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QUASI $\mathcal{N}_{\alpha g^{\#}\psi}$- OPEN AND QUASI $\mathcal{N}_{\alpha g^{\#}\psi}$- CLOSED FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

In this article, we introduce the concept of quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open and quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- closed functions and investigate some of its properties in neutrosophic topological spaces.

1. Introduction

Mapping plays a significant role in the research of classical mathematics, particularly in topology and functional analysis. Closed and open mapping is one such mapping that has been studied for many years by various mathematicians for different types of closed sets.

In 1965, Zadeh [16] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. Later on fuzzy topology was introduced by Chang [5] in 1986. The Intuitionistic fuzzy set was introduced by Atanassov [2, 3] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non membership of each element. After this, intuitionistic fuzzy topology was introduced by Coker [6].

The neutrosophic set was introduced by Smarandache [10, 11] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In

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2012, Salama, Alblowi [12] introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non membership of each element.

Further the fundamental sets like semi open sets, pre open sets, α-open sets, are delivered in neutrosophic topological spaces then theirs properties are well-read by various authors [7, 9]. Recently, Vigneshwaran et al. [15] introduced a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets in neutrosophic topological spaces and studied some of their basic properties.

In this paper, we will continue the study of related functions by involving $\mathcal{N}_{\alpha g^{\#}\psi}$-open and closed sets. We disseminate and characterize the thinking of quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open and quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed functions.

2. Preliminaries

Definition 2.1 [10]**.** A neutrosophic set is an object of the following form

$$\mathcal{A} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s)\mathcal{R}_{\mathcal{A}}(s) : s \in \mathcal{S}\rangle\}$$

where $\mathcal{P}_{\mathcal{A}}(s)$, $\mathcal{Q}_{\mathcal{A}}(s)$ and $\mathcal{R}_{\mathcal{A}}(s)$ denote the degree of membership, the degree of indeterminacy and the degree of nonmembership for each element $s \in \mathcal{S}$ to the set $\mathcal{A}$, respectively.

Definition 2.2 [10]**.** Let $\mathcal{A}$ and $\mathcal{B}$ be neutrosophic sets of the form $\mathcal{A} = \{\langle x, \mathcal{P}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s), \mathcal{R}_{\mathcal{A}}(s) : s \in \mathcal{S}\rangle\}$ and $\mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S}\rangle\}$. Then

(i) $\mathcal{A} \subseteq \mathcal{B}$ if and only if $\mathcal{P}_{\mathcal{A}}(s) \leq \mathcal{P}_{\mathcal{B}}(s)$, $\mathcal{Q}_{\mathcal{A}}(s)$, $\mathcal{Q}_{\mathcal{B}}(s)$ and $\mathcal{R}_{\mathcal{A}}(s) \geq \mathcal{R}_{\mathcal{B}}(s)$;

(ii) $\overline{\mathcal{A}} = \{\langle \mathcal{R}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s), \mathcal{P}_{\mathcal{A}}(s) : s \in \mathcal{S}\rangle\}$;

(iii) $\mathcal{A} \cup \mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s) \vee \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{A}}(s) \wedge \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{A}}(s) \wedge \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S}\rangle\}$;

(iv) $\mathcal{A} \cap \mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s) \wedge \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{A}}(s) \vee \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{A}}(s) \vee \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S}\rangle\}$;

Definition 2.3. [12]. A neutrosophic topology in a nonempty set $\mathcal{X}$ is a family $\mathfrak{J}$ of neutrosophic sets in $\mathcal{X}$ satisfying the following axioms:

(i) $0_N, 1_N \in \mathfrak{J}$;

(ii) $\mathcal{A} \cap \mathcal{B}_i$ for any $\mathcal{A}, \mathcal{B} \in \mathfrak{J}$

(iii) $\mathcal{A} \cap (\mathcal{B})_i$ for any arbitrary family $(\mathcal{A})_i : i \in j \in \mathfrak{J}$.

Definition 2.4. [15]. A subset $\mathcal{A}$ of $(\mathcal{X}, \tau_{\mathcal{N}})$ is called a $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set if $\mathcal{N}acl(\mathcal{A}) \subseteq \mathcal{H}$ whenever $\mathcal{A} \subset \mathcal{H}$ and $\mathcal{H}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- open in $(\mathcal{X}, \tau_{\mathcal{N}})$.

Definition 2.5 [15]. Let U be a neutrosophic set in neutrosophic topological space X. Then $\mathcal{N}_{\alpha g^{\#}\psi}$-$i^*(\mathcal{A}) = \cup\{Q : Q$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$- open set in X and $\mathcal{Q} \subseteq \mathcal{A}\}$ is called aneutrosophic interior of $\mathcal{A}$.

$\mathcal{N}_{\alpha g^{\#}\psi}$-$i^*(\mathcal{A}) = \cup\{P : P$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set in X and $P \supseteq \mathcal{A}\}$ is called a neutrosophic closure of $\mathcal{A}$.

Definition 2.6. [14]. A function $f : (\mathcal{S}, \mathcal{J}) \to (\mathcal{T}, \xi)$ is called

(i) a $\mathcal{N}_{\alpha g^{\#}\psi}$- continuous if $f^{-1}(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- closed in $(\mathcal{S}, \mathfrak{J})$ for every neutrosophic closed set $\mathcal{A}$ of $(\mathcal{S}, \mathfrak{J})$,

(ii) a $\mathcal{N}_{\alpha g^{\#}\psi}$- irresolute if $f^{-1}(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- closed in $(\mathcal{S}, \mathfrak{J})$ for every $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.

Throughout this paper, for a subset $\mathcal{A}$ of a space $(\mathcal{S}, \mathfrak{J})$, $\mathcal{N}_{\alpha g^{\#}\psi}$- open, $\mathcal{N}_{\alpha g^{\#}\psi}$- closed, $\mathcal{N} - i^*(A)$, $\mathcal{N} - c^*(A)$, $\mathcal{N}_{\alpha g^{\#}\psi}$-$i^*(\mathcal{A})$ and $\mathcal{N}_{\alpha g^{\#}\psi}$-c$^*(\mathcal{A})$ denotes a neutrosophic $\alpha g^{\#}\psi$- open, neutrosophic $\alpha g^{\#}\psi$- closed, neutrosophic interior, neutrosophic closure, $\mathcal{N}_{\alpha g^{\#}\psi}$- interior and $\mathcal{N}_{\alpha g^{\#}\psi}$- closure respectively.

3. Quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - Open Functions

Definition 3.1. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is called a quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open if the image of every $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set in $(\mathcal{S}, \mathfrak{J})$ is neutrosophic open in $(\mathcal{T}, \xi)$.

Theorem 3.2. *Every quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open function is neutrosophic open function.*

Proof. Let $\mathcal{A}$ be aneutrosophic open set in $(\mathcal{S}, \mathfrak{J})$. Then $\mathcal{A}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set in $(\mathcal{S}, \mathfrak{J})$. We know that every neutrosophic open set is $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set. Since d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open function, $d(\mathcal{A})$ is neutrosophic open in $(\mathcal{T}, \xi)$. Hence d is neutrosophic open function.

The opposite over the above theorem is not true as shown in the following example.

Example 3.3. Let $\mathcal{S} = \{p, q\}$, $\mathfrak{J} = \{0_N, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \mathfrak{J})$,

$\mathcal{D}_1 = \langle s, (0.3, 0.2), (0.3, 0.2), (0.4, 0.6)\rangle$

$\mathcal{D}_2 = \langle s, (0.2, 0.3), (0.5, 0.4), (0.4, 0.4)\rangle$

$\mathcal{D}_3 = \langle s, (0.3, 0.3), (0.3, 0.2), (0.4, 0.4)\rangle$

$\mathcal{D}_4 = \langle s, (0.2, 0.2), (0.5, 0.4), (0.4, 0.6)\rangle$ and let

$\mathcal{T} = \{p, q\}$, $\xi = \{0_N, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{T}, \xi)$,

$\mathcal{F}_1 = \langle t, (0.4, 0.4), (0.3, 0.2), (0.3, 0.3)\rangle$

$\mathcal{F}_2 = \langle t, (0.3, 0.3), (0.2, 0.2), (0.4, 0.4)\rangle$

$\mathcal{F}_3 = \langle t, (0.4, 0.4), (0.2, 0.2), (0.3, 0.2)\rangle$

$\mathcal{F}_4 = \langle t, (0.3, 0.3), (0.3, 0.2), (0.4, 0.4)\rangle$.

Define $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ by $d(p) = q$, $d(q) = p$.

$\mathcal{N}_{\alpha g^{\#}\psi}$- open sets of $(\mathcal{S}, \mathfrak{J}) = \mathcal{A} = \langle s, (0.4, 0.4), (0.2, 0.2), (0.3, 0.2)\rangle$.

Here $d(\mathcal{A})$ is not neutrosophic open in $(\mathcal{T}, \xi)$.

Therefore d is not quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open function. However d is neutrosophic open function.

Theorem 3.4. *A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open if and only if for every neutrosophic set $\mathcal{A}$ of $(\mathcal{S}, \mathfrak{J})$, $d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A})) \subseteq \mathcal{N} - i^*(d(\mathcal{A}))$.*

Proof. Let d be a quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open function. Now, we have $\mathcal{N} - i^*(\mathcal{A}) \subseteq \mathcal{A}$ and $\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A})$ is an $\mathcal{N}_{\alpha g^{\#}\psi}$- open set. Hence, we obtain $d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A})) \subseteq d(\mathcal{A})$. As $d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A}))$ is neutrosophic open, $d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A})) \subseteq \mathcal{N} - i^*(d(\mathcal{A}))$.

Conversely, assume that $\mathcal{A}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- open set in $(\mathcal{S}, \mathfrak{J})$. Then $d(\mathcal{A}) = d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(\mathcal{A})) \subseteq \mathcal{N} - i^*(d(\mathcal{A}))$ but $\mathcal{N} - i^*(d(\mathcal{A})) \subseteq d(\mathcal{A})$. Consequently, $d(\mathcal{A}) = N - i^*(d(\mathcal{A}))$. Therefore d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open.

Lemma 3.5. *If a function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open then $\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(d^{-1}(\mathcal{A})) \subseteq d^{-1}(\mathcal{N} - i^*(\mathcal{A}))$ for every neutrosophic set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.*

Proof. Let $\mathcal{A}$ be a neutrosophic set of $(\mathcal{T}, \xi)$. Then $\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(d^{-1}(\mathcal{A}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$- open set in $(\mathcal{S}, \mathfrak{J})$ and d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open, then $d(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N} - i^*(d(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N} - i^*(\mathcal{A})$. Thus $\mathcal{N}_{\alpha g^{\#}\psi}\text{-}i^*(d^{-1}(\mathcal{A})) \subseteq d^{-1}(\mathcal{N} - i^*(\mathcal{A}))$.

Theorem 3.6. *A mapping $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- open if and only if for each neutrosophic set $\mathcal{C}$ of $(\mathcal{T}, \xi)$ and for each neutrosophic*

$\mathcal{N}_{\alpha g^{\#}\psi}$-*closed set* $\mathcal{A}$ *of* $(\mathcal{S}, \mathfrak{J})$ *containing* $d^{-1}(\mathcal{C})$ *there is a neutrosophic closed* $\mathcal{B}$ *of* $(\mathcal{T}, \xi)$ *such that* $\mathcal{C} \subseteq \mathcal{B}$ *and* $d^{-1}(\mathcal{B}) \subseteq \mathcal{A}$.

Proof. Assume d is a quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. Let $\mathcal{C}$ be the neutrosophic set of $(\mathcal{T}, \xi)$ and $\mathcal{A}$ is a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{S}, \mathfrak{J})$ such that $d^{-1}(\mathcal{C}) \subseteq \mathcal{A}$. Then $\mathcal{B} = (d^{-1}(\mathcal{A}^c))^c$ is neutrosophic closed set of $(\mathcal{T}, \xi)$ such that $d^{-1}(\mathcal{B}) \subseteq \mathcal{A}$.

Assume is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{S}, \mathfrak{J})$. Then $d^{-1}((d(\mathcal{F}))^c \subseteq \mathcal{F}^c$ and $\mathcal{F}^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$. By hypothesis there is a neutrosophic closed set $\mathcal{B}$ of $(\mathcal{T}, \xi)$ such that $(d(\mathcal{F}))^c \subseteq \mathcal{B}$ and $d^{-1}(\mathcal{B}) \subseteq \mathcal{F}^c$. Therefore $\mathcal{F} \subseteq (d(\mathcal{B}))^c$. Hence $\mathcal{B}^c \subseteq d(\mathcal{F}) \subseteq d((d^{-1}(\mathcal{B}))^c) \subseteq \mathcal{B}^c$ which implies $d(\mathcal{F}) = \mathcal{B}^c$. Since $\mathcal{B}^c$ is neutrosophic open set of $(\mathcal{T}, \xi)$. Hence $d(\mathcal{F})$ is neutrosophic open in $(\mathcal{T}, \xi)$ and thus d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.7. *A function* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$-*open if and only if* $d^{-1}(\mathcal{N} - c^*(\mathcal{A})) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - c^*(d^{-1}(\mathcal{A}))$ *for every neutrosophic set* $\mathcal{A}$ *of* $(\mathcal{T}, \xi)$.

Proof. Suppose that d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open. For any neutrosophic set $\mathcal{A}$ of $(\mathcal{T}, \xi)$, $d^{-1}(\mathcal{A}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - c^*(d^{-1}(\mathcal{A}))$. Therefore by Theorem 3.6, there exists a neutrosophic closed set $\mathcal{B}$ in $(\mathcal{T}, \xi)$ such that $\mathcal{A} \subseteq \mathcal{B}$ and $d^{-1}(\mathcal{B}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - c^*(d^{-1}(\mathcal{A}))$. Therefore, we obtain $d^{-1}(\mathcal{N} - c^*(\mathcal{A})) \subseteq d^{-1}(\mathcal{B}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - c^*(d^{-1}(\mathcal{A}))$.

Conversely, let $\mathcal{A} \subseteq \mathcal{T}$ and $\mathcal{B}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{S}, \mathfrak{J})$ containing $d^{-1}(\mathcal{A})$. Put $W = \mathcal{N} - c^*(\mathcal{A})$, then we have $\mathcal{A} \subseteq W$ and W is neutrosophic closed and $d^{-1}(W) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - c^{*}(d^{-1}(\mathcal{A})) \subseteq \mathcal{B}$. Then by Theorem 3.6, d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open.

Lemma 3.8. *A function* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *and* $e : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *be two functions and* $e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open. If e is continuous, then d is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open.*

Proof. Let $\mathcal{A}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set in $(\mathcal{S}, \mathfrak{J})$, then $(e \circ d)(\mathcal{A})$ is neutrosophic open in $(\mathcal{V}, \omega)$, sincee∘dis quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open. Since e is a one to one continuous function, $d(\mathcal{A}) = e^{-1}(e \circ d(\mathcal{A}))$ is neutrosophic open in $(\mathcal{T}, \xi)$. Hence d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open.

Definition 3.9. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is called a strongly $\mathcal{N}_{\alpha g^{\#}\psi}$ - open if the image of every $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set in $(\mathcal{S}, \mathfrak{J})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - open in $(\mathcal{T}, \xi)$.

Theorem 3.10. *Let* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *and* $e : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *be any two functions. Then*

(i) $e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *is neutrosophic open function if e is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function and d is* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function.*

(ii) $e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *is strongly* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function if e is* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function and d is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function.*

(iii) $e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ *is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function if e is quasi* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function and d is strongly* $\mathcal{N}_{\alpha g^{\#}\psi}$ *- open function.*

Proof.

(i) Let $\mathcal{A}$ be a neutrosophic open set of $(\mathcal{S}, \mathfrak{J})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$ - open function, $d(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set in $(\mathcal{T}, \xi)$. Since e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - open function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is neutrosophic open in (V, ω). Therefore $e \circ d$ is neutrosophic open function.

(ii) Let $\mathcal{A}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{S}, \mathfrak{J})$. Since d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open function, $d(\mathcal{A})$ is neutrosophic open set in $(\mathcal{T}, \xi)$. Since e is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open in (V, ω). Therefore $e \circ d$ is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$-open function.

(iii) Let $\mathcal{A}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{S}, \mathfrak{J})$. Since d is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$-open function, $d(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Since e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is neutrosophic open in (V, ω). Therefore $e \circ d$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-open function.

4. Quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-Closed Functions

Definition 4.1. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is called quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed if the image of every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$ is neutrosophic closed in $(\mathcal{T}, \xi)$.

Theorem 4.2. *Every quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function is neutrosophic closed function.*

Proof. Let $\mathcal{A}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{J})$. Then $\mathcal{A}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$, since every neutrosophic closed set is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set. Since d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function, $d(\mathcal{A})$ is neutrosophic closed in $(\mathcal{T}, \xi)$. Hence d is neutrosophic closed function.

The opposite over the above theorem is not true as shown in the following example.

Example 4.3. Let $\mathcal{S} = \{p, q\}$, $\mathfrak{J} = \{0_N, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \mathfrak{J})$, $\mathcal{D}_1 = \langle s, (0.3, 0.2), (0.3, 0.2), (0.4, 0.6)\rangle$

$\mathcal{D}_2 = \langle s, (0.2, 0.3), (0.5, 0.4), (0.4, 0.4)\rangle$

$\mathcal{D}_3 = \langle s, (0.3, 0.3), (0.3, 0.2), (0.4, 0.4)\rangle$

$\mathcal{D}_4 = \langle s, (0.2, 0.2), (0.5, 0.4), (0.4, 0.6)\rangle$ and let

$\mathcal{T} = \{p, q\}$, $\xi = \{0_N, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{T}, \xi)$,

$\mathcal{F}_1 = \langle t, (0.4, 0.4), (0.3, 0.2), (0.3, 0.3)\rangle$

$\mathcal{F}_2 = \langle t, (0.3, 0.3), (0.2, 0.2), (0.4, 0.4)\rangle$

$\mathcal{F}_3 = \langle t, (0.4, 0.4), (0.2, 0.2), (0.3, 0.2)\rangle$

$\mathcal{F}_4 = \langle t, (0.3, 0.3), (0.3, 0.2), (0.4, 0.4)\rangle$.

Define $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ by $d(p) = q$, $d(q) = p$.

$\mathcal{N}_{\alpha g^{\#}\psi}$- closed sets of $(\mathcal{S}, \mathfrak{J}) = \mathcal{B} = \langle s, (0.3, 0.2), (0.2, 0.2), (0.4, 0.4)\rangle$.

Here $d(\mathcal{B})$ is not neutrosophic closed set in $(\mathcal{T}, \xi)$.

Therefore d is not quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- closed function. However d is neutrosophic closed function.

Theorem 4.4. *A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- closed if and only if for every neutrosophic set $\mathcal{A}$ of $(\mathcal{S}, \mathfrak{J})$, $\mathcal{N} - c^*(d(\mathcal{A})) \subseteq d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A}))$.*

Proof. Assume that d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$- closed function and $\mathcal{A} \subseteq (\mathcal{S}, \mathfrak{J})$. Then $\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set in $(\mathcal{S}, \mathfrak{J})$. Therefore $d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A}))$ is neutrosophic closed in $(\mathcal{T}, \xi)$. Since $d(\mathcal{A}) \subseteq d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A}))$ implies $\mathcal{N} - c^*(d(\mathcal{A})) \subseteq \mathcal{N} - c^*(d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A})))$ $= d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A}))$. This implies, $\mathcal{N} - c^*(d(\mathcal{A})) \subseteq (d(\mathcal{N}_{\alpha g^{\#}\psi} - c^*(\mathcal{A}))$.

Conversely, $\mathcal{A}$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$. Then $\mathcal{A} = \mathcal{N}_{\alpha g^{\#}\psi} - c^{*}(\mathcal{A})$. Therefore, $d(\mathcal{A}) = d(\mathcal{N}_{\alpha g^{\#}\psi} - c^{*}(\mathcal{A}))$. By hypothesis, $\mathcal{N} - c^{*}(d(\mathcal{A})) \subseteq d(\mathcal{N}_{\alpha g^{\#}\psi} - c^{*}(\mathcal{A})) = d(\mathcal{A})$. Hence $\mathcal{N} - c^{*}(d(\mathcal{A})) \subseteq d(\mathcal{A})$. But $d(\mathcal{A}) \subseteq \mathcal{N} - c^{*}(d(\mathcal{A}))$. This implies $d(\mathcal{A})$ is neutrosophic closed set in $(\mathcal{T}, \xi)$. Therefore, d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed.

Lemma 4.5. *A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed then for every neutrosophic set $\mathcal{A}$ of $(\mathcal{T}, \xi)$, $d^{-1}(\mathcal{N} - i^{*}(\mathcal{A})) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - i^{*}(d^{-1}(\mathcal{A}))$.*

Proof. Let $\mathcal{A}$ be a arbitrary neutrosophic set of $(\mathcal{T}, \xi)$. Then $\mathcal{N}_{\alpha g^{\#}\psi} - i^{*}(d^{-1}(\mathcal{A}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$ and d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed. Hence $d(\mathcal{N}_{\alpha g^{\#}\psi} - i^{*}(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N} - i^{*}d((d^{-1}(\mathcal{A}))) \subseteq \mathcal{N} - i^{*}(\mathcal{A}))$. Therefore $d(\mathcal{N}_{\alpha g^{\#}\psi} - i^{*}(d^{-1}(\mathcal{A}))) \subseteq d^{-1}(\mathcal{N} - i^{*}(\mathcal{A}))$.

Definition 4.6. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is said to be strongly $\mathcal{N}_{\alpha g^{\#}\psi}$-closed if the image of every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{T}, \xi)$.

Theorem 4.7. *Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ and $e : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ be any two functions. Then*

(i) *$e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ is neutrosophic closed function if e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function and d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function.*

(ii) *$e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function if e is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function and d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function.*

(iii) *$e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function if e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function and d is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$-closed function.*

Proof.

(i) Let $\mathcal{A}$ be a neutrosophic closed of $(\mathcal{S}, \mathfrak{J})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $d(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed in $(\mathcal{T}, \xi)$. Since e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is closed in $(\mathcal{V}, \omega)$. Therefore $e \circ d$ is closed function.

(ii) Let $\mathcal{A}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed of $(\mathcal{S}, \mathfrak{J})$. Since d is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $d(\mathcal{A})$ is neutrosophic closed in $(\mathcal{T}, \xi)$. Since e is a $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed in $(\mathcal{V}, \omega)$. Therefore $e \circ d$ is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function.

(iii) Let $\mathcal{A}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed of $(\mathcal{S}, \mathfrak{J})$. Since d is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $d(\mathcal{A})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed in $(\mathcal{T}, \xi)$. Since e is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function, $(e \circ d)(\mathcal{A}) = e(d(\mathcal{A}))$ is closed in $(\mathcal{V}, \omega)$. Therefore $e \circ d$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed function.

Theorem 4.8. *If a function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed if and only if for any set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ and for any $\mathcal{N}_{\alpha g^{\#}\psi}$ - open set $\mathcal{B}$ of $(\mathcal{S}, \mathfrak{J})$ containing $d^{-1}(\mathcal{A})$ there exists a neutrosophic open set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ containing $\mathcal{A}$ such that $d^{-1}(\mathcal{A}) \subseteq \mathcal{B}$.*

Proof. Proof is similar to that of Theorem 3.6.

Theorem 4.9. *Let a function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ and $e : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ be any two functions such that $e \circ d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{V}, \omega)$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed.*

(i) *If d is $\mathcal{N}_{\alpha g^{\#}\psi}$ - irresolute onto, then e is neutrosophic closed.*

(ii) *If d is $\mathcal{N}_{\alpha g^{\#}\psi}$ - continuous one to one then d is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed.*

Proof.

(i) Assume that be an arbitrary neutrosophic closed in $(\mathcal{T}, \xi)$. As d is

$\mathcal{N}_{\alpha g^{\#}\psi}$ - irresolute, $d^{-1}(\mathcal{B})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed. Since $e \circ d$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed and d is onto, $(e \circ d)(d^{-1}(\mathcal{B})) = e(\mathcal{B})$, which is neutrosophic closed in $(\mathcal{V}, \omega)$. This implies that e is neutrosophic closed.

(ii) Assume that $\mathcal{B}$ is any $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed in $(\mathcal{S}, \mathfrak{J})$. Since $e \circ d$ is quasi $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed, $(e \circ d)(\mathcal{B})$ is neutrosophic closed in $(\mathcal{V}, \omega)$. Again e is $\mathcal{N}_{\alpha g^{\#}\psi}$ - continuous one to one, $e^{-1}((e \circ d)(\mathcal{B})) = e(\mathcal{B})$, which is $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed in $(\mathcal{T}, \xi)$. This implies that d is strongly $\mathcal{N}_{\alpha g^{\#}\psi}$ - closed.

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