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ASCENDING BI-PENDANT DOMINATION DECOMPOSITION POLYNOMIAL OF PATH AND CYCLE

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Abstract

Let $G = (V, E)$ be a simple connected graph. We introduced Ascending Bi-Pendant Domination Decomposition of Graphs and is defined as a collection $\{G_1, G_2, G_3, \ldots, G_n\}$ of subgraphs of G such that every edge of G is exactly once in G_i, each G_i is connected and $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n$. In this paper, we introduce Ascending Bi-Pendant Domination Decomposition Polynomial of a graph. Also, we have found that Ascending Bi-Pendant Domination Decomposition Polynomial for P_p and C_p.

1. Introduction

Let $G = (V, E)$ be a simple connected graph. All the graphs considered here are finite and undirected. A vertex of degree zero is called an isolated vertex and a vertex of degree one is called a pendant vertex. An edge incident with a pendant vertex is called a pendant edge. Pendant Domination in some

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Generalised Graphs was introduced by Nayaka S. R. Puttaswamy and S. Purushothama [8]. Ascending Domination Decomposition of Subdivision of Graphs was introduced by K. Lakshmiprabha and K. Nagarajan [6]. We introduced the concept of Ascending Pendant Domination Decomposition in [2] and extended this concept for special graphs in [4]. In this paper, we obtained Ascending Bi-Pendant Domination Decomposition Polynomial for P_p and C_p.

Definition 1.1. If $G_1, G_2, G_3, \ldots, G_n$ are connected edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, G_3, \ldots, G_n)$ is said to be decomposition of G.

Definition 1.2. A subset S of vertices in a graph G is called a Dominating Set if every vertex $v \in V$ is either in S or adjacent to some vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by $\gamma(G)$.

Definition 1.3. A Dominating set S in G is called a Pendant Dominating Set if $\langle S \rangle$ contains at least one pendant vertex. The minimum cardinality of a Pendant Dominating Set is called the pendant domination number denoted by $\gamma_{pe}(G)$.

Definition 1.4. A Pendant Dominating set S in G is called a Bi-Pendant Dominating Set if $\langle V \setminus S \rangle$ also contains pendant vertex. The minimum cardinality of a Bi-Pendant Dominating Set is called the bi-pendant domination number denoted by $\gamma_{bpe}(G)$.

Definition 1.5. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The tensor product $G = G_1 \wedge G_2$ is defined as a graph with vertex set $V_1 \times V_2$. Edge set is defined as follows: If $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are two vertices of G with $u_i \in V_1$ and $v_i \in V_2$, $(i = 1, 2)$ then $w_1 w_2 \in E(G)$ if and only if $u_1 u_2 \in E_1$ and $v_1 v_2 \in E_2$.

Definition 1.6 [2]**.** A Decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be Ascending Pendant Domination Decomposition (APDD) if

(i) Each G_i is connected

(ii) $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n.$

Definition 1.7 [5]**.** A Decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be Ascending Bi-Pendant Domination Decomposition (ABPDD) if

(i) Each G_i is connected

(ii) $\gamma_{bpe}(G_{i+1}) = \gamma_{bpe}(G_i) + 1, 1 \leq i \leq n - 1.$

2. Main Results

Definition 2.1. Let G be a graph which admits ABPDD into n-parts. For each $i = 1, 2, \ldots, n,$ let $\mathcal{M}(G, \gamma_{bpe}(G_i))$ be the family of connected subgraphs with $\gamma_{bpe}(G_i)$ and $m(G, \gamma_{bpe}(G_i)) = |\mathcal{M}(G, \gamma_{bpe}(G_i))|.$ Then ABPDD polynomial of a graph G is defined as

$$M(G, x) = \sum_{i=1}^{n} m(G, \gamma_{bpe}(G_i))\, x^{\gamma_{bpe}(G_i)}$$

Remark 2.2. 1. The constant term and the coefficient of x in ABPDD polynomial of any graph G are zero.

Theorem 2.3. *If the path P_p admits ABPDD into n-parts, then*

$$M(P_p, x) = 3 \sum_{i=k}^{k+n-1} (p - (3i - 4))\, x^i$$

Proof. Let G be the path P_p graph.

Suppose that the path P_p admits ABPDD into n-parts.

Then $\gamma_{bpe}(G_{i+1}) = \gamma_{bpe}(G_i) + 1, 1 \leq i \leq n - 1.$

Hence if $\gamma_{bpe}(G_1) = k, k \geq 3,$ then $\gamma_{bpe}(G_2) = k + 1, \gamma_{bpe}(G_3) = k + 2 \ldots$ and $\gamma_{bpe}(G_n) = k + n - 1.$

For each $i = k, k + 1, \ldots, k + n - 1,$ let $\mathcal{M}(P_p, \gamma_{bpe}(G_i))$ be the family of connected subgraphs with $\gamma_{bpe}(G_i) = k + i - 1$ and $m(P_p, \gamma_{bpe}(G_i)) = |\mathcal{M}(P_p, \gamma_{bpe}(G_i))|$.

For $i = 1$,

$\mathcal{M}(P_p, k) = \{H : H$ is connected subgraph of P_p with $\gamma_{bpe}(H) = k\}$

The only possible subgraphsin $\mathcal{M}(P_p, k)$ are P_{3k-4}, P_{3k-3} and P_{3k-2}.

Hence $|\mathcal{M}(P_p, k)| =$ Total number of graphs P_{3k-4}'s in P_p + Total number of graphs P_{3k-3}'s in P_p + Total number of graphs P_{3k-2}'s in P_p.

$$|\mathcal{M}(P_p, k)| = p - (3k - 4 - 1) + p - (3k - 3 - 1) + p - (3k - 2 - 1)$$

$$= 3(p - (3k - 4))$$

Therefore, $m(P_p, k) = 3(p - (3k - 4))$.

For $i = 2$,

$\mathcal{M}(P_p, k + 1) = \{H : H$ is connected subgraph of P_p with $\gamma_{bpe}(H) = k + 1\}$

The only possible subgraphs in $\mathcal{M}(P_p, k + 1)$ are P_{3k-1}, P_{3k} and P_{3k+1}.

Hence $|\mathcal{M}(P_p, k + 1)| =$ Total number of graphs P_{3k-1}'s in P_p + Total number of graphs P_{3k}'s in P_p + Total number of graphs P_{3k+1}'s in P_p.

$$|\mathcal{M}(P_p, k + 1)| = p - (3k - 1 - 1) + p - (3k - 1) + p - (3k + 1 - 1)$$

$$= 3(p - (3k - 4))$$

Therefore, $m(P_p, k + 1) = 3(p - (3k - 1))$.

For $i = 3$, $m(P_p, k + 2) = 3(p - (3k + 2))$.

Continuing in this way,

For $i = n$,

$\mathcal{M}(P_p, k + n - 1) = \{H : H$ is connected subgraph of P_p with $\gamma_{bpe}(H) = k + n - 1\}$

The only possible subgraphs in $\mathcal{M}(P_p, k+n-1)$ are $P_{3k+(3n-7)}$, $P_{3k+(3n-6)}$ and $P_{3k+(3n-5)}$.

Hence $|\mathcal{M}(P_p, k+n-1)| =$ Total number of graphs $P_{3k+(3n-7)}$'s in P_p + Total number of graphs $P_{3k+(3n-6)}$'s in P_p + Total number of graphs $P_{3k+(3n-5)}$'s in P_p.

$$|\mathcal{M}(P_p, k+n-1)| = p-(3k+(3n-7)-1)+p-(3k+(3n-6)-1)$$
$$+p-(3k+(3n-5)-1) = 3(p-(3k+3n-7))$$

Therefore, $m(P_p, k+n-1) = 3(p-(3k+3n-7))$.

Now,

$$M(G, x) = \sum_{i=1}^{n} m(G, \gamma_{bpe}(G_i))\, x^{\gamma_{bpe}(G_i)}$$

$$M(P_p, x) = \sum_{i=1}^{n} m(P_p, \gamma_{bpe}(G_i))\, x^{\gamma_{bpe}(G_i)}$$
$$= m(P_p, \gamma_{bpe}(G_1))\, x^{\gamma_{bpe}(G_1)} + m(P_p, \gamma_{bpe}(G_2))\, x^{\gamma_{bpe}(G_2)} + \dots$$
$$+ m(P_p, \gamma_{bpe}(G_n))\, x^{\gamma_{bpe}(G_n)}$$
$$= m(P_p, k)\, x^k + m(P_p, k+1)\, x^{k+1} + \cdots + m(P_p, k+n-1)\, x^{k+n-1}$$
$$= 3(p-(3k-4))\, x^k + 3(p-(3k-1))\, x^{k+1} + \dots + 3(p-(3k+3n-7))\, x^{k+n-1}$$
$$= 3\sum_{i=k}^{k+n-1} (p-(3i-4))\, x^i$$

Hence the theorem.

Theorem 2.3. *If the Cycle C_p admits ABPDD into n-parts, then*

$$M(C_p,\ x) = 3\sum_{i=k}^{k+n-1} px^i$$

Proof. Let G be the C_p graph.

Suppose that the cycle C_p admits ABPDD into n-parts.

Then $\gamma_{bpe}(G_{i+1}) = \gamma_{bpe}(G_i) + 1,\ 1 \leq i \leq n-1$.

Hence if $\gamma_{bpe}(G_1) = k,\ k \geq 3$, then $\gamma_{bpe}(G_2) = k+1,\ \gamma_{bpe}(G_3) = k+2 \ldots$ and $\gamma_{bpe}(G_n) = k+n-1$.

For each $i = k,\ k+1,\ \ldots,\ k+n-1$, let $\mathcal{M}(C_p,\ \gamma_{bpe}(G_i))$ be the family of connected subgraphs with $\gamma_{bpe}(G_i) = k+i-1$ and $m(C_p,\ \gamma_{bpe}(G_i)) = |\ \mathcal{M}(C_p,\ \gamma_{bpe}(G_i))\ |$.

For $i = 1$,

$$\mathcal{M}(C_p,\ k) = \{H : H \text{ is connected subgraph of } C_p \text{ with } \gamma_{bpe}(H) = k\}$$

The only possible subgraphs in $\mathcal{M}(C_p,\ k)$ are $P_{3k-4},\ P_{3k-3}$ and P_{3k-2}.

Hence $|\ \mathcal{M}(C_p,\ k)\ | =$ Total number of graphs P_{3k-4}'s in C_p + Total number of graphs P_{3k-3}'s in C_p + Total number of graphs P_{3k-2}'s in C_p.

$$|\ \mathcal{M}(C_p,\ k)\ | = p + p + p$$

$$= 3p$$

Therefore, $m(C_p,\ k) = 3p$.

For $i = 2$,

$$\mathcal{M}(C_p,\ k) = \{H : H \text{ is connected subgraph of } C_p \text{ with } \gamma_{bpe}(H) = k+1\}$$

The only possible subgraphs in $\mathcal{M}(C_p,\ k+1)$ are $P_{3k-1},\ P_{3k}$ and P_{3k+1}.

Hence $|\ \mathcal{M}(C_p,\ k+1)\ | =$ Total number of graphs P_{3k-1}'s in C_p + Total number of graphs P_{3k}'s in C_p + Total number of graphs P_{3k+1}'s in C_p.

$$|\,\mathcal{M}(C_p,\, k+1)\,| = p + p + p$$

$$= 3p$$

Therefore, $m(C_p,\, k+1) = 3p$.

For $i = 3$, $m(C_p,\, k+2) = 3p$.

Continuing in this way,

For $i = n$,

$\mathcal{M}(C_p,\, k+n-1) = \{H : H$ is connected subgraph of C_p with $\gamma_{bpe}(H)$ $= k+n-1\}$

The only possible subgraphs in $\mathcal{M}(C_p,\, k+n-1)$ are $P_{3k+(3n-7)}$, $P_{3k+(3n-6)}$ and $P_{3k+(3n-5)}$.

Hence $|\,\mathcal{M}(C_p,\, k+n-1)\,| =$ Total number of graphs $P_{3k+(3n-7)}$'s in $C_p +$ Total number of graphs $P_{3k+(3n-6)}$'s in $C_p +$ Total number of graphs $P_{3k+(3n-5)}$'s in C_p.

$$|\,\mathcal{M}(C_p,\, k+n-1)\,| = p + p + p$$

$$= 3p$$

Therefore, $m(C_p,\, k+n-1) = 3p$.

Now,

$$M(G,\, x) = \sum_{i=1}^{n} m(G,\, \gamma_{bpe}(G_i))\, x^{\gamma_{bpe}(G_i)}$$

$$M(C_p,\, x) = \sum_{i=1}^{n} m(C_p,\, \gamma_{bpe}(G_i))\, x^{\gamma_{bpe}(G_i)}$$

$$= m(C_p,\, \gamma_{bpe}(G_1))\, x^{\gamma_{bpe}(G_1)} + m(C_p,\, \gamma_{bpe}(G_2))\, x^{\gamma_{bpe}(G_2)} + \ldots$$

$$+ m(C_p,\, \gamma_{bpe}(G_n))\, x^{\gamma_{bpe}(G_n)}$$

$$= m(C_p,\ k)\,x^k + m(C_p,\ k+1)\,x^{k+1} + \cdots + m(C_p,\ k+n-1)\,x^{k+n-1}$$

$$= 3px^k + 3px^{k+1} + \ldots + 3px^{k+n-1}$$

$$= 3\sum_{i=k}^{k+n-1} px^i$$

Hence the theorem.

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