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INTUITIONISTIC FUZZY ℓ-FILTER OF ℓ-RING

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Abstract

The paper contains some definitions and results in intuitionistic fuzzy ℓ-filters. Some of its basic properties are investigated.

1. Introduction

Theory of Fuzzy set was introduced in [8] 1965. Fuzzy group was introduced by Rosen field. The concept of Intuitionistic Fuzzy set was introduced [1] as a generalization of the notion of Fuzzy set. M. Mullai [3] has studied the concepts of fuzzy L-ideals and Fuzzy L-filters. The idea of Intuitionistic L-fuzzy semi filters was introduced [2]. Fuzzy set to ℓ-ideal was applied in [4]. Particularly, [7] developed the theory of fuzzy sub lattice. The main purpose of this work is to study the generalization of these concepts for Intuitionistic Fuzzy Lattices. The main motivation in this work is to introduce the concept of Intuitionistic Fuzzy ℓ-filter in ring and establish some results on it.

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2. Preliminaries

Definition 2.1 [3]. A fuzzy subset $\mu : L \to [0, 1]$ of L is called a fuzzy L-filter of L, $\forall g, h \in L$.

(i) $\mu(g \vee h) \leq \max\{\mu(g), \mu(h)\}$

(ii) $\mu(g \wedge h) \leq \min\{\mu(g), \mu(h)\}$

Definition 2.2 [4]. A fuzzy subset μ of an ℓ-ring R, is called a fuzzy ℓ-ring ideal (or) fuzzy ℓ-ideals of R, if for all $g, h \in R$ the following conditions are satisfied.

(i) $\mu(g \vee h) \geq \min\{\mu(g), \mu(h)\}$

(ii) $\mu(g \wedge h) \geq \max\{\mu(g), \mu(h)\}$

(iii) $\mu(g - h) \geq \min\{\mu(g), \mu(h)\}$

(iv) $\mu(gh) \geq \max\{\mu(g), \mu(h)\}$.

Definition 2.3 [1]. Intuitionistic fuzzy set A of a non-empty X is an object of the following form $A = \{\langle X, \mu_A(g), \nu_A(g)\rangle / g \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $g \in X$, respectively and $\forall g \in X, 0 \leq \mu_A(g) + \nu_A(g) \leq 1$.

Definition 2.4 [6]. Intuitionistic Fuzzy subset A of an ℓ-ring R is called a Intuitionistic Fuzzy ℓ-ring ideals or Intuitionistic Fuzzy ℓ-ideals of ℓ-ring R if for all $g, h \in R$.

(i) $\mu_A(g \vee h) \geq \min\{\mu_A(g), \mu_A(h)\}$

(ii) $\mu_A(g \wedge h) \geq \max\{\mu_A(g), \mu_A(h)\}$

(iii) $\mu_A(g - h) \geq \min\{\mu_A(g), \mu_A(h)\}$

(iv) $\mu_A(gh) \geq \max\{\mu_A(g), \mu_A(h)\}$

(v) $\mu_A(g \vee h) \leq \max\{\nu_A(g), \nu_A(h)\}$

(vi) $\mu_A(g \wedge h) \leq \min\{\nu_A(g), \nu_A(h)\}$

(vii) $\nu_A(g - h) \leq \max\{\nu_A(g), \nu_A(h)\}$

(viii) $\nu_A(gh) \leq \min\{\nu_A(g), \nu_A(h)\}$.

3. Intuitionistic Fuzzy ℓ- filter of ℓ- ring

Definition 3.1. An Intuitionistic Fuzzy subset A of ℓ- filter of ℓ- ring R is called an Intuitionistic Fuzzy ℓ- filter of ℓ- ring R if for all $g, h \in R$.

(i) $\mu_A(g \vee h) \leq \max\{\mu_A(g), \mu_A(h)\}$

(ii) $\mu_A(g \wedge h) \leq \min\{\mu_A(g), \mu_A(h)\}$

(iii) $\mu_A(g - h) \leq \max\{\mu_A(g), \mu_A(h)\}$

(iv) $\mu_A(gh) \leq \min\{\mu_A(g), \mu_A(h)\}$

(v) $\nu_A(g \vee h) \geq \min\{\nu_A(g), \nu_A(h)\}$

(vi) $\nu_A(g \wedge h) \geq \max\{\nu_A(g), \nu_A(h)\}$

(vii) $\nu_A(g - h) \geq \min\{\nu_A(g), \nu_A(h)\}$

(viii) $\nu_A(gh) \geq \max\{\nu_A(g), \nu_A(h)\}$.

Example 3.2. Consider the Intuitionistic fuzzy subset A of ℓ- filter of ℓ- ring R defined

$$\mu_A(g) = \begin{cases} .4 \text{ if } g = i \\ .7 \text{ if } g = j \\ .8 \text{ if } g = k, l \end{cases} \qquad \nu_A(g) = \begin{cases} .6 \text{ if } g = i \\ .3 \text{ if } g = j \\ .2 \text{ if } g = k, l \end{cases}$$

$\mu_A(g), \nu_A(g)$ is an Intuitionistic Fuzzy ℓ- filter of ℓ- ring R.

3.1. Characteristics of Intuitionistic Fuzzy ℓ- filter of ℓ- ring

Proposition 3.3. *If A is an Intuitionistic Fuzzy ℓ- filter of ℓ- ring R, then* $\mu_A(0) \leq \mu_A(g) \leq \mu_A(1)$ *and* $\nu_A(0) \geq \nu_A(g) \geq \nu_A(1)$, *for all* $x \in R$ *where* 0 *is the least element and* 1 *is the greatest element in R.*

Proof. Given A is an Intuitionistic Fuzzy ℓ-filter of ℓ-ring R with least element 0 and greatest element 1.

$\mu_A(0) \leq \mu_A(g) \leq \mu_A(1)$

$\nu_A(0) \geq \nu_A(g) \geq \nu_A(1)$, for all $g \in R$

Take $g \in R$ be arbitrary.

$\mu_A(g) = \mu_A(1 \wedge g) \leq \min\{\mu_A(1), \mu_A(g)\} \leq \mu_A(1)$

and $\mu_A(0) = \mu_A(g - g) \leq \max\{\mu_A(g), \mu_A(g)\} \leq \mu_A(g)$

Therefore, $\mu_A(0) \leq \mu_A(g) \leq \mu_A(1)$

$\nu_A(g) = \nu_A(1 \wedge g) \geq \max\{\nu_A(1), \nu_A(g)\} \geq \nu_A(1)$

and $\nu_A(0) = \nu_A(g - g) \geq \min\{\nu_A(g), \nu_A(g)\} \geq \nu_A(g)$

Therefore, $\nu_A(0) \geq \nu_A(g) \geq \nu_A(1)$.

Proposition 3.4. *Let A be an Intuitionistic Fuzzy ℓ-filter of ℓ-ring R,* $\mu_A(g) \geq \mu_A(h)$, $\nu_A(g) \leq \nu_A(h)$ *whenever* $g \leq h$, *where* $g, h \in R$..

Proof. Given A is an Intuitionistic fuzzy ℓ-filter R.

Let $g, h \in R$ be arbitrary.

Assume that $g \leq h$.

$g \wedge h = g$, $g \vee h = h$.

Now, $\mu_A(h) = \mu_A(g \wedge h) \leq \max\{\mu_A(g), \mu_A(h)\} \leq \mu_A(g)$.

Therefore, $\mu_A(h) \leq \mu_A(g)$.

$\nu_A(h) = \nu_A(g \vee h) \geq \min\{\nu_A(g), \nu_A(h)\} \geq \nu_A(g)$.

Therefore, $\nu_A(h), \leq \nu_A(g)$.

Proposition 3.5. *Let A be an Intuitionistic Fuzzy ℓ-filter of ℓ-ring R, then* $\mu_A(g) = \mu_A(-g)$ *and* $\nu_A(-g) = \nu_A(g)$ *for all* $g \in R$.

Proof. Given that A is an Intuitionistic Fuzzy ℓ- filter of ℓ- ring R.

(i) $\mu_A(g) = \mu_A(-g)$

(ii) $\nu_A(-g) = \nu_A(g)$, for all $x \in R$

Take $g \in R$ be arbitrary.

$-g = 0 + (-g) = 0 - g.$

$\mu_A(-g) = \mu_A(0 - g) \leq \max\{\mu_A(0), \mu_A(g)\},$

$\mu_A(-g) \leq \mu_A(g).$

Again $\mu_A(g) = \mu_A(-(g)) \geq \mu_A(-g),$

$\mu_A(g) \geq \mu_A(-g).$

Hence $\mu_A(g) = \mu_A(-g).$

$-g = 0 + (-g) = 0 - g$

$\nu_A(-g) = \nu_A(0 - g) \geq \min\{\nu_A(0), \nu_A(g)\},$

$\nu_A(-g) \geq \nu_A(g).$

Also $\nu_A(g) = \nu_A(-(g)) \leq \nu_A(-g),$

$\nu_A(g) \leq \nu_A(-g),$

$\nu_A(g) = \nu_A(-g).$

Proposition 3.6. *Let A be an Intuitionistic Fuzzy ℓ-filter of ℓ-ring R, then* $\mu_A(g + h) \leq \max\{\mu_A(g), \mu_A(h)\}$, $\nu_A(g + h) \geq \min\{\nu_A(g), \nu_A(h)\}$, *for all* $g, h \in R.$

Proof. Given that A is an Intuitionistic Fuzzy ℓ- filter of ℓ- ring.

To prove:

Let $g, h \in R$ be arbitrary.

$\mu_A(g + h) = \mu_A(g - (-h))$

$\leq \max\{\mu_A(g), \mu_A(-h)\}$

$\leq \max\{\mu_A(g), \mu_A(h)\}$.

$\nu_A(g+h) = \nu_A(g-(-h))$

$\geq \min\{\nu_A(g), \nu_A(-h)\}$

$= \min\{\nu_A(g), \nu_A(h)\}$, for all $g, h \in R$.

Proposition 3.7. *A be an Intuitionistic Fuzzy* ℓ*-filter of* ℓ*-ring R, if* $\mu_A(g) > \mu_A(h)$, $\nu_A(g) < \nu_A(h)$ *for some* $g, h \in R$, *then* $\mu_A(g-h) = \mu_A(g)$, $\mu_A(g-h) = \nu_A(g)$.

Proof. Let A bean Intuitionistic Fuzzy ℓ-filter of ℓ-ring R.

Let $g, h \in R$ be arbitrary.

Assume that $\mu_A(g) > \mu_A(h)$

Now, $\mu_A(g-h) \leq \max\{\mu_A(g), \mu_A(h)\}$,

$\mu_A(g-h) \leq \mu_A(g)$,

$g = g + 0 = g + (-h+h) = g - h + h$.

Therefore, $\mu_A(g) = \mu((g-h)+h) \leq \max\{\mu_A(g-h), \mu_A(h)\} = \mu_A(g-h)$.

$\mu_A(g) \leq \mu_A(g-h)$.

$\mu_A(g-h) = \mu_A(g)$.

Also, assume that $\nu_A(g) < \nu_A(h)$

Now, $\nu_A(g-h) \geq \min\{\nu_A(g), \nu_A(h)\}$

$\nu_A(g-h) \geq \nu_A(g)$

$g = g + 0 = g + (-h+h) = (g-h) + h$.

Therefore, $\nu_A(g) = \nu_A((g-h)+h) \geq \min\{\nu_A(g-h), \nu_A(h)\} = \nu_A(g-h)$.

$\nu_A(g) \geq \nu_A(g-h)$

we have $\nu_A(g) = \nu_A(g-h)$.

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