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EFFICIENT DOMINATION IN OPERATIONS ON FUZZY GRAPHS

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Abstract

In this paper, we study efficient domination set using the concept degree of the vertex in fuzzy graph .We define the efficient domination number $\gamma(G)$ of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on fuzzy graphs like join, direct product, semi product, Cartesian product and composition, further we explain the results with suitable examples.

1. Introduction

The main definition of fuzzy graphs was projected by Kafmann, from the fuzzy relations presented by Zadeh. Though Rosenfeld make known to another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. The idea of domination in fuzzy graphs was studied by A. Somasundaram, and S. Somasundaram. A. Somasundaram the concepts of independent domination, total domination, connected domination of fuzzy graphs present by A. Somasundaram.

In this paper, we study efficient domination set using the concept degree of the vertex in fuzzy graph .We define the efficient domination number $\gamma(G)$

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of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on fuzzy graphs like join, direct product, semi product, Cartesian product and composition, further we explain the results with suitable examples.

2. Preliminaries

A fuzzy graph $G(\sigma, \mu)$ is a defined by the couple of fuzzy membership functions.

$$\sigma : V \to [0, 1], \mu V \times V \to [0, 1],$$

where for all u, vV, every edges in $G(\sigma, \mu)$ fulfills the condition $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

The fuzzy cardinality of any subset $S \subseteq V$ are defined to be the sum of membership value of every elements in S, i.e. $\sum_{v \in S} \sigma(v)$. The order p of a fuzzy graph $G(\sigma, \mu)$ are defined to be the sum of membership values of vertex in $G(\sigma, \mu)$ i.e $\sum_{v \in V} \sigma(x)$. The degree of a vertex v to be $d(v) = \sum_{u \neq v} \mu(uv)$. The size q of a fuzzy graph $G(\alpha, \mu)$ are defined to be the sum of membership values of edges in $G(\sigma, \mu)$ i.e. $\sum_{uv \in E} \mu(uv)$. The minimum degree of G is $\delta(G) = \min \{d(u)/u \in V\}$ and the maximum degree of G is $\Delta(G) = \max \{d(u)/u \in V\}$.

An edge of a fuzzy graph is termed an effective edge or strong edge if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. The neighbourhood of u is defined by set $N(u) = \{v \in V/\mu(u, v) = \sigma(u) \wedge \sigma(v)\}$ and $N[u] = N(u) \cup \{u\}$ is called the closed neighbourhood of u. $d_N(u) = \sum_{v \in N(u)} \sigma(u)$ is so-called the neighbourhood degree of u. The minimum neighbourhood degree of G is $\delta_N(G) = \min \{d_N(v)/v \in V\}$ and the maximum neighbourhood degree of G is $\Delta_N(G) = \max \{d_N(u)/u \in V\}$.

A set D of V is said to be fuzzy dominating set of G if every $v \in V - D$ there exits $u \in D$ such that u dominates v. A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if every node

$v \in D$, $D - \{v\}$ is not a fuzzy dominating set. The fuzzy dominating number $\gamma_f(G)$ of the fuzzy graph G is the minimum cardinality taken over all minimal fuzzy dominating set of G.

Let $G(\sigma, \mu)$ be a fuzzy graph. A set D is subset of V is said to be efficient dominating set of a fuzzy graph G if every $v \in V - D$ there is exactly one $u \in D$ dominates v, i.e., $N(u) \cap D = \{v\}$.

A efficient dominating set D of a fuzzy graph G is called minimal efficient dominating set of G, if every subset of d is not a efficient fuzzy dominating set, i.e., every node $u \in D$, $D - \{u\}$, is not an efficient fuzzy dominating set.

The efficient fuzzy dominating number $\gamma_e(G)$ of the fuzzy graph G is the minimum cardinality taken over all minimal efficient fuzzy dominating set of G.

3. Main Results

Definition 3.1. Let $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The join of G_1 and G_2, denoted by $G_1 + G_2$, is the fuzzy graph on $V_1 \cap V_2$ defined as follows.

$G_1 + G_2 = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ where

$$(\sigma_1, \mu_1)(u) = \begin{Bmatrix} \sigma_1(u) \text{ if } u \in V_1 \\ \sigma_2(u) \text{ if } u \in V_2 \end{Bmatrix}$$

and

$$(\mu_1 + \mu_2)(uv) = \begin{Bmatrix} \mu_1(uv) \text{ if } u \in V_1 \\ \mu_2(uv) \text{ if } u \in V_2 \\ \sigma_1(u) \wedge \sigma_2(v) \text{ if } u \in V_1 \text{ and } v \in V_2 \end{Bmatrix}.$$

Theorem 3.1. *The fuzzy graphs* $G_1(V_1, \sigma_1, \mu_1)$ *and* $G_2(V_2, \sigma_2, \mu_2)$ *with efficient dominating sets* D_1 *and* D_2 *respectively. Then* V_1 *or* V_2 *is the minimum covering of* $G_1 + G_2$.

Proof. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in

$G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 + G_2$ the edges of the forms

$$(\mu_1 + \mu_2)(uv) = \left\{\begin{matrix} \mu_1(uv) \text{ if } uv \in E_1 \\ \mu_2(uv) \text{ if } uv \in E_2 \\ \sigma_1(u) \wedge \sigma_2(v) \text{ if } u \in V_1 \text{ and } V_1 \text{ and } v \in V_2 \end{matrix}\right\}.$$

The edges of the form $uv \in G_1 + G_2$ if $u \in V_1$ and $v \in V_2$ are strong edges in $G_1 + G_2$ the end vertices are belongs to V_1 and V_2 respectively. Therefore V_1 or V_2 efficiently dominates the vertices in $G_1 + G_2$.

Hence proved.

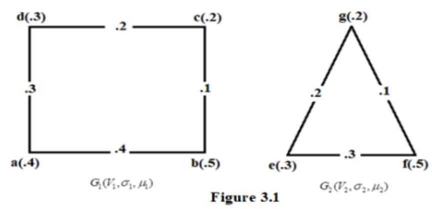
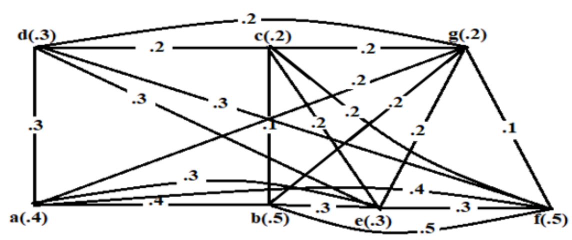


Figure 3.1



$G_1 + G_2$

Example 3.1.

From the above example the efficient dominating set of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are 0.6 and 0.3. The covering sets of the graph $G_1 + G_2$ is $\{a, b, c, d\}$ or $\{e, f, g\}$ and the efficient dominating number is 1.0.

Definition 3.2. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V1 and V2 respectively. Then the Direct product of G_1 and G_2, denoted by $G_1 + G_2$, is the fuzzy graph on $V_1 \times V_2$ defined as follows:

$$G_1 + G_2 = (\sigma_1 + \sigma_2, \mu_2 + \mu_2)$$

where

$$(\sigma_1 + \sigma_2)(\mu_1 \mu_2) = \sigma_1(\mu_1) \wedge \sigma_2(\mu_2) \text{ and}$$

$$(\mu_1 + \mu_2)((u_1u_2), (v_1v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2) \text{ if } (u_1v_1) \in E_1 \text{ and } (u_2v_2) \in E_2.$$

Theorem 3.2. *The fuzzy graphs* $G_1(V_1, \sigma_1, \mu_1)$ *and* $G_2(V_2, \sigma_2, \mu_2)$ *with efficient dominating sets* D_1 *and* D_2 *respectively. Then* $V_1 \times D_2$ *or* $D_1 \times V_2$ *is the minimum efficient dominating set of* $G_1 + G_2$.

Proof. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be fuzzy graphs with efficient dominating sets G_1 and D_2 respectively. Therefore every vertex in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 + G_2$ the edges of the forms $(\mu_1 + \mu_2)((u_1u_2)(v_1v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$ if $(u_1v_1) \in E_1$ and $(u_2v_2) \in K_2$. If the edges $(u_1v_1) \in E_1$ and $(u_2v_1) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

$$\begin{aligned}(\mu_1 + \mu_2)((u_1u_2)(v_1v_2)) &= \mu_1(u_1v_1) \wedge \mu_2(u_2v_2) \\ &= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)\end{aligned}$$

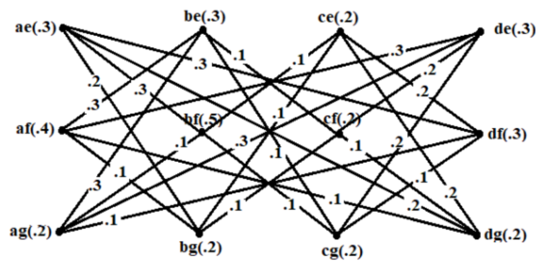
since

$(u_1v_1) \in E_1$ and $(u_2v_2) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$

$$\begin{aligned}(\mu_1 + \mu_2)((u_1u_2)(v_1v_2)) &= \mu_1(u_1v_1) \wedge \mu_2(u_2v_2) \\ &= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2) \\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2) \\ &= (\sigma_1 \bullet \sigma_2)(u_1u_2) \wedge (\sigma_1 \bullet \sigma_2)(v_1v_2).\end{aligned}$$

This implies $(u_1u_2)(v_1v_2) \in G_1 + G_2$ are strong edges whose end vertices belong to $V_1 \times D_2$ or $D_1 \times V_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $V_1 \times D_2$ or $D_1 \times V_2$. Hence proved.

Example 3.2. The direct product of the fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ in Figure 3.1



$G_1 \bullet G_2$

From the above example the efficient dominating set of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are 0.6 and 0.3. The covering sets of the graph $G_1 + G_2$ is $\{ae, be, ce, de\}$ or $\{ae, af, ag, ce, cg, cg\}$ and the efficient dominating number is 1.1.

Definition 3.3. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is two fuzzy graphs on V1 and V2 respectively. Then the semi product of G_1 and G_2 denoted by $G_1 + G_2$, is the fuzzy graph on $V_1 \times V_2$ defined as follows

$$G_1 + G_2 = (\sigma_1 + \sigma_2, \sigma_2, \mu_1 + \mu_2)$$

where

$$(\sigma_1 + \sigma_2)(u_1 u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$$

$$(\mu_1 + \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \text{ and } (u_2 v_2) \in E_2 \\ \mu_1(u_1 v_1) \wedge \mu_2(u_2 v_2) \text{ if } (u_1 v_1) \in E_1 \text{ and } (u_2 v_2) \in E_2 \end{cases}$$

Theorem 3.3. *The fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $V_1 \times D_2$ is the minimum efficient dominating set of $G_1 + G_2$.*

Proof. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 + G_2$ the edges of the forms

$$(\mu_1 + \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \text{ and } (u_2 v_2) \in E_2 \\ \mu_1(u_1 v_1) \wedge \mu_2(u_2 v_2) \text{ if } (u_1 v_1) \in E_1 \text{ and } (u_2 v_2) \in E_2 \end{cases}.$$

Case (i) $(u_1 u_2)(v_1 v_2)$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$. If the edge $(u_2 v_2) \in E_2$ is strong edges in $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

$$(\mu_1 + \mu_2)((u_1\, u_2)(v_1\, v_2)) = \sigma_1(u_1) \wedge \sigma_2(u_2 v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

since $(u_2 v_2) \in E_2$ is a strong edges in $G_2(V_2, \sigma_2, \mu_2)$.

$$(\mu_1 + \mu_2)((u_1\, u_2)(v_1\, v_2)) = \sigma_1(u_1) \wedge \mu_2(u_2 v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(u_1) \wedge \sigma_2(v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2)$$

$$\therefore u_1 = v_1$$

$$= \sigma_1 + \sigma_2(u_1 u_2) \wedge \sigma_1 + \sigma_2(v_1 v_2).$$

This implies $(u_1 u_2)(v_1 v_2) \in G_1 + G_2$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$ are strong edges whose end vertices belong to $V_1 \times D_2$. Since $(u_2 v_2) \in E_2$ is a strong edge such that $u_2 \in D_2$ or $v_2 \in D_2$. Therefore the set $V_1 \times D_2$ is efficiently dominated the all other vertices in $G_1 + G_2$.

Case (ii). If the edges $(u_1 v_1) \in E_1$ and $(u_2 v_2) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

$$(\mu_1 + \mu_2)((u_1 u_2)(v_1\, v_2)) = \mu_1(u_1 v_1) \wedge \mu_2(u_2 v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

sin $(u_1v_1) \in E_1$ and $(u_2v_2) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$

$$(\mu_1 + \mu_2)((u_1u_2)(v_1v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

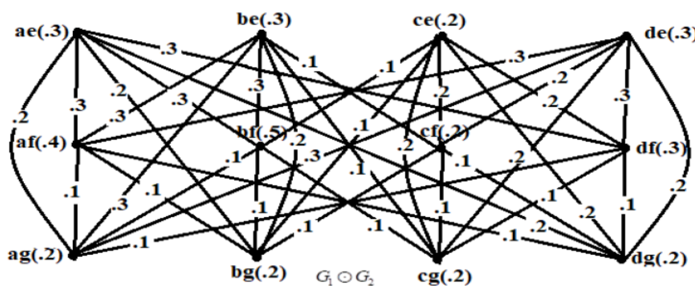
$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2)$$

$$= \sigma_1 + \sigma_2(u_1u_2) \wedge \sigma_1 + \sigma_2(v_1v_2)$$

This implies $(u_1u_2)(v_1v_2) \in G_1 + G_2$ are strong edges whose one end vertices belong to $V_1 \times D_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $V_1 \times D_2$.

From case (i) and (ii) $V_1 \times D_2$ is the minimum efficient dominating set of $G_1 + G_2$. Hence proved.

Example 3.3. The semi product of the fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ in Figure 3.1



From the above example the efficient dominating set of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are 0.6 and 0.3. The covering sets of the graph $G_1 + G_2$ is $\{ae, be, ce, de\}$ and the efficient dominating number is 1.1.

Definition 3.4. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively. Then the Cartesian product of G_1 and G_2 denoted by $G_1 \times G_2$, is the fuzzy graph on $V_1 \times V_2$ defined as follows

$$G_1 \times G_2 = (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$$

where

$$(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$$

and

$$(\mu_1 \times \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1) \text{ if } u_2 = v_2 \\ 0 \quad \text{otherwise.} \end{cases}$$

Theorem 3.4. *The fuzzy graphs* $G_1 = (V_1 \sigma_1, \mu_1)$ *and* $G_2 = (V_2 \sigma_2, \mu_2)$ *with efficient dominating sets* D_1 *and* D_2 *respectively. Then* $V_1 \times D_2$ *or* $D_1 \times V_2$ *is the minimum efficient dominating set of* $G_1 \times G_2$.

Proof. Let $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$ be fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 \times G_2$ the edges of the forms

$$(\mu_1 \times \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1) \text{ if } u_2 = v_2. \end{cases}$$

Case (i) $(u_1 u_2)(v_1 v_2)$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$. If the edge $(u_2 v_2) \in E_2$ is strong edges in $G_2 = (V_2, \sigma_2, \mu_2)$ therefore we get

$$\begin{aligned}(\mu_1 \times \mu_2)((u_1, u_2)(v_1, v_2)) &= \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)\end{aligned}$$

since $(u_2 v_2) \in E_2$ is a strong edges in

$$\begin{aligned}(\mu_1 \times \mu_2)((u_1 u_2)(v_1 v_2)) &= \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2) \\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(u_1) \wedge \sigma_2(v_2) \\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2) \\ &\therefore u_1 = v_1 \\ &= (\sigma_1 \times \sigma_2)(u_1 u_2) \wedge (\sigma_1 \times \sigma_2)(v_1 v_2).\end{aligned}$$

This implies $(u_1 u_2)(v_1 v_2) \in G_1 \times G_2$ if $u_1 = v_1$ and $(u_2 u_1) \in E_2$ are strong edges whose end vertices belong to $V_1 \times D_2$. Since $(u_2 v_2) \in E_2$ is a strong edge such that $u_2 \in D_2$ or $u_2 \in D_2$. Therefore the set $V_1 \times D_2$ is efficiently dominated the all other vertices in $G_1 \times G_2$.

Case (ii) $(u_1 u_2)(v_1 v_2)$ if $u_2 = v_2$ and $(u_1 v_1) \in E_1$. If the edge $(u_1 v_1) \in E_1$ is strong edges in $G_1(V_1, \sigma_1, \mu_1)$ therefore we get

$$(\mu_1 \times \mu_2)((u_1, u_2)(v_1, v_2)) = \sigma_1(u_2) \wedge \mu_1(u_1 v_1)$$

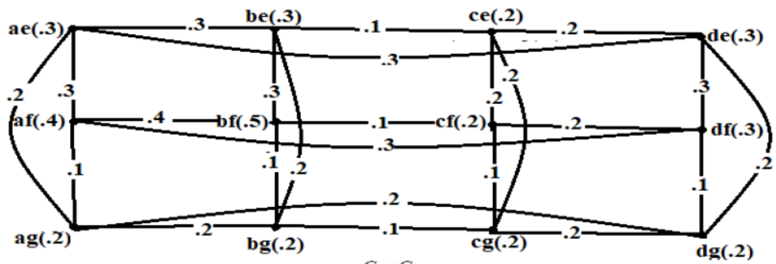
$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2)$$

since $(u_1 v_1) \in E_1$ is a strong edges in $G_1(V_1, \sigma_1, \mu_1)$.

$$\begin{aligned}(\mu_1 \times \mu_2)((u_1 u_2)(v_1 v_2)) &= \sigma_2(u_2) \wedge \mu_1(u_1 v_1)\\ &= \sigma_1(u_1) \wedge \sigma_1(u_1) \wedge \sigma_2(u_2)\\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(u_1) \wedge \sigma_2(v_2)\\ &= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2)\\ &\therefore u_2 = v_2\\ &= (\sigma_1 \times \sigma_2)(u_1 u_2) \wedge (\sigma_1 \times \sigma_2)(v_1 v_2).\end{aligned}$$

This implies $(u_1 u_2)(v_1 v_2) \in G_1 \times G_2$ if $u_2 = v_2$ and $(u_1 v_1) \in E_1$ are strong edges whose end vertices belong to $D_1 \times V_1$. Since $(u_1 v_1) \in E_1$ is a strong edge such that $u_1 \in D_1$ or $v_1 \in D_1$. Therefore the set $D_1 \times V_2$ is efficiently dominated the all other vertices in $G_1 \times G_2$.

Example 3.4. The Cartesian product of the fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ in Figure 3.1



$G_1 \times G_2$

From the above example the efficient dominating set of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are 0.6 and 0.3. The covering sets of the graph $G_1 + G_2$ is $\{ae, be, ce, de\}$ or $\{ae, ce, af, cf, ag, eg\}$ and the efficient dominating number is 1.1.

Definition 3.5. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively. Then the composition of G_1 and G_2 denoted by $(\sigma_1 \circ \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$, is the fuzzy graph on $V_1 \times V_2$ defined as follows

$$G_1 \circ G_2 = (\sigma_1 \circ \sigma_2, \mu_1 \circ \mu_2),$$

where

$$(\sigma_1 \circ \sigma_2)(u_1 u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$$

and

$$(\mu_1 \circ \mu_2)((u_1 u_2), (v_1 v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \text{ and } (u_2 v_2) \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1) \text{ if } u_2 = v_2 \text{ and } (u_1 v_1) \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1) \text{ otherwise.} \end{cases}$$

Theorem 3.5. *The fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $V_1 \times D_1$* or *$D_1 \times V_1$ is the minimum efficient dominating set of $G_1 \circ G_2$.*

Proof. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in

$G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 \circ G_2$ the edges of the forms

$$(\mu_1 \circ \mu_2)((u_1 u_2), (v_1 v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) \text{ if } u_1 = v_1 \text{ and } (u_2 v_2) \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1) \text{ if } u_2 = v_2 \text{ and } (u_1 v_1) \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1) \text{ otherwise.} \end{cases}$$

Case (i) $(u_1 u_2)(v_1 v_2)$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$. If the edge $(u_1 v_2) \in E_2$ is strong edges in $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

$$(\mu_1 \circ \mu_2)((u_1 u_2)(v_1 v_2)) = \sigma_1(u_1) \wedge \mu_2(u_2 v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

since $(u_2 v_2) \in E_2$ is a strong edges in $G_2(V_2, \sigma_2, \mu_2)$.

$$(\mu_1 \circ \mu_2)((u_1 u_2)(v_1 v_2)) = \sigma_1(u_1) \wedge \mu_2(u_2 v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2)$$

$$\therefore u_1 = v_1$$

$$= (\sigma_1 \circ \sigma_2)(u_1 u_2) \wedge (\sigma_1 \circ \sigma_2)(v_1 v_2).$$

This implies $(u_2 u_2)(v_1 v_2) \in G_1 \circ G_2$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$ are strong edges whose end vertices belong to $V_1 \times D_2$. Since $(u_2 v_2) \in E_2$ is a strong edge such that $u_2 \in D_2$ or $v_2 \in D_2$. Therefore the set $V_1 \times D_2$ is efficiently dominated the all other vertices in $G_1 \circ G_2$.

Case (ii) $(u_2 u_2)(v_1 v_2)$ if $u_2 = v_2$ and $(u_1 v_1) \in E_1$. If the edge $(u_1 v_1) \in E_1$ is strong edges in $G_1(V_1, \sigma_1, \mu_1)$ therefore we get

$$(u_1 \times u_2)((u_1 u_2)(v_1 v_2)) = \sigma_2(u_2) \wedge \mu_1(u_1 v_1)$$

$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2)$$

since $(u_1 v_1) \in E_1$ is a strong edges in $G_1(V_1, \sigma_1, \mu_1)$.

$$(\mu_1 \circ \mu_2)((u_1 u_2)(v_1 v_2)) = \sigma_2(u_2) \wedge \mu_1(u_1 v_1)$$

$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(u_1) \wedge \sigma_2(u_2)$$

$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(v_1) \wedge \sigma_2(v_2)$$

$$\therefore u_2 = v_2$$

$$= (\sigma_1 \circ \sigma_2)(u_1 u_2) \wedge (\sigma_1 \circ \sigma_2)(v_1 v_2).$$

This implies $(u_2 u_2)(v_1 v_2) \in G_1 \circ G_2$ if $u_2 = v_2$ and $(u_1 v_1) \in E_1$ are strong edges whose end vertices belong to $D_1 \times V_2$. Since $(u_1 v_1) \in E_1$ is a strong edge such that $u_1 \in D_1$ or $v_1 \in D_1$. Therefore the set $D_1 \times V_2$ is efficiently dominated the all other vertices in $G_1 \circ G_2$.

Case (iii) If the edges $(u_1 v_1) \in E_1$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ therefore we get

$$(\mu_1 \circ \mu_2)((u_1 u_2)(v_1 v_2)) = \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1)$$

$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

since $(u_1 v_1) \in E_1$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$.

$$(\mu_1 \circ \mu_2)((u_1 u_2)(v_1 v_2)) = \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1)$$

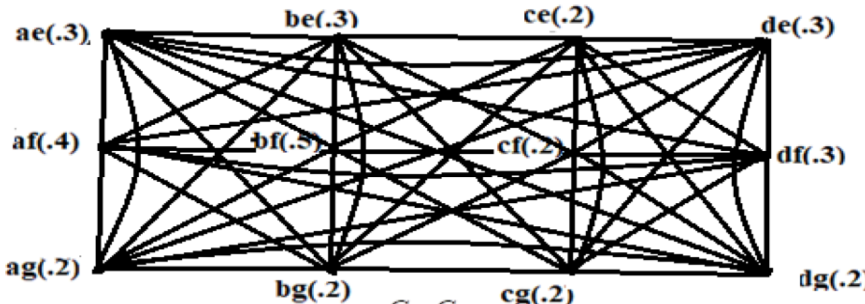
$$= \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

$$= (\sigma_1 \circ \sigma_2)(u_1 u_2) \wedge (\sigma_1 \circ \sigma_2)(v_1 v_2).$$

This implies $(u_2 u_2)(v_1 v_2) \in G_1 \circ G_2$ are strong edges whose one end vertices belong to $D_1 \times V_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $D_1 \times V_2$.

From case (i), (ii) and (ii) $V_1 \times D_2$ or $D_1 \times V_2$ is the minimum efficient dominating set of $G_1 \circ G_2$. Hence proved.

Example 3.5. The Cartesian product of the fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ in Figure 3.1.



$G_1 \circ G_2$

Edges	Membership value
(ae)(af)	.3
(ae)(ag)	.2
(af)(ag)	.1
(be)(bf)	.3
(be)(bg)	.2
(bf)(bg)	.1
(ce)(cf)	.2
(ce)(cg)	.2
(cf)(cg)	.1
Edges	Membership value
(de)(df)	.3
(de)(dg)	.2
(df)(dg)	.1
(ae)(be)	.3
(ae)(de)	.3
(be)(ce)	.1
(ce)(de)	.2
(bf)(cf)	.1
(af)(df)	.3
Edges	Membership value
(af)(bf)	.4

(cf)(df)	.2
(ag)(bg)	.2
(ag)(dg)	.2
(bg)(cg)	.1
(cg)(dg)	.2
(ae)(bf)	.3
(ae)(df)	.3
(ae)(bg)	.2

Edges	Membership value
(ae)(dg)	.2
(af)(be)	.3
(af)(de)	.3
(af)(bg)	.1
(be)(cf)	.1
(af)(dg)	.1
Edges	Membership value
(af)(dg)	.1
(be)(cg)	.1
(bf)(ce)	.1
(bf)(cg)	.1
(bg)(ce)	.1
(bg)(cf)	.1
Edges	Membership value
(ce)(df)	.2
(ce)(dg)	.2
(cf)(de)	.2
(cf)(dg)	.1
(cg)(de)	.2
(cg)(df)	.1

From the above example the efficient dominating set of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ are 0.6 and 0.3. The covering sets of the graph $G_1 \circ G_2$ is $\{ae, be, ce, de\}$ or $\{ae, ce, af, cf, ag, cg\}$ and the efficient dominating number is 1.1.

4. Conclusion

In this paper, we study efficient domination set using the concept degree of the vertex in fuzzy graph .We define the efficient domination number $\gamma(G)$ of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on fuzzy graphs like join, direct product, semi product, Cartesian product and composition, Also we explain the results with suitable examples. Further we study the efficient domination in intuitionistic fuzzy graphs.

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