in the closed neighborhood of that other vertex. A function $f : V(G) \to [0, 1]$ is called Fractional dominating functions if for every $v \in V(G), f(N[v]) \geq 1$. The details about Graphs, Dominating Set, Minimum Dominating Set, Fractional Domination Set and Fractional Domination Number can be found in [ 3-9]. Greedy Dominating Sequence is a sequence which dominates each vertex in the graph [10].

## 2. Dominating Sequence in Graph

In this paper, It describes dominating sequence of graph using a greedy algorithm. The algorithm steps are given below:

**Step 1.** Select vertices $v_1, v_2, v_3, \ldots, v_n$ so that for $i = 1, 2, 3, \ldots, n$.

**Step 2.** Choose vertex '$i$' which dominates more vertices in the neighborhood of remaining vertices in the given graph.

**Step 3.** Continuing this process until each and every vertex are dominated by choosing a sequence of vertex which are in the order of the graph.

From this iteration process, it can be got $v_1, v_2, v_3, \ldots, v_n$ as the greedy dominating sequence. The number of vertices in this sequence is known as the greedy domination number of the graph. It is denoted as $\gamma_g(G)$. Fractional Domination Number of the molecular graph of Anthracene is found by Priyanga, Raji and Jayalalitha [6]. Here for every $v \in V(G)$, $f(N[v]) = 1$. The function $f : V(G) \to [0, 1]$ is Minimum Dominating function and fractional dominating function for the molecular graph of Anthracene. Domination number $= \gamma(G) = 4$ and Fractional Domination Number $= \gamma f(G) = 4$.

## 3. Greedy Dominating Sequence in Molecular Graph of Anthracene

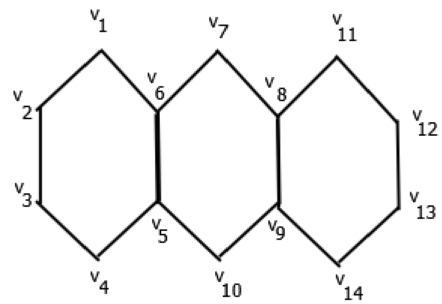


**Figure 1.** Anthracene.

Here the molecular graph of Anthracene consists of 14 vertices. They are $v_1, v_2, v_3, \ldots, v_{14}$ and its order is 14. It is known that $\gamma_f(G) \leq \gamma(G)$ for any graph $G$. We have to check whether the relationship $\gamma_f(G) \leq \gamma(G) \leq \gamma_g(g)$ for any graph $G$. Using greedy algorithm steps discussed above, the following steps are proceeded. First, $v_3$ is chosen. It dominates the vertices $v_2$ and $v_4$ Then take $v_6$. So are $v_1$, $v_5$ and $v_5$ dominated by $v_6$. $v_9$ covers $v_8$, $v_{10}$ and $v_{14}$. Now choose $v_{12}$ which dominates $v_{11}$ and $v_{13}$. From this, we get the greedy dominating sequence $\{v_3, v_6, v_9, v_{12}\}$. Hence $\gamma_g(G) =$ the number of vertices in this sequence $= 4$. Therefore it can be concluded that $\gamma_f(G) \leq \gamma(G) \leq \gamma_g(G)$ for any graph $G$.

## 4. Greedy Dominating Sequence in Molecular Graph of Linear Benzenoid Chain

**Proposition 4.1.** *If the number of vertices in the molecular graph of Linear Benzenoid Chain is* $4n + 2$ *vertices where n is the number of hexagons. Then there exists a relationship between Domination Number, Fractional Domination Number and Greedy Domination Number.*

**Proof.** Let the number of vertices in the molecular graph of Linear Benzenoid Chain be $4n + 2$ vertices where $n$ is the number of hexagons.

To Prove: There is a relation between Domination Number, Fractional Domination Number and Greedy Domination Number.

If $n = 1$, there are 6 vertices. Four vertices are dominated by two vertices. So Domination Number $= 2,$ Fractional Domination Number $= 2$ and also Greedy Domination Number $= 2.$

If $n = 2,$ there are 10 vertices. It has eight vertices of degree 2 and two vertices of degree 3. Here one vertex of degree 3 dominates three vertices and the remaining seven vertices are dominated by two vertices of degree 2. Hence Domination Number $= 3,$ Fractional Domination Number $= 3$ and also Greedy Domination Number $= 3.$

If $n = 3,$ there are 14 vertices. It has ten vertices of degree 2 and four vertices of degree 3. Here two vertices of degree 3 dominates six vertices and the remaining twelve vertices are dominated by two vertices of degree 2. Hence Domination Number $= 4,$ Fractional Domination Number $= 4$ and also Greedy Domination Number $= 4.$

Continuing this process, we can get Domination Number, Fractional Domination Number and Greedy Domination Number for the molecular graph of Linear Benzenoid Chain $= n + 1$ for $n$ hexagons. It can be concluded that $\gamma_f(G) = \gamma(G) = \gamma_g(G)$ for the molecular graph of Linear Benzenoid Chain.

## 5. Conclusion

This paper summarizes that dominating sequence in the molecular graph of Linear Benzenoid Chain and its greedy domination number are discussed. Dominating Sequence in the molecular graph of organic compounds will be applied in various real life situations in future.

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