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ON ALMOST NANO PRE-CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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Abstract

The main goal of this paper is to introduce and study the properties of almost nano pre-continuous functions and obtained some of their basic results. Further, we have given appropriate examples to understand the abstract concepts clearly.

1. Introduction

Lellis Thivagar and Richard [1] introduced a nano topological space with respect to a subset of a universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He also introduced weak form of nano-open sets namely nano α-open sets, nano semi-open sets and nano pre-open sets. Sathishmohan et al [4] defined nano preneighbourhoods, nano pre-interior, nano pre-limit point, nano pre-derived set, nano pre-frontier and nano pre-regular in nano topological spaces and obtained some of its properties.

Also in [7], they introduced and investigated the properties of nano semi preneighbourhoods, nano semi pre-interior, nano semi pre-frontier, nano semi preexterior, nano-dense and nano-submaximal. Further, the authors [5] introduced and examined the properties of nano-T0 space, nano semi-T0 space, nano pre-T0 space, nano-T1 space, nano semi-T1 space, nano pre-T1 space, nano-T2 space, nano semi-T2 space, nano pre-T2 space and obtain some of its basic results. And they have [6] defined and examined the

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properties of nano pre-irresolute, almost nano preirresolute, quasi nano pre-irresolute, nano semi-regular, nano pre-regular, strongly nano pre-regular, almost nano pre-regular and obtain some relationship between the existing sets. They also introduced and investigated the properties of nano pre-normal, almost nano pre-normal, nano mildly pre-normal, completely nano pre-normal spaces and derived the relationship between them. The structure of this manuscript is as follows:

In section 2, we recall some existing definitions and remarks which are more important to prove our main results.

In section 3, we induct and study some theorems which satisfy the conditions of almost nano pre-continuous.

2. Preliminaries

Definition 2.1 [1]**.** If $(U, \tau R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

The nano interior of A is defined as the union of all nano-open subsets of A is contained in A and is denoted by $\text{Nint}\,(A)$. That is, $\text{Nint}\,(A)$ is the largest nano-open subset of A.

The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by $Ncl\,(A)$. That is, $Ncl(A)$ is the smallest nanoclosed set containing A.

Definition 2.2 [3]**.** Let $(U, \tau R(X))$ and $(V, \tau R(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau R(X)) \to (V, \tau R(Y))$ is

(i) nano semi-continuous, $f^{-1}(A)$ is nano semi-open on U for every nano open set in V.

(ii) nano pre-continuous, $f^{-1}(A)$ is nano pre-open on U for every nano open set in V.

Lemma 2.3 [4]**.** *Let* $\{Bi \mid i \in I\}$ *be a collection of nano pre-open sets in a nano topological space* U, *then* $\cup Bi \in NPO(U)$.

Lemma 2.4 [4]. *For every subset* $W \subset U$, *we have the following.*

(i) $(U - W)^{*} = U - W^{*}$.

(ii) $(U - W)^{*} = U - W^{*}$.

Lemma 2.5 [5]. $NPO(U, \tau) = NPO(U, \tau\alpha)$; $NPF(U, \tau) = NPF(U, \tau\alpha)$.

Lemma 2.6 [6]. $NRO(U, \tau) = NRO(U, \tau\alpha)$; $NRF(U, \tau) = NRF(U, \tau\alpha)$.

Lemma 2.7 [5]. *If* $P \in NPO(X)$ and $Q \in NPO(U)$ then $Q \in NPO(X)$.

Lemma 2.8 [2]. $NSO(U, X) \cup NPO(U, X) \subset N\beta O(U, X)$.

3. Almost Nano pre-continuous Functions

In this section we have introduced almost nano pre-continuous functions and studied some properties.

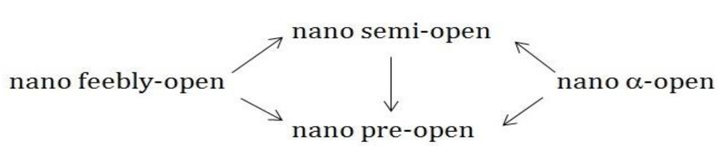
Definition 3.1. A function $f : U \to V$ is said to be almost nano continuous at $x \in U$ if for each nano open set X containing $f(x)$, there exists a nano open set Y containing x such that $f(Y) \subset \text{Nint}(Ncl(X))$.

Definition 3.2. A subset $A \subset U$ is said to be feebly nano open if $A \subset \text{Nscl}(\text{Nint}(A))$.

Theorem 3.3. *Let A be a subset of U. Then A is feebly nano open iff* $A \in \tau\alpha$.

The complement of a feebly nano open set is called f eebly nano closed. The family of all feebly nano open sets of U is denoted by $FNO(U)$.

Remark.



Example 3.4. Let $U = \{a, b, c, d\}$, $U/R = \{\{a, d\}, \{b\}, \{c\}\}$, $X = \{a, b\}$ and $\tau R(X) = \{U, \varnothing, \{b\}, \{a, b, d\}, \{a, d\}\}$ be a nano topology on U.

Let $X = \{b, c\}$ is nano semi-open but not feebly nano open.

(ii) Let $X = \{b, d\}$ is nano pre-open but not feebly nano open.

Lemma 3.5. *If $A \subset U$. Then A is feebly nano closed iff $A \in FN\alpha(U)$.*

Definition 3.6. A function $f : U \to V$ is called:

Almost nano semi-continuous if the inverse image of every nano regular openset of V is nano semi-open in U.

Almost nano pre-continuous if the inverse image of every nano regular open setof V is nano pre-open in U.

Almost nano pre semi $(N\alpha)$- continuous if the inverse image of every nano regular open set of V is nano pre semi-open in U.

A function $f : U \to V$ which is almost nano pre-continuous at each $x \in U$ is said to be almost nano pre-continuous.

Clearly every almost nano continuous as well as nano pre-continuous function is almost nano pre-continuous.

Remark

$$\begin{tikzcd}
\text{nano continuous} \arrow[r] \arrow[d] & \text{NS-continuous} \arrow[r] \arrow[d] & \text{NP-continuous} \arrow[r] \arrow[d] & \text{N}\alpha\text{-continuous} \arrow[d] \arrow[dl] \\
\text{AN-continuous} \arrow[r] & \text{ANS-continuous} \arrow[r] & \text{ANP-continuous} \arrow[r] & \text{AN}\alpha\text{-continuous}
\end{tikzcd}$$

Example 3.7. Let $U = \{a, b, c, d\} = V$, $U/R = \{U, \varnothing, \{a, d\}, \{b\}, \{c\}\}$, $\tau R(X) = \{U, \varnothing, \{b\}, \{a, b, d\}, \{a, d\}\}$ and $V/R = \{U, \varnothing\{a\}, \{b\}, \{c, d\}\}$, $\tau R0(x) = \{U, \varnothing, \{b\}, \{b, c, d\}, \{c, d\}\}$. $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. Let $x = \{c\} \in U$, there exists a nano open set $X = \{a, b, d\}$ contains $f(x)$, then there exists a nano pre-open set $Y = \{a, b, c\}$ containing x, is almost nano pre-continuous but not almost nano semi-continuous (resp. nano pre-continuous).

Theorem 3.8. *A function $f : U \to V$ is almost nano α-continuous iff it is both almost nano semi-continuous and almost nano pre-continuous.*

Theorem 3.9. *A function $f : U \to V$ is almost nano α-continuous iff it is almost feebly nano-continuous.*

Corollary 3.10. *A function $f : U \to V$ is an almost feebly nano-continuous iff it is almost nano pre-continuous and almost nano semi-continuous.*

Theorem 3.11. *For a function $f : U \to V$ the following are equivalent.*

f is almost nano pre-continuous.

For each $x \in U$ and for each nano regular open set X containing $f(x)$, there exists a nano pre-open set Y containing x such that $f(Y) \subset X$.

The inverse image of every nano regular open set of V is nano pre-open in U.

The inverse image of every nano regular closed set of V is nano pre-closed setin U.

$f^{-1}(A) \subset (f^{-1}(N \operatorname{int}(Ncl(A))))$ *for every nano open subset A of V.*

$(f^{-1}(Ncl(N \operatorname{int}(F)))) \subset f^{-1}(F)$ *for every nano closed subset F of V.*

Proof. (a) $\Rightarrow$ (b). Obvious.

$\Rightarrow$ (c). Let X be a nano regular open subset of V. Let $x \in f^{-1}(X)$. Then $f(x) \in X$. By (b), there exists a nano pre-open set Y_x containing x such that $f(Y_x) \in X$. Therefore, $x \in Yx \subset f^{-1}(X)$. Then, we obtain that $f^{-1}(X) = \cup\{Y_x \mid x \in f^{-1}(X)\}$. This implies that $f^{-1}(X)$ is nano pre-open by Lemma 2.5.

$\Rightarrow$ (d). Obvious.

$\Rightarrow$ (e). Let X be a nano open of V and $x \in f^{-1}(X)$. Then $f(x) \in X \subset N \operatorname{int}(Ncl(X))$. Thus $x / \in f^{-1}(V - N \operatorname{int}(Ncl(X)))$, $V - N \operatorname{int}(Ncl(X))$ being a nano regular closed subset of V and by (d), there is a nano pre-open set Y containing x such that $Y \cap f^{-1}(V - N \operatorname{int}(Ncl(X))) = \varphi$. Then, $Y \subset U - [f^{-1}(V - N \operatorname{int}(Ncl(X)))] = f^{-1}(N \operatorname{int}(Ncl(X)))$. Hence $x \in [f^{-1}(N \operatorname{int}(Ncl(X)))]^*$. As x is an arbitrary point of $f^{-1}(X)$ and hence $f^{-1}(X) \subset [f^{-1}(N \operatorname{int}(Ncl(X)))]^*$.

$\Rightarrow$ (f). Let f be a nano closed subset of V. Then by (e), $f^{-1}(V - F) \subset [f^{-1}(N\operatorname{int}(Ncl(V - F)))]^{*}$. And, also $N\operatorname{int}(Ncl(V - F) = Y - Ncl(N\operatorname{int}(F)))$. Then by Lemma 2.6, $[f^{-1}(N\operatorname{int}(Ncl(V - F)))]^{*} = U - [f^{-1}(Ncl(N\operatorname{int}(F)))]$. This implies that $f^{-1}(V - F) \subset U - [f^{-1}(Ncl(N\operatorname{int}(F)))]^{*}$. Hence $[f^{-1}(Ncl(N\operatorname{int}(F)))]^{*} \subset f^{-1}(F)$ for every nano closed subset F of V.

$\Rightarrow$ (a). Let $x \in X$ and A be a nano regular open subset of V containing $f(x)$. Then $V - A$ is nano regular closed set. Then by $(f), [f^{-1}(V - A)]^{*} = [f^{-1}(Ncl(N\operatorname{int}(V - A)))]^{*} \subset f^{-1}(V - A) = U - f^{-1}(A)$. Thus, $f^{-1}(a)$ is nano pre-open. Therefore f is almost nano pre-continuous.

Theorem 3.12. *If f is an almost nano pre-continuous function of a space U into a nano semi-regular space V, then f is nano pre-continuous.*

Proof. Let $x \in U$ and let X be a nano open subset of V containing $f(x)$. As V is nano semi-regular space, there exists a nano open set G in V containing $f(x)$ such that $G \subset N\operatorname{int}(Ncl(G)) \subset X$. By almost nano pre-continuous of f, there exists a nano pre-open set Y containing x such that $f(Y) \subset N\operatorname{int}(Ncl(G)) \subset Y$. Then it follows that f is nano pre-continuous.

Definition 3.13. Let $f : (U, \tau R(X) \to (V, \tau R, (Y))$ be a function. Then a function $f\alpha : (U, \tau\alpha) \to (V, \tau)$ (respectively $f\alpha : (U, \tau\alpha) \to (V, \tau\alpha)$, $f\alpha : (U, \tau\alpha) \to (V, \tau\alpha)$ associated with $f : (U, \tau) \to (V, \tau)$ is defined as follows.

$f\alpha(x) = f(x)$ (respectively $f*(x) = f(x)$, $f*(x) = f(x)$ for each $x \in U$.

Next we prove the following.

Lemma 3.14. *Let* (X, τ) *be a nano topological space then* $(\tau\alpha)\alpha = \tau\alpha$.

Theorem 3.15. *Let* $f : (U, \tau R(X)) \to (V, \tau R, (Y))$ *be a function. Then,*

*f is almost nano pre-*continuous *iff* $f\alpha$ *is almost nano pre-continuous.*

$f\alpha$ *is almost nano pre-continuous iff* $(f\alpha)\alpha$ *is almost nano pre-continuous.*

f is almost nano pre-continuous iff $f\alpha$ *is almost nano pre-continuous.*

f is almost nano semi-continuous iff $f\alpha$ *is almost nano pre-continuous.*

f is almost nano pre-continuous iff $(f\alpha)\alpha$ *is almost nano pre-continuous.*

Proof. (a) Let f be an almost nano pre-continuous and $X \in NRO(V, \tau R, (Y))$. Since f is almost nano pre-continuous, $f^{-1}(X) \in NPO(U, \tau R(X))$. But by Lemma 2.7, $f^{-1}(X) \in NPO(U, \tau\alpha)$. Therefore $f\alpha$ is an almost nano pre-continuous.

Conversely, suppose $f\alpha$ is an almost nano pre-continuous and $X \in NRO(V, \tau R, (Y))$. By hypothesis, $f^{-1}(X) \in NPO(U, \tau\alpha) = NPO(U, \tau\alpha)$. Thus f is an almost nano pre-continuous.

Suppose $f\alpha$ is an almost nano pre-continuous and $X \in (V, \tau R, (Y))$. Then, $(f\alpha^{-1}(X) \in NPO(U, (\tau\alpha)\alpha))$. But by Lemma 3.14, $(\tau\alpha)\alpha = \tau\alpha$. Hence $(f\alpha^{-1}(X)) \in NPO(U, (\tau\alpha)\alpha)$. This shows that $(f\alpha)\alpha$ is an almost nano pre-continuous.

Conversely, suppose $(f\alpha)\alpha$ is an almost nano pre-continuous. Let $X \in NRO(V, \tau R, (Y))$. Since $(f\alpha)\alpha$ is an almost nano pre-continuous, $(f\alpha)\alpha^{-1}(X) \in NPO(U, (\tau\alpha)\alpha) = NPO(U, \tau\alpha)$. Thus, $f\alpha$ is an almost nano pre-continuous.

Let f be almost nano pre-continuous and $X \in NRO(V, \tau')$. By using Lemma 2.7 and 2.8, $NRO(V, \tau\alpha) = NRO(V, \tau)$. Since f is almost nano pre-continuous, $f^{-1}(X) \in NPO(U, \tau)$. This shows that $f\alpha$ is almost nano pre-continuous. Conversely, let $f\alpha$ is almost nano pre-continuous and $X \in NRO(V, \tau\alpha)$. Again by Lemma 2.8, $X \in NRO(V, \tau'\alpha)$. Since $f\alpha$ is almost nano pre-continuous, $f\alpha^{-1}(X) \in NRO(U, \tau)$. This shows that f is almost nano pre-continuous.

(d) It is similar to the proof of (a).

(e) It is similar to the proof of (b).

Theorem 3.16. *Let* $f : (U, \tau R(X) \to (V, \tau R, (Y)))$ *be a function. Then the following are equivalent.*

f is an almost nano pre-continuous.

$fs : (U, \tau R(X) \to (V, \tau R, (Y)))$ *is almost nano pre-continuous*

$f\alpha$ is almost nano pre-continuous.

$(f *)\alpha$ *is almost nano pre-continuous.*

Theorem 3.17. *Let* $f : U \to V$ *be an almost nano pre-continuous. Then for each nano open set G of* V, $(f^{-1}(G))* \subset f^{-1}(Ncl(G))$.

Proof: Let G be a nano open set of V. Then $Ncl(G)$ is nano regular closed set in V. Using Theorem 3.11 $[f^{-1}(Ncl(G))]$ is nano pre-closed in U. As $f^{-1}(G) \subset f^{-1}(Ncl(G))$, $(f^{1}(G))* \subset f^{-1}(Ncl(G))$.

Theorem 3.18. *For a function* $f : U \to V$, *the following are equivalent. f is almost nano pre-continuous.*

$(f^{-1}(X))* \subset f^{-1}(Ncl(X))$ *for each* $X \in NSPO(V)$.

$(f^{-1}(X))* \subset f^{-1}(Ncl(X))$ *for each* $X \in NSO(V)$.

$f^{-1}(X) \subset (f^{-1}(N\operatorname{int}(Ncl(X)))^{*}$ for each $X \in NPO(V)$.

Proof. (a) $\Rightarrow$ (b). Let $X \in NSPO(V)$. Then $Ncl(X) \in NRF(V)$ by Theorem 3.15 (c). Since f is almost nano pre-continuous, $f^{-1}(Ncl(X)) \in NPF(U)$. Hence $(f^{-1}(X))* \subset (f^{-1}(Ncl(X)))^{*} = f^{-1}(Ncl(X))$.

$\Rightarrow$ (c). Obvious since $NSO(V) \subset NSPO(V)$.

$\Rightarrow$ (a). Let F be a nano regular closed set of V. Then $F = Nrcl(F)$ and $F \in NSO(V)$. Then by (c), we obtain that $(f^{-1}(F))* \subset f^{-1}(Nrcl(F)) = f^{-1}(F)$. This implies that $f^{-1}(F) \in NPF(U)$ and hence f is almost nano pre-continuous. (a) $\Rightarrow$ (d). Let $X \in NPO(V)$. Then $X \subset N\operatorname{int}(Ncl(X))$ and $N\operatorname{int}(Ncl(X))$ is nano regular open. Since f is almost nano pre-continuous,

$f^{-1}(N\operatorname{int}(Ncl(X)))$ is nano pre-open in U and hence $f^{-1}(X) \subset f^{-1}(N\operatorname{int}(Ncl(X))) = (f^{-1}(N\operatorname{int}(Ncl(X)))^{*}$.

$\Rightarrow$ (a). Let X be a nano regular open set of V. Then $X \in NPO(V)$ and by (d), $f^{-1}(X) \subset (f^{-1}(N\operatorname{int}(Ncl(X))))^{*} = (f^{-1}(X))^{*}$. But, $(f^{-1}(X))^{*} \subset (f^{-1}(X))$. This implies $f^{-1}(X) \in NPO(U)$. Therefore f is almost nano pre-continuous.

We need the following.

Lemma 3.19. *For a subset Y of V, the following are true.* (a) $Ncl\alpha(Y) = Ncl(Y)$ *for each* $Y \in NSPO(V)$.

$Y^{*} = Ncl(Y)$ *for each* $Y \in NSO(V)$.

$Nscl(Y) = N\operatorname{int}(Ncl(Y))$ *for each* $Y \in NPO(V)$.

In view of Theorem 3.18 and Lemma 3.19, we state the following.

Corollary 3.20. *For a function* $f : U \to V$, *the following are equivalent.*

(a) *f is almost nano pre-continuous.*

$(f^{-1}(Y))^{*} \subset f^{-1}(Ncl\alpha(Y))$ *for each* $Y \in NSPO(V)$

$(f^{-1}(Y))^{*} \subset f^{-1}(Y^{*})$ *for each* $Y \in NSO(V)$.

$f^{-1}(Y) \subset (f^{-1}(Nscl(Y)))^{*}$ *for each* $Y \in NPO(V)$.

References

[1] Lellis Thivagar and C. Richard, On nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention 1(1) (2013), 31-37.

[2] A. Revathy and G. Illango, On nano β-open sets, International Journal of Engineering, Contemporary Mathematical and Science 1(2) (2015), 1-6.

[3] P. Sathishmohan, V Rajendran, A Devika and R Vani, On nano semi-continuity and nano precontinuity, International Journal of Applied Research 3(2) (2017), 76-79.

[4] P. Sathishmohan, V. Rajendran, P. K. Dhanasekaran and C. Vignesh Kumar, More on nano prenegihbourhoods in nano topological spaces, Journal of Applied Science and Computations 5(10) (2018), 899-907.

[5] P. Sathishmohan, V. Rajendran and P. K. Dhanasekaran, Further properties of nano pre-T_0, nano pre-T_1 and nano pre-T_2 spaces, Malaya Journal of Matematik 7(1) (2019), 34-38.

[6] P. Sathishmohan, V. Rajendran, P. K. Dhanasekaran and C. Vignesh Kumar, Nano pre-regular and strongly nano pre-regular spaces, International Journal of Recent Technology and Engineering 7(5S) (2019), 464-466.

[7] P. Sathishmohan, V. Rajendran, C. Vignesh Kumar and P. K. Dhanasekaran, On nano semi pre neighbourhoods on nano topological spaces, Malaya Journal of Mathematik 6(1) (2018), 294-298.