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A STUDY OF CAUSAL INFLUENCE OF SELF-BELIEFS ON EMOTIONAL INTELLIGENCE USING FUZZY RELATIONAL MAP MODEL AND WASPAS METHOD IN PICTURE FUZZY ENVIRONMENT

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Abstract

Cognitive processes such as decision making, reasoning and approximation involve a great deal of uncertainty and vagueness. The existing multicriteria decision making methods have their own advantages and limitations. Some of the methods do not consider the weights of the attributes or the interrelationship among the attributes which may have an impact on the decision-making process and decisions. In order to overcome these limitations researchers have developed hybrid methods that incorporate the weight of the attributes and its influence on other factors. One of the approaches is to include the weight obtained from the steady state vector of the dynamical system which is represented by the decision matrix. This study attempts to construct a hybrid approach using fuzzy relational map model and Weighted Aggregated Sum Product Assessment (WASPAS) ranking method with picture fuzzy information. This hybrid approach is applied to explore the causal influences of self-beliefs on the traits of emotional intelligence in undergraduate students. The problem is studied in the picture fuzzy environment including the weight information and considering the interrelationship among the attributes.

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1. Introduction

Fuzzy relational map is a soft computing technique that combines fuzzy logic and neural network theory to study the behaviour of complex systems [1]. The functioning of fuzzy relational map (FRM) is similar to the dynamics of fuzzy cognitive map (FCM). FRM with a set of fuzzy rules is helpful in modelling the dynamics of real-world qualitative systems. The ability to deal with non-monotonic and/or asymmetric causal relations improves the efficiency of the FRMs in the analysis of complex systems existing in psychology and social sciences.

Fuzzy sets, introduced by L. A. Zadeh in 1965, are efficient in representing the uncertainty and imprecision that is present in the data provided by the experts. The ordinary fuzzy sets are characterized by a single value which is the degree of membership value of an element belonging to a set. Atanassov developed a new kind of fuzzy sets called intuitionistic fuzzy sets that are characterized by the degree of non-membership values along with the degree of membership values. The intuitionistic fuzzy sets perform better than the ordinary fuzzy sets in quantifying the imprecise information and uncertainty of the linguistic expressions. Since the introduction of fuzzy sets, researchers have come up with several other advanced fuzzy sets to quantify the uncertainty that is inherent of human cognitive processes. Picture fuzzy sets are a little more advanced than the intuitionistic fuzzy sets in a sense that they are characterized by an additional membership value called neutral membership such that the sum of all the three membership degree values is less than or equal to one [2, 3].

Fuzzy relational map is a dynamical system that represents the causal relationship between two disjoint sets of concepts, wherein the concepts and the causal relationships are represented by the fuzzy sets [1]. The inference of FRM is obtained as steady state vectors which are the output values of the dynamic system for the user provided input data. From these output values decisions or conclusions that predict the behavior of the system can be arrived at. The fuzzy relational map constructed based on ordinary fuzzy sets are not capable of including the hesitancy of experts and hence are not able to model the system comprehensively. Replacing the ordinary fuzzy sets with enhanced fuzzy sets in FRMs results in efficient model that includes relatively more information.

Multi attribute decision making (MADM) methods are significant tools to solve complex problems which have different alternatives with respect to varied attributes. Diversified MADM approaches are found in the literature applied to problems with different levels of complexity and predictability. Weighted Aggregated Sum Product Assessment (WASPAS) is one of the MADM methods proposed by Zavadskas et al. in 2012 [4]. In essence, WASPAS method is a combination of two methods, namely, Simple Additive Weighting (SAW) and Weighted Product method (WPM). An advantage of this method is that this is a simple and straightforward method involving mathematically uncomplicated calculations. Though this method is simple in approach, it is capable of producing accurate results as it includes the weights of the criteria into the calculations.

Emotions play a vital role in determining one's performance in profession, life and relationship. Emotion is the currency with which human beings do all sorts of transactions in their environment. A good knowledge or intelligence about one's own disposition in different settings is required to be successful in personal and professional life. Emotional intelligence is the capacity to work well with one's own emotions and that of the others in all situations [5]. In this paper, Fuzzy Relational Map model based on picture fuzzy sets is implemented to study the influence of beliefs on emotional intelligence and calculate the weight of the attributes from the steady state vector of fuzzy inference process. The picture fuzzy information is analyzed with fuzzy relational map and WASPAS ranking method.

2. Picture Fuzzy Sets-Preliminaries

The concept of Picture Fuzzy Sets (PFS) is a generalisation of intuitionistic fuzzy sets. Single valued Picture Fuzzy Sets (PFS) [2], [3] of the universe of discourse X is given by

$$Ap(x) = \{\langle\, x,\ \mu_{Ap}(x),\ \eta_{Ap}(x),\ \upsilon_{Ap}(x)\, \rangle \mid x \in X\} \tag{1}$$

Where $\mu_{Ap}(x), \eta_{Ap}(x), \upsilon_{Ap}(x) \in [0, 1]$, $0 \leq \mu_{Ap}(x) + \eta_{Ap}(x) + \upsilon_{Ap}(x) \leq 1$. Here $\mu_{Ap}(x)$, $\eta_{Ap}(x)$ and $\upsilon_{Ap}(x)$ denote the degree of positive membership, neutral membership and negative membership of x in Ap respectively. $\pi_{Ap}(x) = 1 - \mu_{Ap}(x) - \eta_{Ap}(x) - \upsilon_{Ap}(x)$ is the degree of refusal membership of

x in Ap. If there is only one element in X, then $Ap(x) = \{\langle x, \mu_{Ap}(x), \eta_{Ap}(x), v_{Ap}(x) \rangle \mid x \in X\}$ is called Picture Fuzzy Number (PFN). A PFN is usually denoted as $(\mu_{Ap}, \eta_{Ap}, v_{Ap})$.

Operational laws complying with the mathematical rules are the tools that enable working with new kinds of sets. Wei. et al. introduced some operations of PFNs and developed several picture fuzzy aggregation operators based on the operational laws but these operations did not satisfy the constraint condition $0 \leq \mu_{Ap}(x) + \eta_{Ap}(x) + v_{Ap}(x) \leq 1$ of picture fuzzy sets in all the situations [6]. To overcome this issue Wang et al. introduced several operations based on probability and proposed aggregating operators. Some of the operations and aggregating operators used in this paper are as follows [6].

Let $a = (\mu_a, \eta_a, v_a)$ and $b = (\mu_b, \eta_b, v_b)$ be the two PFNs, then,

$$a \oplus b = ((1 - (1 - \mu_a)(1 - \mu_b)), \eta_a\eta_b, ((v_a + \eta_a)(v_b + \eta_b) - \eta_a\eta_b)) \tag{2}$$

$$a \otimes b = (((\mu_a + \eta_a)(\mu_b + \eta_b) - \eta_a\eta_b), \eta_a\eta_b, (1 - (1 - v_a)(1 - v_b))) \tag{3}$$

$$\lambda a = ((1 - (1 - \mu_a)^{\lambda}), \eta_a^{\lambda}, ((v_b + \eta_b)^{\lambda} - \eta_a^{\lambda})), \lambda > 0 \tag{4}$$

$$a^{\lambda} = (((\mu_a + \eta_a)^{\lambda} - \eta_a^{\lambda}), \eta_a^{\lambda}, (1 - (1 - v_a)^{\lambda})), \lambda > 0 \tag{5}$$

Let $P_i = (\mu_i, \eta_i, v_i), (i = 1, 2, \ldots, n)$ be a collection of PFNs, $w_i \in [0, 1]$ be the weight of P_i and $\sum_{i=1}^{n} w_i = 1$. Then the Picture Fuzzy Weighted Geometric (PFWG) operator [6] is defined as follows.

$$PFWG\,(P_1, P_2, \ldots, P_n) = \otimes_{i=1}^{n} P_i^{w_i} = P_1^{w_1} \otimes P_2^{w_2} \ldots \otimes P_n^{w_n}$$

$$= (\prod_{i=1}^{n} (\mu_i + \eta_i)^{w_i} - \prod_{i=1}^{n} \eta_i^{w_i}, \prod_{i=1}^{n} \eta_i^{w_i}, 1 - \prod_{i=1}^{n} (1 - v_i)^{w_i}) \tag{6}$$

Wang et al. [6] proposed the following score and accuracy functions to compare two PFNs. Let $a = (\mu_a, \eta_a, v_a)$ be a PFN, then the score function $S(a)$ and accuracy function $H(a)$ of are defined as follows.

$$S(a) = (\mu_a - \eta_a) \tag{7}$$

$$H(a) = \mu_a + \eta_a + v_a. \tag{8}$$

The ranking method based on the score function and accuracy function is defined as follows.

[1] If $S(a) > S(b)$ then $a > b$;

[2] If $S(a) = S(a)$ and $H(a) > H(a)$ then $a > b$

3. Fuzzy Relational Map in Picture Fuzzy Environment

In FRMs based on ordinary fuzzy sets, the limitation is that the hesitancy of the experts is not considered [8, 9]. In case of picture fuzzy environment, both the concepts and causal relationships are denoted by single-valued picture fuzzy sets which are characterized by a membership, non-membership and neutral membership values. Picture fuzzy sets enable the inclusion of hesitancy which is the influential part of the dynamic system and inference process.

Let $C = \{C_1, C_2, \ldots, C_n\}$ be a finite set of attributes and $A = \{A_1, A_2, \ldots, A_m\}$ be a set of feasible alternatives which are completely disjoint in nature. The causal relationship between the concepts is characterized by fuzzy relational map and is mathematically represented by an adjacency matrix of order $m \times n.$ The FRM calculates the successive states iteratively in discrete steps during the reasoning process until convergence. The states of the inference process are represented by a state vector S^k. Each state vector S^k is an n-tuple representing the node values of the concepts, that is $S_i^k \in [0, 1], i = 1, 2, \ldots, n$ at k-th iteration. Let $w = (w_1, w_2, \ldots, w_n)$ be the weight vector of all attributes which satisfies the condition $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1.$ The value of each node is computed using equation (9) and the user provided input $S^{(0)}$ based initiates the inference process.

$$S_i^{k+1} = f\left(s_i^k + \sum_{j=1}^{N} S_j^k \cdot w_{ji}\right) \tag{9}$$

where w_{ji} is the weight of the edge from the node j to node i, s_i^k, $s_i^{k+1} \in [0, 1]$ are state vectors at k and $k+1$ iterations and f is a sigmoid function which thresholds the output value within [0, 1].

Elpiniki, et al., introduced intuitionistic fuzzy cognitive map which is an extension of traditional FCM in 2009 [8]. To overcome the limitations in the calculation of state vector Elpiniki, et al., further enhanced the equation to compute the state vector in [9] as given in equation (10)

$$S_i^{k+1} = f(S_i^k + \sum_{j=1}^{N} S_j^k \cdot w_{ji}^{\mu}(1 - w_{ji}^{\pi})) \tag{10}$$

where w_{ji}^{μ}, $w_{ji}^{\pi} \in [0, 1]$ represent the influence weight and the hesitancy weight corresponding to the edge from node j to node i. These weights are estimated by defuzzification of the aggregated, linguistically expressed, concept influences and hesitancies, respectively. The weight factor $w_{ji}^{\mu}(1 - w_{ji}^{\pi})$ in equation (10) preserves the sign of the influence [10]. If $w_{ji}^{\pi} = 1$, then the expression $(1 - w_{ji}^{\pi})$ becomes zero and this implies that the two concepts are totally unrelated. If the hesitancy of expert reduces, then its influence of on causal relation between the concepts also reduces. In particular if $w_{ji}^{\pi} = 0$, then the state vector depends on the influence weight w_{ji}^{μ} totally.

4. Picture Fuzzy FRM-WASPAS Method: A Hybrid Approach

To analyze the causal relationship between the sets of attributes and alternatives and rank the alternatives in the order of influence, an integrated approach of Picture Fuzzy FRM-WASPAS method is applied adapted. This method consists of the following steps.

Step 1. The individual experts provide the relationship between the set of attributes and alternatives in terms of linguistic expressions.

Step 2. The individual expert's opinion-based evaluation matrix is transformed into picture fuzzy values using the picture fuzzy sets.

Step 3. The weights of the individual experts are calculated using picture fuzzy entropy method [11] given by the equation (11) with $\sigma_i \in [0, 1]$ and $\sum_{k=1}^{i} \sigma_i = 1$.

$$\sigma_k = \frac{1 - \zeta_k}{i - \sum_{k=1}^{i} \zeta_k}, \ k = 1, 2, \ldots, l \tag{11}$$

where ζ_k denotes the picture fuzzy entropy of information provided by the k-th expert and

$$\zeta_k = \sum_{i=0}^{n} \sum_{j=0}^{m} \left(1 - \frac{1}{2}(| \mu_{p_{ij}^k} - \eta_{p_{ij}^k} | + | \eta_{p_{ij}^k} - \upsilon_{p_{ij}^k} | + | \mu_{p_{ij}^k} - \upsilon_{p_{ij}^k} |)\right). \tag{12}$$

Step 4. The evaluation matrix provided by each expert is aggregated using Picture Fuzzy Weighted Geometric (PFWG) operator given in equation (6). The aggregated matrix is a matrix where each entry is represented by a single-valued picture fuzzy set.

Step 5. The steady state vector obtained in the fuzzy inference process of FRM is used as weights of the attributes $(\widetilde{w}_j)$.

Step 6. The Weighted Sum Model (WSM) is calculated as follows.

$$\widetilde{Q}_i^{(1)} = \sum_{j=1}^{n} \widetilde{x}_{ijw} = \sum_{j=1}^{n} \widetilde{x}_{ij} \cdot \widetilde{w}_j \tag{13}$$

Equation (13) is divided into two parts as addition and multiplication parts.

Step 6. 1. The multiplication part of equation (13) based on equation (4) is given as follows:

$$\widetilde{x}_{ijw} = \widetilde{x}_{ij} \cdot \widetilde{w}_j = ((1 - (1 - \mu_{\widetilde{x}_{ij}})^{\widetilde{w}_j}), \eta_{\widetilde{x}_{ij}}^{\widetilde{w}_j}, ((\upsilon_{\widetilde{x}_{ij}} + \eta_{\widetilde{x}_{ij}})^{\widetilde{w}_j} - \eta_{\widetilde{x}_{ij}}^{\widetilde{w}_j}))$$

Step 6.2. The addition part of equation (13) based on equation (2) is given as follows.

$$\widetilde{x}_{i1w} \oplus \widetilde{x}_{i2w} = ((1 - (1 - \mu_{\widetilde{x}_{i1w}})(1 - \mu_{\widetilde{x}_{i2w}})), \eta_{\widetilde{x}_{i1w}} \cdot \eta_{\widetilde{x}_{12w}}$$

$$((\upsilon_{\widetilde{x}_{i1w}} + \eta_{\widetilde{x}_{i2w}})(\upsilon_{\widetilde{x}_{i1w}} + \eta_{\widetilde{x}_{i2w}}) - \eta_{\widetilde{x}_{i1w}} \cdot \eta_{\widetilde{x}_{i2w}}))$$

Step 7. The Weighted Product Model (WPM) is calculated as follows.

$$\widetilde{Q}_i^{(2)} = \prod_{j=1}^{n} \widetilde{x}_{ij}^{\widetilde{w}_j} \tag{14}$$

Equation (14) is separated into two parts as addition and multiplication parts.

Step 7. 1. The Exponential part of equation (14) based on equation (5) is given as follows:

$$\widetilde{x}_{ij}^{\widetilde{w}_j} = \left(\left((\mu_{\widetilde{x}_{ij}} + \eta_{\widetilde{x}_{ij}})^{\widetilde{w}_j} - \eta_{\widetilde{x}_{ij}}^{\widetilde{w}_j}\right), \eta_{\widetilde{x}_{ij}}^{\widetilde{w}_j}, (1 - (1 - \upsilon_{\widetilde{x}_{ij}})^{\widetilde{w}_j})\right)$$

Step 7. 2. The multiplication part of equation (14) based on equation (3) is calculated as follows:

$$\widetilde{x}_{i1}^{\widetilde{w}_1} \otimes \widetilde{x}_{i2}^{\widetilde{w}_2} = (((\mu_{\widetilde{x}_{i1}^{\widetilde{w}_1}} + \eta_{\widetilde{x}_{i1}^{\widetilde{w}_1}})(\mu_{\widetilde{x}_{i2}^{\widetilde{w}_2}} + \eta_{\widetilde{x}_{i2}^{\widetilde{w}_2}}) - \eta_{\widetilde{x}_{i1}^{\widetilde{w}_1}} \cdot \eta_{\widetilde{x}_{i2}^{\widetilde{w}_2}}), \eta_{\widetilde{x}_{i1}^{\widetilde{w}_1}} \cdot \eta_{\widetilde{x}_{i2}^{\widetilde{w}_2}},$$

$$(1 - (1 - \upsilon_{\widetilde{x}_{i1}^{\widetilde{w}_1}})(1 - \upsilon_{\widetilde{x}_{i2}^{\widetilde{w}_2}})))$$

Step 8. For different values of λ the $\widetilde{Q}_i$ value is calculated using the equation (15).

$$\widetilde{Q}_i = \lambda \widetilde{Q}_i^{(1)} + (1 - \lambda)\widetilde{Q}_i^{(2)} \tag{15}$$

Step 9. The $\widetilde{Q}_i$ is defuzzified using the score function given in equation (7).

Step 10. The alternatives are ranked in the order of decreasing score values. If two alternatives have equal score values then their accuracy value is calculated as given in equation (8) and used to rank the alternatives.

5. Description of the Problem

In the recent past, emotional intelligence has gained importance in personal and professional life. It has been considered as a significant factor that is responsible for an individual's success along with academics, skills and abilities. Though this idea of emotional intelligence was in usage since 1950, Salovey and Mayer coined the term 'emotional intelligence' and used it in their research. The paper published by Salovey and Mayer in 1990 attracted the attention of the research world towards emotional intelligence [12]. Since then, several studies have been carried out in studying emotional intelligence competencies and its influence on life's success. It has been proved time and again that individuals with high emotional intelligence enjoy greater subjective well-being [13].

Emotional intelligence is one of the psychological constructs that defines the personality and behaviour of an individual in social context. Frijida et al. in [14] claims "... thinking, no matter how well articulated, is not sufficient for action. ... Emotions are prime candidates for turning a thinking being into an actor". The ability to handle one's own emotions and understand the emotions of others is called emotional intelligence and the good news is that it can be developed through consistent practice and conscious exercise from the early years in students. Besides the existing speculations about emotional intelligence, one of the main objectives of studying emotional intelligence construct is to developing methods and strategies to prevent the societal problems such as underachievement, unemployment, poverty, violence, public health and criminality [13]. In this study, an attempt is made to explore the causal influence of self-beliefs on emotional intelligence in undergraduate students.

5.1. Emotional Intelligence Assessment Index System

The current study of emotional intelligence (EI) is dominated by two conceptually different approaches: the trait and the ability approach [15]. The trait approach considers EI as dispositional tendencies like personality traits or self-efficacy beliefs while, the ability approach conceptualizes EI as a cognitive ability based on the processing of emotion information. In this paper, the trait approach of EI is considered to study the factors that influence EI positively. The 15 traits of emotional intelligence described in [13] is taken as elements of domain space, the set of attributes. Table 1 presents the list of traits of emotional intelligence considered in this study [13].

Table 1. Traits of emotional intelligence.

Factors of EI	Traits	Brief Explanation
Well-being	Self-esteem	Seeing oneself as elf-confident and successful
	Happiness	Feeling cheerful and satisfied with one's life
	Optimism	Ability to see the brighter side of life.
Self-control	Emotional control	Capable of controlling one's emotions
	Stress-management	Ability to withstand and deal with stress
	Impulse control	Reflective and less likely to give into urges.
Emotionality	Emotion perception	Ability to observe feelings (Self and others)
	Emotion expression	Capability to communicate feelings wisely
	Relationships	Capable of having fulfilling relationships
	Empathy	Capable of understanding others' perspectives.
Sociability	Social awareness	Ability to network with excellent social skills
	Emotion	Ability to influence the feelings of others
	management Assertiveness	Willingness to stand for what is right.
Adaptability	-	Flexible and willing to adapt to new conditions.
Self-motivation	-	Driven and unlikely to give up during adversity.

Psychologist Dr. Albert Ellis, who developed the 'Rational Emotive Behavior Theory' popularly known as the 'ABCDE' theory of emotions, in the 1950s confirms the fact that emotional behaviours such as emotional upset (milder form) or emotional disturbance (extreme form) are basically caused by self-beliefs. According to this theory, the activating events (A) do not upset people, but their beliefs (B), meaning the things they tell themselves lead them to feel negative emotions [16]. Beneath every quality, skill or competency there is a bunch of beliefs that runs the human brain, consciously and unconsciously, to achieve the characteristic that defines a person. An emotional intelligent person may repeat belief statements such as "I am feeling angry now" (Self-awareness), "This is not my time to act" (Self-regulation), "I have the power to move on" (Impetus), "I can understand what you may go through" (Compassion), "I can be assertive" (Social Skills)" and so on. These beliefs are not uniform and may vary person to person in degree of intensity and influence. Consequently, belief clusters are considered instead of single strand of belief statement to study the causal influence. Daniel Goleman, the American Psychologist who popularized the notion of emotional intelligence characterized emotional intelligence in terms of several characteristics [17]. Inspired by those characteristics the experts agreed upon taking the following set of factors that might influence emotional intelligence in an individual: self-awareness-being alert to one's feelings, self-regulation-managing one's feelings, impetus-using feelings to achieve one's goals, compassion-turning into how others feel, social skills-handling feelings well in interactions with others.

5.2. Construction of Evaluation Matrix

The problem considered in this study is assessing the influence of self-beliefs on the emotional intelligence of freshmen in College as perceived by their teachers. The teachers from their understanding based on the interaction with their students, interview and self-report questionnaire completed the decision matrix with the linguistic terms (Table 2). The linguistic terms are converted into single-valued picture fuzzy numbers.

5.3. Analysis of the Problem Using FRM-WASPAS Method

The relationship between the set of attributes and alternatives provided by the individual experts in terms of linguistic expressions is presented in Table 3. In this study all the attributes are treated like benefit attributes that the cost criterion is represented by lower linguistic term. The individual expert's opinion-based evaluation matrix is transformed into Picture fuzzy values using the single-valued picture fuzzy sets in Table 2. The weights of importance of individual experts are found to be $(0.2984, 0.3343, 0.3673)^T$. The evaluation matrix provided by each expert is aggregated using Picture fuzzy weighted geometric operator (PFWG) and the aggregated matrix in picture fuzzy numbers is given in Table 4. The normalized weights of the attributes $(\widetilde{w}_j)$ are calculated using Picture FRM inference on the decision matrix and is given in Table 5. The Weighted Sum Model (WSM) and the Weighted Product Model (WPM) are calculated using equations (13) and (14) respectively. Taking $\lambda = 0.5$ the $\widetilde{Q}_i$ value is calculated using equation (15). The picture fuzzy values of $\widetilde{Q}_i$ are defuzzified using the score function given in equation (7). The alternatives are ranked based on the score values (Table 7).

Table 2. Linguistic terms and their corresponding PFN.

Linguistic term	PFN
Absolutely more influence (AMI)	(0.9, 0.0, 0.1)
Very high influence (VHI)	(0.8, 0.1, 0.1)
High influence (HI)	(0.7, 0.1, 0.2)
Slightly more influence (SMI)	(0.6, 0.2, 0.2)
Medium influence (MI)	(0.5, 0.2, 0.2)
Slightly low influence (SLI)	(0.4, 0.3, 0.3)
Low influence (LI)	(0.3, 0.3, 0.4)
Very low influence (VLI)	(0.2, 0.3, 0.5)
Absolutely low influence (ALI)	(0.1, 0.4. 0.5)

Table 3. Evaluation Matrix of experts in Linguistic terms.

	B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5
T_{11}	SMI	HI	HI	SMI	HI	VHI	VHI	HI	HI	MI	HI	HI	HI	SMI	MI
T_{12}	SMI	HI	HI	VHI	HI	HI	SMI	HI	HI	MI	HI	MI	SMI	MI	MI
T_{13}	HI	SMI	VHI	HI	HI	VHI	MI	VHI	SLI	MI	HI	MI	VHI	MI	MI
T_{21}	AMI	HI	SMI	MI	SMI	VHI	VHI	LI	LI	SMI	VHI	HI	VLI	LI	SMI
T_{22}	VHI	SMI	HI	SMI	HI	VHI	VHI	LI	HI	HI	VHI	MI	LI	MI	HI
T_{23}	VHI	SMI	HI	SMI	SMI	VHI	HI	LI	SMI	SMI	VHI	HI	SLI	MI	SMI

T_{31}	VHI	SMI	HI	SMI	SMI	VHI	VHI	HI	HI	HI	VHI	SMI	LI	HI	HI
T_{32}	AMI	HI	SMI	HI	VHI	VHI	HI	SLI	HI	HI	VHI	HI	SLI	HI	HI
T_{33}	HI	SMI	HI	VHI	HI	HI	SMI	SLI	HI	HI	HI	MI	LI	VHI	HI
T_{34}	SMI	SMI	HI	VHI	VHI	SMI	MI	SLI	VHI	HI	MI	MI	LI	AMI	HI
T_{41}	VHI	SMI	SMI	HI	VHI	VHI	HI	SMI	SMI	HI	HI	LI	MI	HI	HI
T_{42}	VHI	MI	HI	SMI	MI	HI	HI	MI	SMI	HI	HI	HI	MI	MI	HI
T_{43}	HI	HI	HI	VHI	VHI	HI	HI	VHI	MI	VHI	VHI	VHI	HI	MI	VHI
T_{51}	VHI	SMI	VHI	SMI	SMI	HI	HI	VHI	MI	HI	HI	SMI	VHI	LI	HI
T_{61}	VHI	SMI	VHI	HI	HI	SMI	HI	AMI	SLI	LI	SMI	SMI	VHI	LI	LI

Table 4. Aggregated Evaluation Matrix in terms of PFS.

	B_1	B_2	B_3	B_4	B_5
T_{11}	(0.71, 0.12, 0.17)	(0.73, 0.10, 0.17)	(0.70, 0.10, 0.20)	(0.64, 0.16, 0.20)	(0.57, 0.16, 0.20)
T_{12}	(0.68, 0.12, 0.20)	(0.60, 0.16, 0.20)	(0.67, 0.13, 0.20)	(0.66, 0.13, 0.17)	(0.57, 0.16, 0.20)
T_{13}	(0.73, 0.10, 0.17)	(0.53, 0.20, 0.20)	(0.80, 0.10, 0.10)	(0.54, 0.19, 0.23)	(0.57, 0.16, 0.20)
T_{21}	(0.90, 0.00, 0.10)	(0.73, 0.10, 0.17)	(0.35, 0.27, 0.39)	(0.36, 0.27, 0.35)	(0.60, 0.20, 0.20)
T_{22}	(0.80, 0.10, 0.10)	(0.63, 0.16, 0.17)	(0.44, 0.22, 0.35)	(0.60, 0.16, 0.20)	(0.70, 0.10, 0.20)
T_{23}	(0.80, 0.10, 0.10)	(0.68, 0.12, 0.20)	(0.48, 0.22, 0.31)	(0.56, 0.20, 0.20)	(0.60, 0.20, 0.20)
T_{31}	(0.80, 0.10, 0.10)	(0.67, 0.16, 0.17)	(0.57, 0.15, 0.28)	(0.68, 0.12, 0.20)	(0.68, 0.12, 0.20)
T_{32}	(0.90, 0.00, 0.10)	(0.70, 0.10, 0.20)	(0.46, 0.27, 0.27)	(0.70, 0.10, 0.20)	(0.73, 0.10, 0.17)
T_{33}	(0.70, 0.10, 0.20)	(0.56, 0.20, 0.20)	(0.47, 0.22, 0.31)	(0.77, 0.10, 0.13)	(0.70, 0.10, 0.20)
T_{34}	(0.56, 0.20, 0.20)	(0.53, 0.20, 0.20)	(0.47, 0.22, 0.31)	(0.90, 0.00, 0.10)	(0.73, 0.10, 0.17)
T_{41}	(0.76, 0.10, 0.14)	(0.54, 0.18, 0.28)	(0.56, 0.20, 0.20)	(0.67, 0.13, 0.20)	(0.73, 0.10, 0.17)
T_{42}	(0.73, 0.10, 0.17)	(0.65, 0.12, 0.20)	(0.57, 0.16, 0.20)	(0.56, 0.20, 0.20)	(0.65, 0.12, 0.20)
T_{43}	(0.74, 0.10, 0.16)	(0.74, 0.10, 0.16)	(0.73, 0.10, 0.17)	(0.59, 0.16, 0.17)	(0.80, 0.10, 0.10)
T_{51}	(0.73, 0.10, 0.17)	(0.64, 0.16, 0.20)	(0.80, 0.10, 0.10)	(0.46, 0.23, 0.28)	(0.68, 0.12, 0.20)
T_{61}	(0.67, 0.16, 0.17)	(0.64, 0.16, 0.20)	(0.90, 0.00, 0.10)	(0.47, 0.22, 0.31)	(0.44, 0.22, 0.35)

Table 5. Weights of Attributes.

Attribute	T_{11}	T_{12}	T_{13}	T_{21}	T_{22}	T_{23}	T_{31}	T_{32}
FRM method	0.0671	0.0674	0.0672	0.0657	0.0667	0.0660	0.0662	0.0670
Entropy Method	0.0662	0.0736	0.0703	0.0726	0.0690	0.0717	0.0607	0.0557
Attribute	T_{33}	T_{34}	T_{41}	T_{42}	T_{43}	T_{51}	T_{61}	-
FRM method	0.0670	0.0667	0.0666	0.0676	0.0670	0.0663	0.0656	-
Entropy Method	0.0703	0.0643	0.0671	0.0750	0.0542	0.0643	0.0649	-

Table 6. WSM, WPM values in PFS.

	FRM method		Entropy method	
	WSM ($\tilde{Q}_i^{(1)}$)	WPM ($\tilde{Q}_i^{(2)}$)	WSM ($\tilde{Q}_i^{(1)}$)	WPM ($\tilde{Q}_i^{(2)}$)
B_1	(0.76, 0.00, 0.24)	(0.85, 0.00, 0.15)	(0.76, 0.00, 0.24)	(0.85, 0.00, 0.15)
B_2	(0.64, 0.14, 0.19)	(0.64, 0.14, 0.19)	(0.64, 0.14, 0.20)	(0.64, 0.14, 0.20)
B_3	(0.64, 0.00, 0.36)	(0.76, 0.00, 0.24)	(0.63, 0.00, 0.36)	(0.75, 0.00, 0.24)
B_4	(0.64, 0.00, 0.34)	(0.77, 0.00, 0.21)	(0.64, 0.00, 0.35)	(0.77, 0.00, 0.21)
B_5	(0.66, 0.13, 0.19)	(0.65, 0.13, 0.20)	(0.65, 0.13, 0.20)	(0.65, 0.13, 0.20)

Table 7. Ranking by FRM-WASPAS method.

	$\lambda(\tilde{Q}_i^{(1)})$	$(1-\lambda)(\tilde{Q}_i^{(2)})$	$\tilde{Q}_i$	Score	Rank
B_1	(0.51, 0.00, 0.49)	(0.61, 0.00, 0.39)	(0.81, 0.00, 0.19)	0.8090	1
B_2	(0.40, 0.38, 0.20)	(0.40, 0.38, 0.20)	(0.64, 0.05, 0.19)	0.5885	5
B_3	(0.40, 0.00, 0.60)	(0.50, 0.00, 0.49)	(0.70, 0.00, 0.29)	0.7019	3
B_4	(0.40, 0.00, 0.59)	(0.52, 0.00, 0.46)	(0.71, 0.00, 0.27)	0.7092	2
B_5	(0.42, 0.36, 0.21)	(0.41, 0.36, 0.21)	(0.66, 0.05, 0.20)	0.6073	4

5.4. A comparative analysis

In this comparison process, the same aggregated decision matrix (Table 4) is used to compute the weight values of the attributes by picture fuzzy entropy method given in equation (11). These weights based on picture fuzzy entropy method (Table 5) are used in WASPAS method to rank the alternatives. Same ranking is attained (Table 8) in this approach too. However, the ranking obtained in picture FRM-WASPAS method is relatively reliable as the interaction among the attributes and the alternatives are taken into consideration.

Table 8. Ranking with weights from Entropy method.

	$\lambda(\widetilde{Q}_i^{(1)})$	$(1-\lambda)(\widetilde{Q}_i^{(2)})$	$\widetilde{Q}_i$	Score	Rank
B_1	(0.51, 0.00, 0.49)	(0.61, 0.00, 0.39)	(0.81, 0.00, 0.19)	0.8090	1
B_2	(0.40, 0.38, 0.20)	(0.40, 0.38, 0.20)	(0.64, 0.00, 0.05)	0.5842	5
B_3	(0.40, 0.00, 0.60)	(0.50, 0.00, 0.49)	(0.70, 0.00, 0.29)	0.6983	3
B_4	(0.40, 0.00, 0.59)	(0.51, 0.00, 0.47)	(0.20, 0.00, 0.27)	0.7046	2
B_5	(0.42, 0.36, 0.21)	(0.41, 0.37, 0.21)	(0.65, 0.05, 0.06)	0.5972	4

6. Results and Discussion

The decision matrix describes the causal relationship between the traits of emotional intelligence and the clusters of beliefs that determine the competencies of emotional intelligence. The decision matrix is treated like a fuzzy relational map and the steady state vector of the system was computed for user provided defuzzified input values. The normalized output vector is used as weight of the attributes in WASPAS model applied on the comprehensive evaluative matrix of the experts' opinion. The rankings of the alternatives are: $B_1 \succ B_5 \succ B_3 \succ B_2 \succ B_4$ and are consistent with respect to WSM, WPM and WASPAS models. The weights of attributes are also obtained using entropy method and they are used in WASPAS model. The ranking is found to be in the same order. It can be inferred from the ranking order that B_1 (Self-awareness) is the most influential B_4 (Compassion) is the least influential belief clusters.

7. Conclusion

The advantage of the hybrid approach adopted in this study are:1) The weights obtained from the steady state vector of the fuzzy inference are more reliable as the interaction among the attributes and influence of alternatives on the attributes are taken into account during the reasoning process in contrast to the other methods of weight determination where the weight values of attributes are obtained independently. 2) The calculations are relatively less in this case and the computations can be carried out with less effort. 3) The introduction of more and more enhanced fuzzy sets that can quantify the imprecise information makes the system more efficient and the results obtained are more reliable. The picture fuzzy set, an extension of ordinary fuzzy sets, is more comprehensive than the other fuzzy sets as they include more components of information. This allows the experts to express their opinion with a higher level of confidence, understanding and freedom. The system can further be improved by assuming interval-valued, triangular or trapezoidal membership values in the place of single valued fuzzy sets.

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