



# A CODING TECHNIQUE WITH ATOMIC NUMBERS USING THE PRIME CORDIAL LABELING ON F26A GRAPH

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## Abstract

In this paper, a method of generating new graph labeling associated with the existing graph labeling is introduced by assigning non-negative integers assumed by a parameter of any branch of knowledge to the vertices of any graph by maintaining the other conditions of a particular graph labeling. Here by taking the Atomic Number of Elements and the Prime Cordial Labeling, a new graph labeling is obtained and it is named as Prime Cordial Labeling - Chemistry Atomic Number of Elements (PCL-CANE). Any Graph can be checked if it is a PCL-CANE. The graph F26A, a symmetric bipartite cubic graph with 26 vertices and 39 edges is taken for discussion and it is proved to be a PCL-CANE graph. A technique of coding a message with F26A and PCL-CANE using Graph Message Jumbled (GMJ) code is also included as an application.

## 1. Introduction

The Authors of the paper decided to work on a suitable graph with the existing Prime Cordial Labeling, associating it with any branch in general and Chemistry in particular. The source of enlightenment is attributed to 'A dynamic survey of graph labelings', an Electronic Journal of Combinatorics by J. A. Gallian [1], a few Research papers on graph labelings by S. K. Vidya [5] et al and G. Uma Maheswari [7] et al. on coding techniques made the Authors strike at an application.

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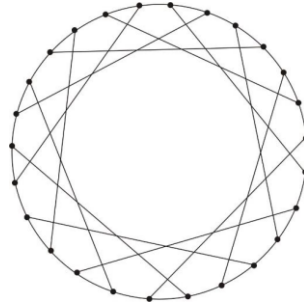
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A Simple graph  $G(V, E)$  is taken for discussion.

## 2. Pre-Requisites

**Definition 2.1: F 26A** [2]. In the Mathematical Field of Graph Theory, the F26A Graph is a symmetric bipartite cubic graph with 26 vertices and 39 edges. It has a chromatic number 2, chromatic index 3, diameter 5, radius 5 and girth 6. It is also a 3-vertex connected and 3-edge connected graph.



**Definition 2.2: Prime Cordial Labeling** [4]. A Prime Cordial Labeling of a graph  $G$  with the vertex set  $V(G)$  is a bijection  $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$  such that each edge  $uv$  is assigned the label 1 if  $\gcd(f(u), f(v)) = 1$  and 0 if  $\gcd(f(u), f(v)) > 1$ , where the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits the prime cordial labelling is called a prime cordial graph.

**Definition 2.3: Atomic Number of Elements** [6]. The Atomic Number or proton number of a chemical element is the number of protons found in the nucleus of every atom of that element. The Atomic number uniquely identifies a chemical element. It is identical to the charge number of the nucleus.

**Definition 2.4: GMJ Code** [7]. By assigning numbers to the 26 alphabets of English in a different manner, choosing a suitable labelled graph with a given clue mathematical or non-mathematical, finding the number in the graph for each letter of each word of the given message and presenting the letter codes in a unique way in some form, writing it in a horizontal string or in any other way and creating a picture with the codes after shuffling the

order of the letters in order to increase the secrecy of the coded message is named as GMJ Coding method.

#### **GMJ CODE STANDS FOR:**

(i) A Graph Message Jumble code-A coding technique to communicate a message through Graphs, Jumbling letters is named as GMJ Code.

(ii) It also refers to the name of one of the Authors (Gabriel Margaret Joan) who has conceived this method of coding. 2.5 A Method of generating new graph labelings associated with Already existing labelings. For any graph labeling the following three are required.

(i) A set of numbers from which vertex labels are chosen.

(ii) A rule that assigns a value to each edge.

(iii) A condition that these values must satisfy.

By taking  $A$ , a set of non-negative integers assumed by a parameter in any branch of knowledge and a function  $f, f : V(G) \rightarrow A$  as an into or an onto and injective function such that (ii) and (iii) are specified as in the existing labeling. Here a new labeling in chemistry is framed using the Prime Cordial Labeling.

**2.6 Definition of PCL-CANE.** By taking  $A$  as the set of Atomic Number of 118 Elements in the definition of Prime Cordial Labeling, a new labeling called Prime Cordial Labeling-Chemistry Atomic Number of Elements denoted by PCL-CANE is obtained.

If  $p < n(A)$ , the function  $f$  is into.

$p = n(A)$ , the function  $f$  is onto.

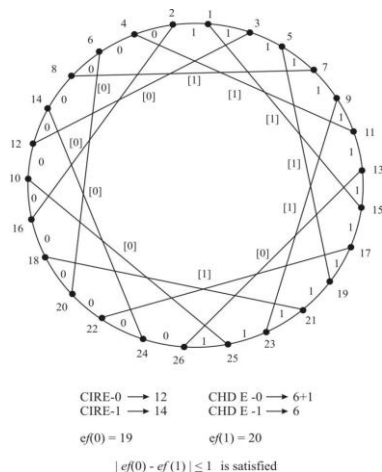
$p < n(A)$ , No function is possible, as each vertex of the graph must be assigned a unique number.

### **3.0 Description of The Work Done.**

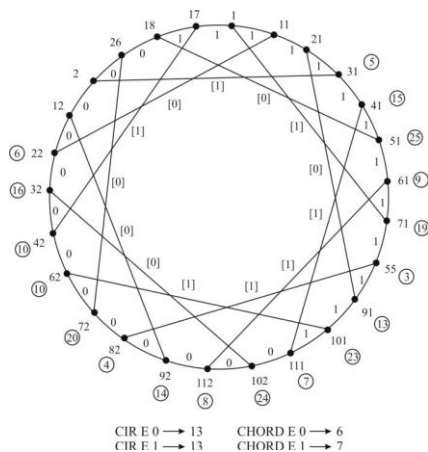
The work done in this paper is divided into two parts

3.1 Proving F26A graph is a PCL-CANE graph with respect to two different assignments of Atomic number of Elements to the vertices.

In the first assignment, the numbers 1 to 26 are allotted skipping an integer 22 times. This labeling is named as PCL-CANE 1J (one jump). The vertices are assigned the numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 26, 24, 22, 20, 18, 16, 10, 12, 14, 8, 6, 4, 2 respectively. 1J implies one jump is made in fixing numbers to the vertices. Here  $e_f(0) = 19$  and  $e_f(1) = 20$ .



In the second assignment, any 26 Atomic numbers from 1 to 118 are taken and are allotted skipping 9 integers for 17 times and is called PCL - CANE (9J).



1, 11, 21, 31, 41, 51, 61, 71, 55, 91, 101, 111, 102, 112, 92, 82, 72, 62, 42, 32, 22, 12, 2, 26, 18, 17 is the allotment It is made sure that the numbers chosen leave the remainder 1 to 25 only once on division by 26 and 26 or any

multiples of 26 is taken as 26. The usage of the word mod.26 is avoided here for 0 does not correspond to the Atomic number of any Element. Here  $ef(0) = 20$ ,  $ef(1) = 19$ . Thus F26A is a PCL-CANE(1J) and PCL-CANE (9J) Graph are proved.

3.2 In the second part, a coding technique with a combination of GMJ Coding, F26A, PCL-CANE (1J) And PCL-CANE (9J) is developed and two illustrations are presented. According to the GMJ Code, clues, mathematical or non mathematical are to be provided in order to guess the graph, the graph labeling and the numbering of alphabets.

**Clue1. (For the graph):** How many Alphabets are there in the French language? (F26A, The name of the graph is understood).

**Clue2. (For the graph labeling):** The most striking quality of Albert Einstein was being Amiable (Most striking-Prime, Amiable-Cordial, Albert Einstein-Atom bomb-Atomic Number). Hence PCL-CANE labeling is guessed).

**Clue3. (For numbering of Alphabets):** CANE-BF The Atomic numbers 1 to 118 are allotted to the alphabets in the following manner, moving from the back and from the front (BF) That is

Z-> 1, 27, 53, 79, 105

A-> 2, 28, 54, 80, 106

Y-> 3, 29, 55, 81, 107

B-> 4, 30, 56, 82, 108 and so on.

### 3.3 Method of coding:

(i) The Message is written in English.

(ii) For a letter in the message, the number allotted by CANE-BF is noted and it is located in the graph which is labelled.

(iii) The letter is replaced in an innovative way and the coding of letters is over.

(iv) After coding all the letters of the message in the same way the Cipher text can be written in a horizontal string in order, or jumbling them in any

order making decoding a little more difficult.

A	2	28	54	80	106
B	4	30	56	82	108
C	6	32	58	84	110
D	8	34	60	86	112
E	10	36	62	88	114
F	12	38	64	90	116
G	14	40	66	92	118
H	16	42	68	94	
I	18	44	70	96	
J	20	46	72	98	
K	22	48	74	100	
L	24	50	76	102	
M	26	52	78	104	
N	25	51	77	103	
O	23	49	75	101	
P	21	47	73	99	
Q	19	45	71	97	
R	17	43	69	95	
S	15	41	67	93	
T	13	39	65	91	117
U	11	37	63	89	115
V	9	35	61	87	113
W	7	33	59	85	111
X	5	31	57	83	109
Y	3	29	55	81	107
Z	1	27	53	79	105

The Message to be coded: “YOUR FORMULATION OF THE NEW DRUG IS INDEED THE PINNACLE OF SUCCESS”.

**ILLUSTRATION 1(PCL-CANE1J).** The 26 vertices are numbered starting with the first vertex  $v_1$  in the clockwise direction. There 13 chords first chord joins  $v_1$  and  $v_8$ .  $v_1$  and  $v_8$  are called head and tail respectively and

denoted by  $h_1$  and  $t_1$ . Similarly the same notation is maintained for the other chords.

Coding of the word: YOUR

$Y \rightarrow 3, 29, 55, 81, 107$

$Y \rightarrow 3, 3 = h_2$

$O \rightarrow 23, 49, 75, 101.$

Here  $O \rightarrow 23, 23 = t_5$

$U \rightarrow 11, 37, 63, 89, 115.$

Here  $U \rightarrow 11, 11 = h_6$

$R \rightarrow 17, 43, 69, 95.$

Here  $R \rightarrow 17, 17 = h_8$ . so the cipher text of "YOUR" is  $h_2 t_5 h_6 h_8$ .

Similarly all the other words are coded. YOUR FORMULATION OF THE NEW DRUG  $h_2 t_5 h_6 h_8 t_2 t_5 h_6 h_{11} t_{13} h_7 t_9 t_5 t_2 h_7 h_{13} t_{10} h_{10} t_{10} h_4 t_4 h_8 h_6 t_{11}$  IS INDEED PINNACLE OF SUCCESS  $t_9 h_1 t_9 h_{10} t_4 t_{10} t_4 h_9 t_9 h_{10} t_{13} t_{12} h_{11} t_{10} t_5 t_2 h_1 h_6 t_{12} t_{12} t_{10} h_1 h_1$  Therefore the coded message in cipher text is  $h_2 t_5 h_6 h_8 t_2 t_5 h_8 t_7 h_6 h_{11} t_{13} h_7 t_9 t_5 t_2 h_7 h_{13} t_{10} h_{10} t_{10} h_4 t_4 h_8 h_6 t_{11} t_9 h_1 t_9 h_{10} t_4 t_{10} t_4 h_{10} h_{10} t_{13} t_{13} h_{13} t_{12} h_{11} t_{10} t_5 t_2 h_1 h_6 t_{12} t_{12} t_{10} h_1 h_1$  Another way of coding is suggested here, Just the position of the number with respect to the numbering of vertices can be used.

**Illustration2 (PCL-CANE 9J).** Using the same procedure and notations the word 'YOUR' takes the following form.

$Y \rightarrow 55, Y = h_8$

$O \rightarrow 101, U \rightarrow h_9$

$U \rightarrow 11, U \rightarrow h_2$

$R \rightarrow 17, R \rightarrow t_{13}$

$YOUR \rightarrow h_8 h_9 h_2 t_{13}$

Another way of coding is suggested here, Just the position of the number with respect to the numbering of vertices can be used.

$y \rightarrow F9A$ ,  $O \rightarrow F11A$ ,  $U \rightarrow F2A$ ,  $R \rightarrow F0A$ . After coding all the letters, one finds the Cipher text to be too difficult to decode. A very high degree of secrecy is provided in the coding method. It is to be mentioned that not only the sender but also the receiver must possess sufficient knowledge about graph labelings and atomic number of elements.

### Conclusion

Having provided a coding technique using GMJ code new PCL-CANE labelings on the graph F26A, the Authors intend to formulate new techniques of coding, totally different from the one discussed here by working on different graphs with different labelings. It is suggested that, A Study can be done of any property satisfied by the graph F26A is relevant or applicable to the set of elements whose Atomic numbers are used by PCL-CANE Labelings.

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