Advances and Applications in Mathematical Sciences
Volume 20, Issue 8, June 2021, Pages 1425-1432

SIMILARITY MEASURES WITH VECTOR-LENGTH UNDER FUZZY ENVIRONMENT

D. STEPHEN DINAGAR and E. FANY HELENA

PG and Research Department of Mathematics
T.B.M.L. College (Affiliated to Bharathidasan University)
Porayar, Tamil Nadu, India
E-mail: dsdina@rediffmail.com
fanyhelena3@gmail.com

Abstract

In this article, we have proposed a similarity measures based on vector- length with the aid of trapezoidal intuitionistic fuzzy numbers. Distinct procedure as Type 1, Type 2 and Type 3 procedures and few relevant properties have also been discussed .Suitable illustrations are given for the proposed method. Finally a comparison have been made to justify the three types of similarities.

1. Introduction

Many real-world applications make use of similarity measure to see how two objects are related together. Over the last decades, many studies have been done on the concept of similarity measure between two intuitionistic fuzzy numbers. In [1] Atanassov defined various operators on intuitionistic fuzzy set which further enriched the theory for its applications to various area of decision sciences. This generalization of fuzzy set to intuitionistic fuzzy set gave a new dimension to optimization under uncertainty and envisaged a new area of optimization under intuitionistic fuzzy environment. On the one hand, the similarity measures were defined based on distance models, such as the hamming distance similarity method [2]. In [3] Li introduced a new similarity measures between the intuitionistic fuzzy set. Stephen Dinagar and Fany Helena [5, 6] proposed a similarity measures for generalized trapezoidal intuitionistic fuzzy number based on valued

2010 Mathematics Subject Classification: Primary 03A55; Secondary 94D05, 76M55.
Keywords: Trapezoidal Intuitionistic Fuzzy Number, Vector-Length, Similarity Measures.
Received February 25, 2020; Accepted July 25, 2020

ambiguity and centroid ranking. Stephen Dinagar and Fany Helena [7, 8] introduced and studied about the similarity measures of TrIFNs using centroids of horizontal and vertical axes and value and ambiguity indices and also the Similarity Measures of Generalized Interval-Valued Trapezoidal Intuitionistic Fuzzy number. In this work, by the above references the similarity measures proposed using the vector-length.

This paper is organized as follows; we gave the basic definitions of similarity measures and trapezoidal intuitionistic fuzzy number in section 2. Section 3 we revised similarity measures of TFNs using vector-length. In section 4 we proposed similarity measure for TrIFNs using the vector-length and some relevant properties are discussed followed by illustrations. And also, comparison is done between the three types of similarities. Finally conclusion is given in section 5.

2. Preliminaries

Definition 2.1. A real function $S : F(U) \times F(U) \to [0, 1]$ is defined as the fuzzy similarity measure of $F(U)$ if S satisfies following properties.

(1) $\forall A, B \in F(U), S(A, B) = S(B, A)$.

(2) If $A \in F(U)$, then $S(A, A^c) = 0$.

(3) $\forall A \in F(U), S(A, B) = 1$.

(4) $\forall A, B$ and $C \in F(U)$, if $A \subseteq B \subseteq C$, then $S(A, B) \geq S(A, C); S(B, C) \geq S(A, C)$.

Definition 2.2. A trapezoidal intuitionistic fuzzy number A with parameters $b_1 \leq a_1, b_2 \leq a_2, a_3 \leq b_3, a_4 \leq b_4$ is denoted as $A^i = [(a_1, a_2, a_3, a_4)(b_1, b_2, b_3, b_4)]$ is the set of real numbers $\Re$ is an intuitionistic fuzzy number whose membership function and non membership function are given as

$$\mu_{A^i} = \begin{cases} \left(\dfrac{x - a_1}{a_2 - a_1}\right) & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \left(\dfrac{a_4 - x}{a_4 - a_3}\right) & \text{if } a_3 \le x \le a_4 \\ 1 & \text{othewise.} \end{cases} \tag{1}$$

$$v_{A^i} = \begin{cases} \left(\dfrac{b_2 - x}{b_2 - b_1}\right) & \text{if } b_1 \le x \le b_2 \\ 0 & \text{if } b_2 \le x \le b_3 \\ \left(\dfrac{x - b_4}{b_3 - b_4}\right) & \text{if } b_3 \le x \le b_4 \\ 1 & \text{othewise.} \end{cases} \tag{2}$$

Definition 2.3. Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and $Y = \{y_1, y_2, y_3, \ldots, y_n\}$ be two vector length n, where all the coordinates are positive three important similarity measures are defined as

$$J(Y, Z) = \frac{xy}{\| x \|_2^2 + \| y \|_2^2 - xy} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} x_i y_i} \tag{3}$$

$$E(Y, Z) = \frac{2xy}{\| x \|_2^2 + \| y \|_2^2} = \frac{2\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} \tag{4}$$

$$C(Y, Z) = \frac{xy}{\| x \|_2^2 + \| y \|_2^2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}. \tag{5}$$

3. Similarity Measures of Fuzzy Numbers [4]

Definition 3.1. Let $A^i = (a_1, a_2, a_3, a_4)$ and $B^i = (b_1, b_2, b_3, b_4)$ be two triangular fuzzy numbers where $0 \le a_1 \le a_2 \le a_3 \le 1, 0 \le b_1 \le b_2 \le b_3 \le 1;$ the similarity measures between two TFNs using the vector length where $n = 1, 2, 3$ can be defined as follows

$$S^J(A^i, B^i) = \frac{\sum_{i=1}^{3} a_i b_i}{\sum_{i=1}^{3} a_i^2 + \sum_{i=1}^{3} b_i^2 - \sum_{i=1}^{3} a_i b_i} \tag{6}$$

$$S^E(A^i, B^i) = \frac{2\sum_{i=1}^{3} a_i b_i}{\sum_{i=1}^{3} a_i^2 + \sum_{i=1}^{3} b_i^2} \tag{7}$$

$$S^C(A^i, B^i) = \frac{\sum_{i=1}^{3} a_i b_i}{\sqrt{\sum_{i=1}^{3} a_i^2}\sqrt{\sum_{i=1}^{3} b_i^2}}. \tag{8}$$

4. Proposed Similarity Measures for Intuitionistic Fuzzy Number

Definition 4.1. Let $A^i = [(a_1, a_2, a_3, a_4)(a_5, a_6, a_7, a_8)]$ and $B^i = [(b_1, b_2, b_3, b_4)(b_5, b_6, b_7, b_8)]$ be two trapezoidal intuitionistic fuzzy numbers where $a_5 \le a_1 \le a_6 \le a_2 \le a_7 \le a_4 \le a_8$ and $b_5 \le b_1 \le b_6 \le b_2 \le b_3 \le b_7 \le b_4 \le b_8$ the similarity measures between two TrIFNs can be defined using the vector length n where $n = 1, 2, 3, 4, 5, 6, 7, 8$ as follows

Type 1. Similarity

$$S_1(A^i, B^i) = \frac{\sum_{i=1}^{8} a_i b_i}{\sum_{i=1}^{8} a_i^2 + \sum_{i=1}^{8} b_i^2 - \sum_{i=1}^{8} a_i b_i} \tag{9}$$

Type 2. Similarity

$$S_2(A^i, B^i) = \frac{2\sum_{i=1}^{8} a_i b_i}{\sum_{i=1}^{8} a_i^2 + \sum_{i=1}^{8} b_i^2}. \tag{10}$$

Type 3. Similarity

$$S_3(A^i, B^i) = \frac{\sum_{i=1}^{8} a_i b_i}{\sqrt{\sum_{i=1}^{8} a_i^2}\sqrt{\sum_{i=1}^{8} b_i^2}}. \tag{11}$$

4.2. Some Relevant Properties for the Proposed Method

The similarity satisfies the following properties

(1) $0 \le S(A^i, B^i) \le 1$

(2) $S(A^i, B^i) = S(B^i, A^i)$

(3) If $A^i = B^i$ (i.e) $a^i = \tilde{b}^i$, $i = 1, 2, 3, 4, 5, 6, 7, 8$; $S(A^i, B^i) = 1$.

Proof.

First, we prove $S(A^i, B^i)$ and satisfies the above properties as follows

P_1 : It is obvious that $S_1(A^i, B^i) \ge 0$, we need to prove

$$S_1(A^i, B^i) \le 1.$$

By using the basic mathematical equation

$$2a_i b_i \le a_i^2 b_i^2.$$

We get

$$\sum_{i=1}^{8} a_i^2 + \sum_{i=1}^{8} b_i^2 - \sum_{i=1}^{8} a_i b_i \geq 2a_i b_i - a_i b_i \geq a_i b_i.$$

We get $S_1(A^i, B^i) \leq 1$.

P_2 : It is obvious that the equation is true.

P_3 : When $A^i = B^i$ that is $\tilde{a}^i = \tilde{b}^i$ for $i = 1, 2, 3, 4, 5, 6, 7, 8$

$$S_1(A^i, B^i) = \frac{\sum_{i=1}^{8} a_i a_i}{\sum_{i=1}^{8} a_i^2 + \sum_{i=1}^{8} a_i^2 - \sum_{i=1}^{8} a_i a_i} = 1.$$

We can prove $S_2(A^i, B^i)$ and $S_3(A^i, B^i)$ in a same way. It satisfy the above properties.

4.3. Illustrations

Example 1. The similarity measure between two TrIFNs

$A^i = [(0.4, 0.56, 0.7, 0.82)(0.3, 0.45, 0.74, 0.89)]$ and $B^i = [(0.39, 0.58, 0.76, 0.87)(0.24, 0.49, 0.8, 0.94)]$ is

Type 1. Similarity

Using (9) we get, $S_1(A^i, B^i) = 0.9324$.

Type 2. Similarity

Using (10) we get, $S_2(A^i, B^i) = 0.9550$.

Type 3. Similarity

Using (11) we get, $S_3(A^i, B^i) = 0.9664$.

Example 2. The similarity measure between two TrIFNs

$A^i = [(0.468, 0.59, 0.73, 0.84)(0.34, 0.53, 0.81, 0.95)]$ and

B^i = [(0.47, 0.56, 0.674, 0.79) (0.31, 0.51, 0.78, 0.89)] is

Type 1. Similarity

Using (9) we get, $S_1(A^i, B^i) = 0.9966$.

Type 2. Similarity

Using (10) we get, $S_2(A^i, B^i) = 0.9983$.

Type 3. Similarity

Using (11) we get, $S_3(A^i, B^i) = 0.9998$.

4.4. Comparison between Type 1, Type 2 and Type 3

While evaluating the performance of the proposed similarity measures, we compared the three types of similarity measures. Among them Type 1 has the smaller value from this we conclude that it will be the high degree of similarity. Also the Type 3 it will have the low degree of similarity as it has the greater value.

Table 1. Similarity Measures.

Sets	Type 1	Type 2	Type 3
Set A	0.9324	0.9650	0.9664
Set B	0.9966	0.9983	0.9998

5. Conclusion

Thus we have defined distinct procedures to calculate the similarity measures between intuitionistic fuzzy numbers using the vector length notion. Using the proposed notion, three types of similarity measures are defined. And also some relevant properties are discussed. Illustrations are given for the justification of the proposed method. Finally comparison is done between the three types of similarity measure to justify the degree of similarity. Moreover in future we may extend this to any another fuzzy numbers.

References

[1] K. T. Atanassov, New operators defined over intuitionistic fuzzy sets, Fuzzy Sets and Systems 61 (1994), 137-142.

[2] Z. Liang and P. Shi, Similarity measures on intuitionistic fuzzy sets, Pattern Recognition Letters 24 (2003), 2678-2693.

[3] H. W. Liu, New similarity measures between intuitionistic fuzzy sets and between elements, Mathematical and Computer Modeling 42 (2005), 61-70.

[4] Liyuan Zhang, Xuanhua and Tao Li, Some similarity measures of triangular fuzzy number and their applications in multiple criteria group decision making, Journal of Applied Mathematics (2013), 1-7.

[5] D. Stephen Dinagar and E. Fany Helena, Similarity Measure between Trapezoidal Intuitionistic fuzzy numbers with value and Ambiguity, Compliance Engineering Journal 10(9) (2019), 137-147.

[6] D. Stephen Dinagar and E. Fany Helena, Similarity Measure on Generalized Trapezoidal Intuitionistic fuzzy Numbers with Centroid ranking, A Journal of Composition Theory, ISSN: 0731-6755, Vol. XII, Issue 10, 2019, pp. 265-271.

[7] D. Stephen Dinagar and E. Fany Helena, Similarity Measures of Intuitionistic Trapezoidal Fuzzy Number using Centriods of Horizontal and Vertical Axes and Value and Ambiguity Indices, Advances and Applications in Mathematical Science (in communication).

[8] D. Stephen Dinagar and E. Fany Helena, Similarity Measures on Generalized Interval-Valued Trapezoidal Intuitionistic Fuzzy number, Malaya Journal of Matematik 1 (2020), 313-318.